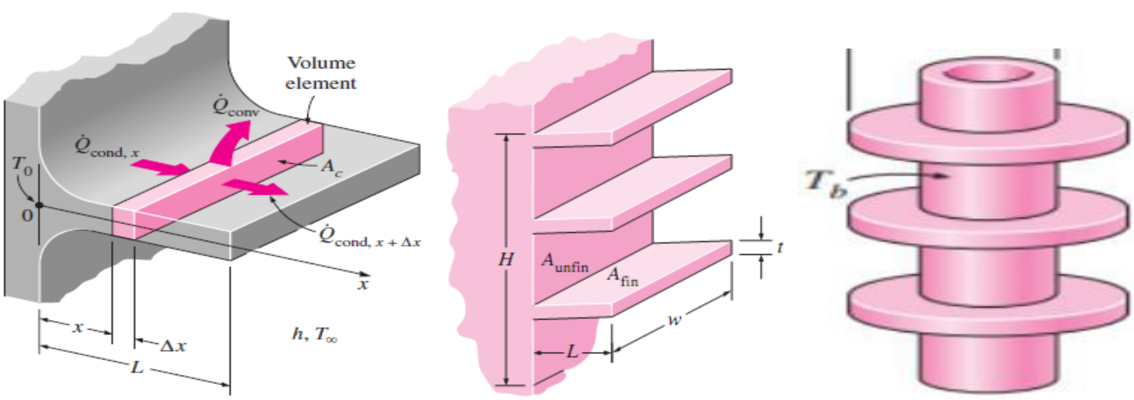
Fin Equation:

Consider a volume element of a fin at location ***x*** having a length of *x*, cross-sectional area of A_c, and a perimeter of ***p***. Under steady conditions, the energy balance on this volume element can be

$$\begin{pmatrix}\text{Rate of } \textit{heat} \\ \textit{conduction} \text{ into} \\ \text{the element at } x\end{pmatrix} = \begin{pmatrix}\text{Rate of } \textit{heat} \\ \textit{conduction} \text{ from the} \\ \text{element at } x + \Delta x\end{pmatrix} + \begin{pmatrix}\text{Rate of } \textit{heat} \\ \textit{convection} \text{ from} \\ \text{the element}\end{pmatrix}$$



Or

$$\dot{Q}_{cond,\,x} = \dot{Q}_{cond,\,x+\Delta x} + \dot{Q}_{conv}$$

Where

$$\dot{Q}_{conv} = h(p\,\Delta x)(T - T_\infty)$$

Substituting and dividing by Δx , we obtain: [**perimeter: *p* = 2 (*w*+*t*)**]

$$\frac{\dot{Q}_{cond,\,x+\Delta x} - \dot{Q}_{cond,\,x}}{\Delta x} + hp(T - T_\infty) = 0$$

Sameer.Saad.Raheem@uomus.edu.iq

Taking the limit as $\Delta x \rightarrow 0$ gives

$$\frac{d\dot{Q}_{\text{cond}}}{dx} + hp(T - T_\infty) = 0$$

In general, the cross-sectional area ***Ac*** and the perimeter ***p*** of a fin vary with ***x***, which makes this differential equation difficult to solve. In the special case of *constant cross section* and *constant thermal conductivity,* the differential equation reduces to,

$$\frac{d^2\theta}{dx^2} - a^2\theta = 0$$

Where,

$$a^2 = \frac{hp}{kA_s}$$

$$a = \sqrt{\frac{hp}{kA_s}}$$

and ($\theta = T - T_\infty$) is the *temperature excess.* At the fin base ($\theta_b = T_b - T_\infty$)

$$M = \sqrt{hpkA_c}\theta_b$$

At the fin tip we have several possibilities,

1- **Infinitely Long Fin : ($T_{fin\ tip} = T_\infty$),**

at the fin base where x = 0 the value of θ will be θ_b. Let, p is the perimeter, and A_c is the cross sectional area. The rates of heat transfer for both cases are given to be

The temperature distribution along the fin **for very long fins is**,

$$\frac{T_x - T_\infty}{T_b - T_\infty} = e^{-ax} = e^{-x\sqrt{hp/kA_c}}$$

The rates of heat transfer:

$$Q_{long\ fin} = -kA_c \frac{dT}{dx}|_{x=0} = \sqrt{hPkA_c}(T_b - T_\infty)$$

Sameer.Saad.Raheem@uomus.edu.iq

Alternatively, the rate of heat transfer from the fin could also be determined by considering heat transfer from a differential volume element of the fin and integrating it over the entire surface of the fin. That is,

$$\dot{Q}_{\text{fin}} = \int_{A_{\text{fin}}} h[T(x) - T_\infty]\, dA_{\text{fin}} = \int_{A_{\text{fin}}} h\theta(x)\, dA_{\text{fin}}$$

2- Insulated Fin Tip: ($Q_{\text{fin tip}} = 0$)

for fins with negligible heat transfer at the fin are given by,

- The condition at the fin base remains the same, the relation for **the temperature distribution is,**

$$\frac{T_x - T_\infty}{T_b - T_\infty} = \frac{\cosh a\,(L - x)}{\cosh aL}$$

- The rate of heat transfer from the fin can be determined again **from Fourier's law of heat conduction,**

$$Q_{insulated\ tip} = -kA_c \frac{dT}{dx}\Big|_{x=0} = \sqrt{hPkA_c}\,(T_b - T_\infty)\tanh aL$$

Sameer.Saad.Raheem@uomus.edu.iq

- Note that the heat transfer relations for the very long fin and the fin with negligible heat loss at the tip differ by the factor **tanh aL**, which approaches **1 as L becomes very large**.

3- Fin subjected to Convection (and Radiation) at Tip:

Fins exposed to convection at their tips **can be treated as fins with insulated tips** by using the **corrected length (L_c)** instead of the actual fin length.

$$L_c = L + \frac{A_c}{p}$$

Primary References

- *Holman, J.P., Heat Transfer, 10th Edition, McGraw-Hill.*
- *Kern, D.Q., Process Heat Transfer, McGraw-Hill.*

Additional Reference

- *Çengel, Y.A. & Ghajar, A.J., Heat and Mass Transfer: Fundamentals and Applications, McGraw-Hill.*