Analysis of heat exchangers

Heat exchangers usually operate for long periods of time with no change in their operating conditions. Therefore, they can be modeled as ***steady-flow*** devices with the following ***assumptions***:

1. The ***mass flow rate*** of each fluid remains constant.
2. The ***fluid properties*** such as ***temperature*** and ***velocity*** at any inlet or outlet remain the same.
3. The ***kinetic*** and ***potential energy changes*** are negligible.
4. ***Axial heat conduction*** along the tube is usually insignificant and can be considered ***negligible.***
5. The ***specific heat*** of a fluid, in general, changes with temperature. But, in a specified temperature range, it can be treated as a constant, at some average value.
6. Finally, the outer surface of the heat exchanger is assumed to be ***perfectly insulated***, so that there is no heat loss to the surrounding medium, and any heat transfer occurs between the two fluids only.

Under these assumptions, and others, the ***first law of thermodynamics*** requires that the ***rate of heat transfer from the hot fluid be equal to the rate of heat transfer to the cold one***. That is,

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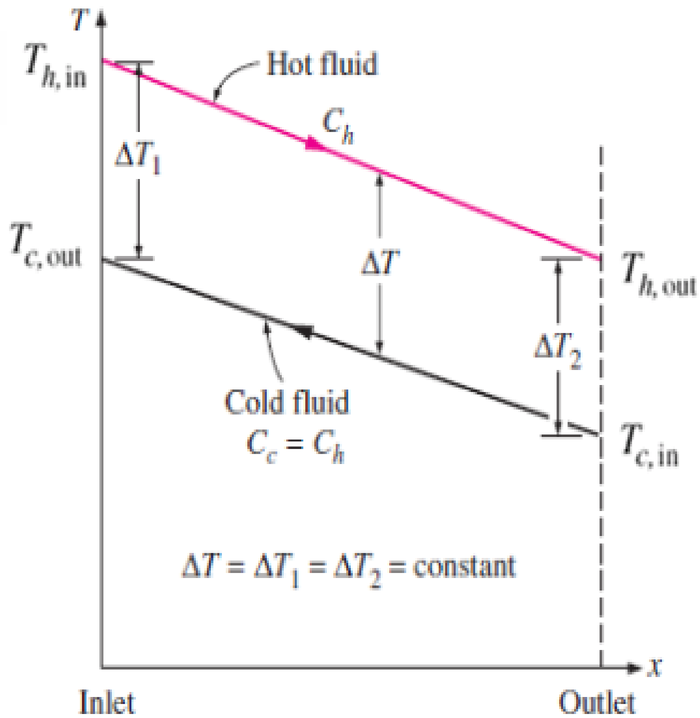

$$\dot{Q} = \dot{m}_c C_{pc}\left(T_{c,out} - T_{c,in}\right) \quad and \quad \dot{Q} = \dot{m}_h C_{ph}\left(T_{h,out} - T_{h,in}\right)$$

Where the subscripts ***c*** and ***h*** stand for ***cold*** and ***hot*** fluids, respectively, and

$\dot{m}_c, \dot{m}_h$ = mass flow rates

C_{pc}, C_{ph} = specific heats

$T_{c,\text{out}}, T_{h,\text{out}}$ = outlet temperatures

$T_{c,\text{in}}, T_{h,\text{in}}$ = inlet temperatures

The ***heat capacity rate*** is defined for the hot and cold fluid streams as,

$$C_h = \dot{m}_h C_{ph} \quad and \quad C_c = \dot{m}_c C_{pc} \qquad \textbf{(W/°C, or kW/°C)}$$

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The heat capacity rate of a fluid stream represents the rate of heat transfer needed to change the temperature of the fluid stream by 1°C as it flows through a heat exchanger.

With the definition of the heat capacity rate above:

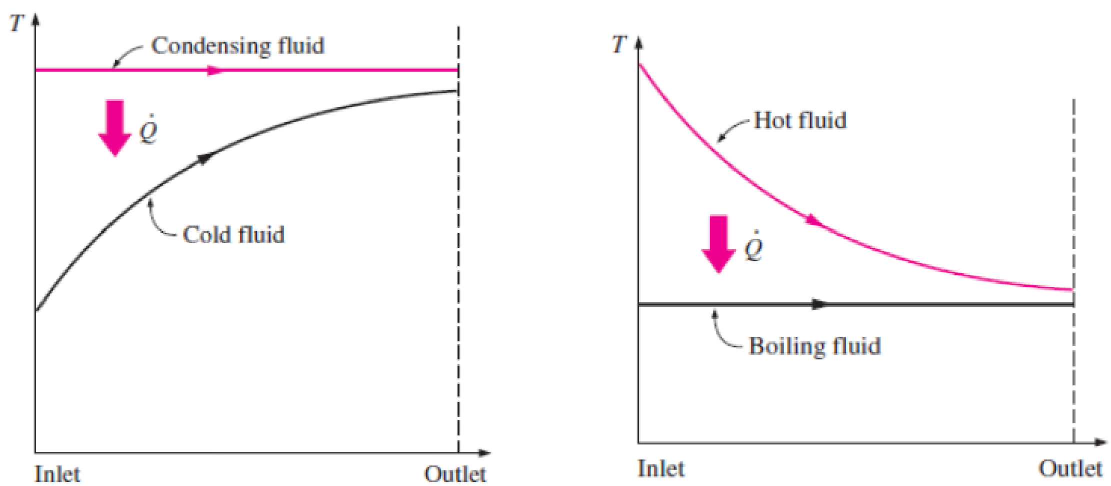
$$\dot{Q} = C_c\left(T_{c,out} - T_{c,in}\right) \qquad and \qquad \dot{Q} = C_h\left(T_{h,in} - T_{h,out}\right)$$

That is, the heat transfer rate in a heat exchanger is equal to the heat capacity rate of either fluid multiplied by the temperature change of that fluid.

Two ***special types*** of heat exchangers commonly used in practice are ***condensers*** and ***boilers*** *(or* ***evaporator****)*. One of the fluids in a condenser or a boiler undergoes a ***phasechange process***, and the rate of heat transfer is expressed as:

$$\dot{Q} = \dot{m} \cdot h_{fg}$$

Where $\dot{m}$ is the rate of evaporation or condensation of the fluid and h_{fg} is the enthalpy of vaporization of the fluid at the specified temperature or pressure.



(a) Condenser ($C_h \to \infty$) (b) Boiler ($C_c \to \infty$)

Variation of fluid temperatures in a heat exchanger

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The rate of heat transfer in a heat exchanger can also be expressed in an analogous manner to ***Newton's law of cooling*** as:

$$\dot{Q} = UA_s \, \Delta T_m$$

Where ***U*** is the overall heat transfer coefficient, *As* is the heat transfer area, and □ *Tm* is an appropriate ***average temperature difference between the two fluids***.

Example-1:

A double-pipe (shell and tube) heat exchanger is constructed of a stainless steel (***k***=15.1W/m·°C) inner tube of inner diameter D_i =1.5cm and outer diameter D_o=1.9cm and an outer shell of inner diameter 3.2cm. The convection heat transfer coefficient is given to be h_i=800W/m^2.°C on the inner surface of the tube and h_o=1200 W/m^2·°C on the outer surface. For a fouling factor of $R_{f,i}$=0.0004 m^2·°C/W on the tube side and $R_{f,o}$=0.0001m^2·°C/W on the shell side, ***determine* (a)** the ***thermal resistance*** of the heat exchanger ***per unit length*** and ***(*b*) the overall heat transfer coefficients,* Ui** *and* **Uo** based on the inner and outer surface areas of the tube, respectively.

Solution:

The thermal resistance for an unfinned shell-and-tube heat exchanger with fouling on both heat transfer surfaces is given by:

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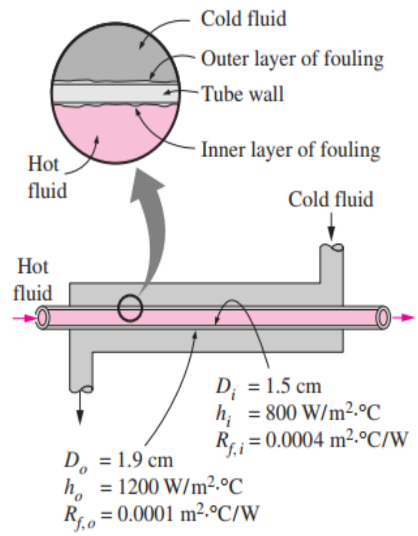
$$R = \frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = \frac{1}{h_iA_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{f,o}}{A_o} + \frac{1}{h_oA_o}$$

where

$$A_i = \pi D_i L = \pi(0.015 \text{ m})(1 \text{ m}) = 0.0471 \text{ m}^2$$
$$A_o = \pi D_o L = \pi(0.019 \text{ m})(1 \text{ m}) = 0.0597 \text{ m}^2$$

Substituting, the total thermal resistance is determined to be

$$R = \frac{1}{(800 \text{ W/m}^2\cdot{}^\circ\text{C})(0.0471 \text{ m}^2)} + \frac{0.0004 \text{ m}^2\cdot{}^\circ\text{C/W}}{0.0471 \text{ m}^2} + \frac{\ln(0.019/0.015)}{2\pi(15.1 \text{ W/m}\cdot{}^\circ\text{C})(1 \text{ m})} + \frac{0.0001 \text{ m}^2\cdot{}^\circ\text{C/W}}{0.0597 \text{ m}^2} + \frac{1}{(1200 \text{ W/m}^2\cdot{}^\circ\text{C})(0.0597 \text{ m}^2)}$$
$$= (0.02654 + 0.00849 + 0.0025 + 0.00168 + 0.01396)^\circ\text{C/W}$$
$$= \mathbf{0.0532^\circ C/W}$$



(*b*) Knowing the total thermal resistance and the heat transfer surface areas, the overall heat transfer coefficient based on the inner and outer surfaces of the tube are determined again from Eq. above:

$$U_i = \frac{1}{RA_i} = \frac{1}{(0.0532\ ^\circ\text{C/W})(0.0471 \text{ m}^2)} = \mathbf{399\ W/m^2\cdot{}^\circ C}$$

and

$$U_o = \frac{1}{RA_o} = \frac{1}{(0.0532\ ^\circ\text{C/W})(0.0597 \text{ m}^2)} = \mathbf{315\ W/m^2\cdot{}^\circ C}$$

H.W.

Water at an average temperature of 107°C and an average velocity of 3.5 m/s flows through a 5m long stainless steel tube (***k*** = 14.2 W/m·°C) in a boiler. The inner and outer diameters of the tube are D_i =1.0 cm and D_o=1.4 cm, respectively, and the fouling factor $R_{f,i}$ = 0.0005 m^2·°C/W on the inner surface of the tube. If the convection heat transfer coefficient at the outer surface of the tube where boiling is taking place is h_o=8400 W/m^2·°C. ***Determine the overall heat transfer coefficient U_i*** of this boiler based on the inner surface area of the tube.

The properties of water at 107· C · 110· C from the tables:

$$\upsilon = \mu / \rho = 0.268 \times 10^{-6}\ \text{m}^2/\text{s}$$

$$k = 0.682\ \text{W/m}^2.\text{K}$$

$$\text{Pr} = 1.58$$

(Answer: U_i = 1337 W/m^2·°C)

Hint: use the correlations of forced convection to find (h_o)

Primary References

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Additional Reference

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