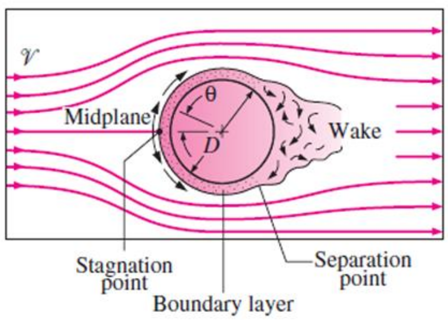
FLOW ACROSS CYLINDERS AND SPHERES

Flow across cylinders and spheres is frequently encountered in practice. For example, the tubes in a shell-and-tube heat exchanger involve both *internal flow* through the tubes and *external flow* over the tubes, and both flows must be considered in the analysis of the heat exchanger. Also, many sports such as soccer, tennis, and golf involve flow over spherical balls.

The characteristic length for a circular cylinder or sphere is taken to be the *external diameter D*. Thus, the Reynolds number is defined as $\mathrm{Re} = \mathcal{V}D/\nu$ where $\mathcal{V}$ is the uniform velocity of the fluid as it approaches the cylinder or sphere. The critical Reynolds number for flow across a circular cylinder or sphere is about $\mathrm{Re}_{cr} \approx 2 \times 10^5$. That is, the boundary layer remains laminar for about $\mathrm{Re} \lesssim 2 \times 10^5$ and becomes turbulent for $\mathrm{Re} \gtrsim 2 \times 10^5$.



The nature of the flow across a cylinder or sphere strongly affects the total drag coefficient C_D. Both the *friction drag* and the *pressure drag* can be significant. The high pressure in the vicinity of the stagnation point and the low pressure on the opposite side in the wake produce a net force on the body in the direction of flow. The drag force is primarily due to friction drag at low Reynolds numbers ($\mathrm{Re} < 10$) and to pressure drag at high Reynolds numbers ($\mathrm{Re} > 5000$). Both effects are significant at intermediate Reynolds numbers.

The average drag coefficients C_D for cross flow over a smooth single circular cylinder and a sphere are given in Figure 7–17. The curves exhibit different behaviors in different ranges of Reynolds numbers:

- For $Re \lesssim 1$, we have creeping flow, and the drag coefficient decreases with increasing Reynolds number. For a sphere, it is $C_D = 24/Re$. There is no flow separation in this regime.
- At about $Re = 10$, separation starts occurring on the rear of the body with vortex shedding starting at about $Re \approx 90$. The region of separation increases with increasing Reynolds number up to about $Re = 10^3$. At this point, the drag is mostly (about 95 percent) due to pressure drag. The drag coefficient continues to decrease with increasing Reynolds number in this range of $10 < Re < 10^3$. (A decrease in the drag coefficient does not necessarily indicate a decrease in drag. The drag force is proportional to the square of the velocity, and the increase in velocity at higher Reynolds numbers usually more than offsets the decrease in the drag coefficient.)

- In the moderate range of $10^3 < Re < 10^5$, the drag coefficient remains relatively constant. This behavior is characteristic of blunt bodies. The flow in the boundary layer is laminar in this range, but the flow in the separated region past the cylinder or sphere is highly turbulent with a wide turbulent wake.
- There is a sudden drop in the drag coefficient somewhere in the range of $10^5 < Re < 10^6$ (usually, at about 2×10^5). This large reduction in C_D is due to the flow in the boundary layer becoming *turbulent,* which moves the separation point further on the rear of the body, reducing the size of the wake and thus the magnitude of the pressure drag. This is in contrast to streamlined bodies, which experience an increase in the drag coefficient (mostly due to friction drag) when the boundary layer becomes turbulent.

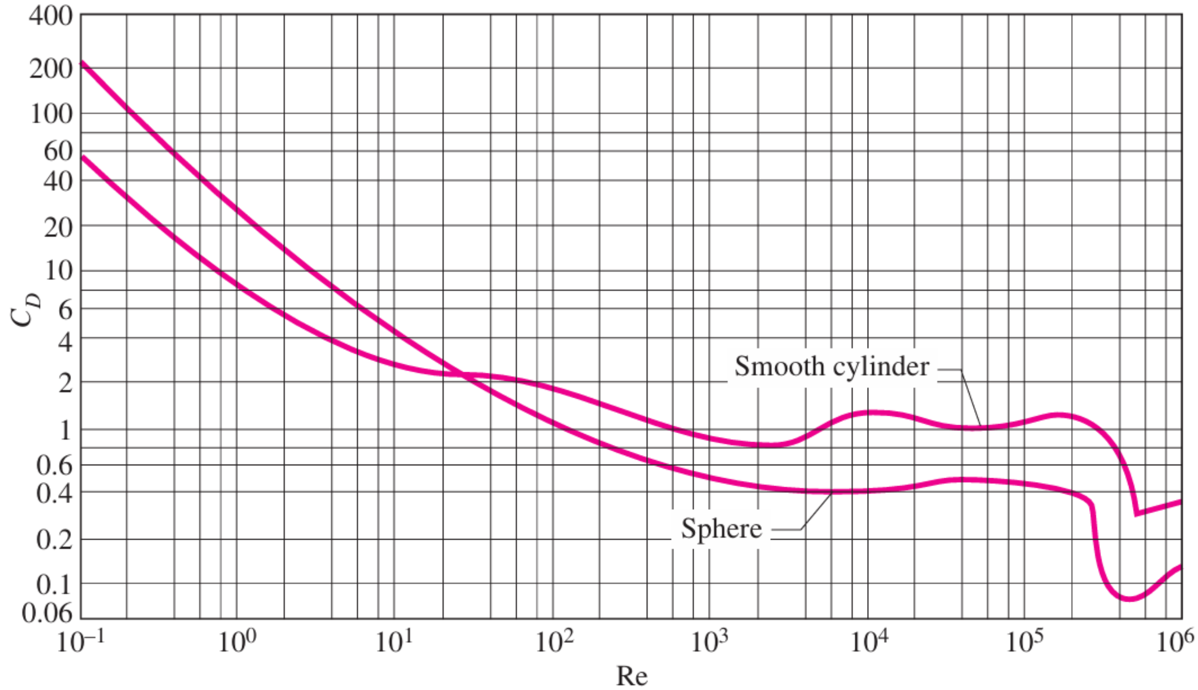
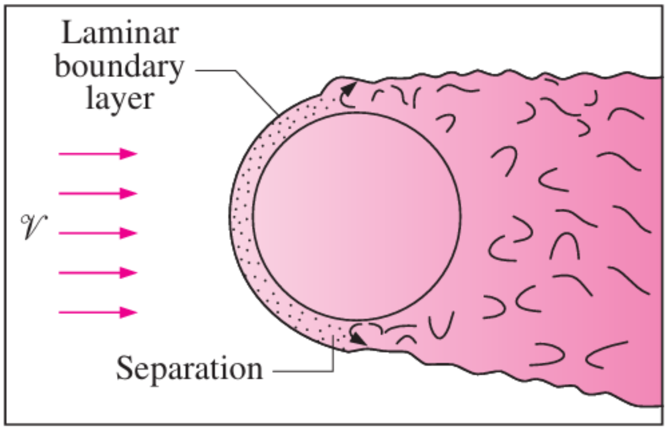


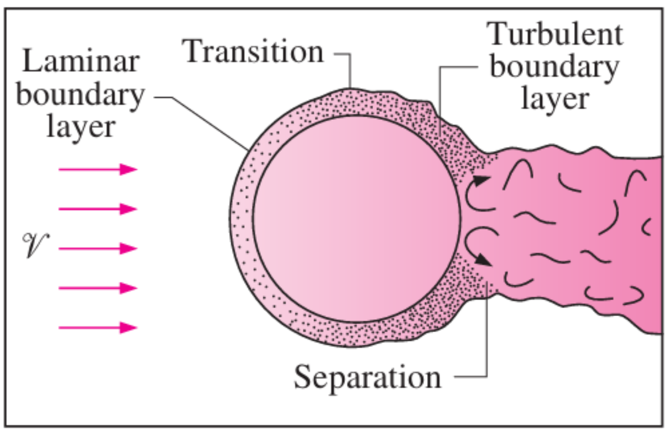
FIGURE 7–17

Average drag coefficient for cross flow over a smooth circular cylinder and a smooth sphere (from Schlichting, Ref. 10).

Flow separation occurs at about $\theta \approx 80°$ (measured from the stagnation point) when the boundary layer is *laminar* and at about $\theta \approx 140°$ when it is *turbulent* (Fig. 7–18). The delay of separation in turbulent flow is caused by the rapid fluctuations of the fluid in the transverse direction, which enables the turbulent boundary layer to travel further along the surface before separation occurs, resulting in a narrower wake and a smaller pressure drag. In the range of Reynolds numbers where the flow changes from laminar to turbulent, even the drag force F_D decreases as the velocity (and thus Reynolds number) increases. This results in a sudden decrease in drag of a flying body and instabilities in flight.



(*a*) Laminar flow ($Re < 2 \times 10^5$)



(*b*) Turbulence occurs ($Re > 2 \times 10^5$)

Heat Transfer Coefficient

Flows across cylinders and spheres, in general, involve *flow separation,* which is difficult to handle analytically. Therefore, such flows must be studied experimentally or numerically. Indeed, flow across cylinders and spheres has been studied experimentally by numerous investigators, and several empirical correlations have been developed for the heat transfer coefficient.

The discussions above on the local heat transfer coefficients are insightful; however, they are of little value in heat transfer calculations sinc calculation of heat transfer requires the *average* heat transfer coefficier the entire surface. Of the several such relations available in the literature for the average Nusselt number for cross flow over a cylinder, we present the one proposed by Churchill and Bernstein:

$$Nu_{cyl} = \frac{hD}{k} = 0.3 + \frac{0.62\,Re^{1/2}\,Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}}\left[1 + \left(\frac{Re}{282{,}000}\right)^{5/8}\right]^{4/5} \qquad (7\text{-}35) \qquad 1$$

This relation is quite comprehensive in that it correlates available data well for Re Pr > 0.2. The fluid properties are evaluated at the *film temperature* $T_f = \frac{1}{2}(T_\infty + T_s)$, which is the average of the free-stream and surface temperatures.

For flow over a *sphere,* Whitaker recommends the following comprehensive correlation:

$$Nu_{sph} = \frac{hD}{k} = 2 + [0.4\,Re^{1/2} + 0.06\,Re^{2/3}]\,Pr^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4} \qquad 2$$

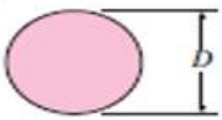
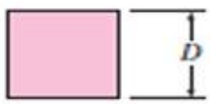
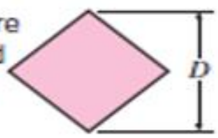
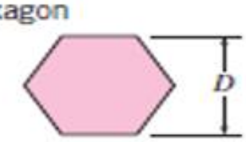
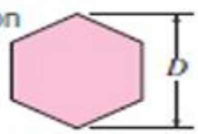
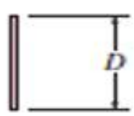
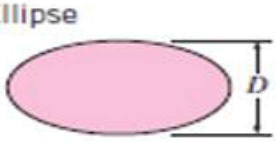
which is valid for $3.5 \le Re \le 80{,}000$ and $0.7 \le Pr \le 380$. The fluid properties in this case are evaluated at the free-stream temperature T_∞, except for μ_s, which is evaluated at the surface temperature T_s. Although the two relations above are considered to be quite accurate, the results obtained from them can be off by as much as 30 percent.

The average Nusselt number for flow across cylinders can be expressed compactly as

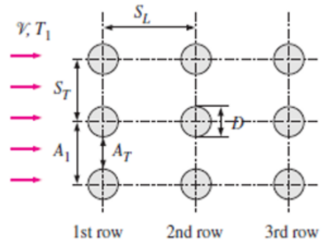
$$Nu_{cyl} = \frac{hD}{k} = C\,Re^m\,Pr^n \qquad 3$$

Table 1

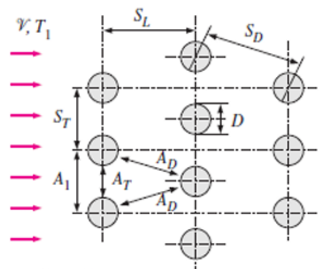
Empirical correlations for the average Nusselt number for forced convection over circular and noncircular cylinders in cross flow (from Zukauskas, and Jakob,

Cross-section of the cylinder	Fluid	Range of Re	Nusselt number
Circle 	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$Nu = 0.989Re^{0.330}\,Pr^{1/3}$ $Nu = 0.911Re^{0.385}\,Pr^{1/3}$ $Nu = 0.683Re^{0.466}\,Pr^{1/3}$ $Nu = 0.193Re^{0.618}\,Pr^{1/3}$ $Nu = 0.027Re^{0.805}\,Pr^{1/3}$
Square 	Gas	5000–100,000	$Nu = 0.102Re^{0.675}\,Pr^{1/3}$
Square (tilted 45°) 	Gas	5000–100,000	$Nu = 0.246Re^{0.588}\,Pr^{1/3}$
Hexagon 	Gas	5000–100,000	$Nu = 0.153Re^{0.638}\,Pr^{1/3}$ 4
Hexagon (tilted 45°) 	Gas	5000–19,500 19,500–100,000	$Nu = 0.160Re^{0.638}\,Pr^{1/3}$ $Nu = 0.0385Re^{0.782}\,Pr^{1/3}$
Vertical plate 	Gas	4000–15,000	$Nu = 0.228Re^{0.731}\,Pr^{1/3}$
Ellipse 	Gas	2500–15,000	$Nu = 0.248Re^{0.612}\,Pr^{1/3}$

FLOW ACROSS TUBE BANKS



(a) In-line



(b) Staggered

The tubes in a tube bank are usually arranged either *in-line* or *staggered* in the direction of flow, as shown in Figure 7–26. The outer tube diameter D is taken as the characteristic length. The arrangement of the tubes in the tube bank is characterized by the *transverse pitch* S_T, *longitudinal pitch* S_L, and the *diagonal pitch* S_D between tube centers. The diagonal pitch is determined from

$$S_D = \sqrt{S_L^2 + (S_T/2)^2} \tag{7-38}$$

As the fluid enters the tube bank, the flow area decreases from $A_1 = S_T L$ to $A_T = (S_T - D)L$ between the tubes, and thus flow velocity increases. In staggered arrangement, the velocity may increase further in the diagonal region if the tube rows are very close to each other. In tube banks, the flow characteristics are dominated by the maximum velocitiy $\mathcal{V}_{max}$ that occurs within the tube bank rather than the approach velocity $\mathcal{V}$. Therefore, the Reynolds number is defined on the basis of maximum velocity as

$$\text{Re}_D = \frac{\rho \mathcal{V}_{max} D}{\mu} = \frac{\mathcal{V}_{max} D}{\nu} \tag{7-39}$$

The maximum velocity is determined from the conservation of mass re-
Several correlations, all based on experimental data, have been proposed for the average Nusselt number for cross flow over tube banks. More recently, Zukauskas has proposed correlations whose general form is

$$Nu_D = \frac{hD}{k} = C\,Re_D^m\,Pr^n(Pr/Pr_s)^{0.25} \quad \text{(7-42)}$$

where the values of the constants C, m, and n depend on value Reynolds number. Such correlations are given in Table 7–2 explicitly for $0.7 < Pr < 500$ and $0 < Re_D < 2 \times 10^6$. The uncertainty in the values of Nusselt number obtained from these relations is ±15 percent. Note that all properties except Pr_s are to be evaluated at the arithmetic mean temperature of the fluid determined from

$$T_m = \frac{T_i + T_e}{2} \quad 8$$

where T_i and T_e are the fluid temperatures at the inlet and the exit of the tube bank, respectively.

since $\rho \mathcal{V} A_1 = \rho \mathcal{V}_{max}(2A_D)$ or $\mathcal{V} S_T = 2\mathcal{V}_{max}(S_D - D)$.

TABLE 7–2

Nusselt number correlations for cross flow over tube banks for $N > 16$ and $0.7 < Pr < 500$ (from Zukauskas, Ref. 15, 1987)*

Arrangement	Range of Re_D	Correlation
In-line	0–100	$Nu_D = 0.9\,Re_D^{0.4}Pr^{0.36}(Pr/Pr_s)^{0.25}$
	100–1000	$Nu_D = 0.52\,Re_D^{0.5}Pr^{0.36}(Pr/Pr_s)^{0.25}$
	$1000–2 \times 10^5$	$Nu_D = 0.27\,Re_D^{0.63}Pr^{0.36}(Pr/Pr_s)^{0.25}$
	$2 \times 10^5–2 \times 10^6$	$Nu_D = 0.033\,Re_D^{0.8}Pr^{0.4}(Pr/Pr_s)^{0.25}$
Staggered	0–500	$Nu_D = 1.04\,Re_D^{0.4}Pr^{0.36}(Pr/Pr_s)^{0.25}$
	500–1000	$Nu_D = 0.71\,Re_D^{0.5}Pr^{0.36}(Pr/Pr_s)^{0.25}$
	$1000–2 \times 10^5$	$Nu_D = 0.35(S_T/S_L)^{0.2}\,Re_D^{0.6}Pr^{0.36}(Pr/Pr_s)^{0.25}$
	$2 \times 10^5–2 \times 10^6$	$Nu_D = 0.031(S_T/S_L)^{0.2}\,Re_D^{0.8}Pr^{0.36}(Pr/Pr_s)^{0.25}$

*All properties except Pr_s are to be evaluated at the arithmetic mean of the inlet and outlet temperatures of the fluid (Pr_s is to be evaluated at T_s).

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{A_s h}{\dot{m}C_p}\right) \qquad \text{(7-46)}$$

where $A_s = N\pi DL$ is the heat transfer surface area and $\dot{m} = \rho\mathcal{V}(N_T S_T L)$ is the mass flow rate of the fluid. Here N is the total number of tubes in the bank, N_T is the number of tubes in a transverse plane, L is the length of the tubes, and $\mathcal{V}$ is the velocity of the fluid just before entering the tube bank. Then the heat transfer rate can be determined from 13

$$\dot{Q} = hA_s\Delta T_{ln} = \dot{m}C_p(T_e - T_i) \qquad \text{(7-47)}$$

The second relation is usually more convenient to use since it does not require the calculation of ΔT_{ln}.

over tube banks is still internal flow through the shell) is the *logarithmic mean temperature difference* ΔT_{ln} defined as

$$\Delta T_{ln} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)} \qquad 10$$

We will also show that the exit temperature of the fluid T_e can be determined from

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Table 3

Correction factor F to be used in $Nu_{D,N_L} = F Nu_D$ for $N_L < 16$ and $Re_D > 1000$ (from Zukauskas, Ref 15, 1987).

N_L	1	2	3	4	5	7	10	13
In-line	0.70	0.80	0.86	0.90	0.93	0.96	0.98	0.99
Staggered	0.64	0.76	0.84	0.89	0.93	0.96	0.98	0.99

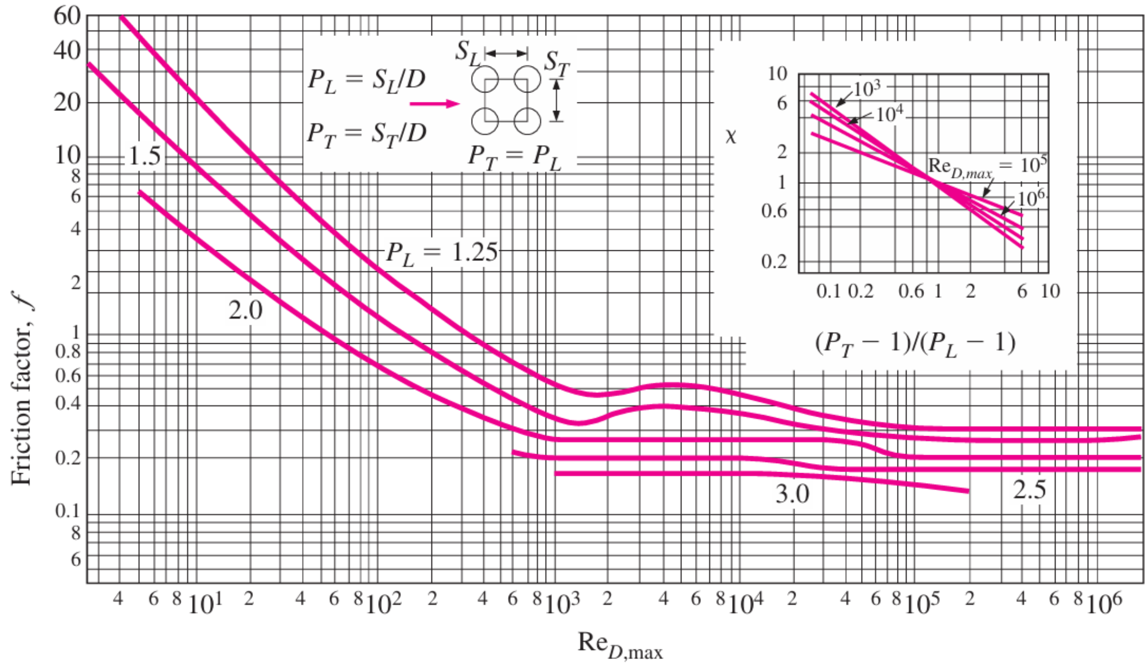
square and equilateral triangle arrangements. Also, pressure drop occurs in the flow direction, and thus we used N_L (the number of rows) in the ΔP relation.

The power required to move a fluid through a tube bank is proportional to the pressure drop, and when the pressure drop is available, the pumping power required can be determined from

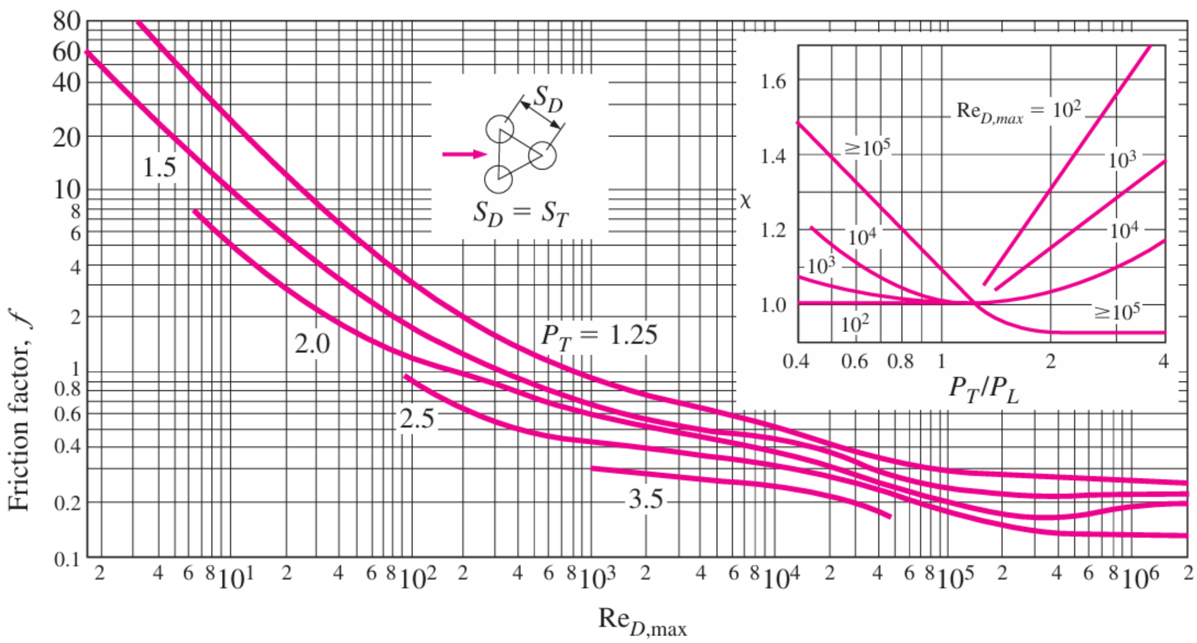
$$\dot{W}_{pump} = \dot{V}\Delta P = \frac{\dot{m}\Delta P}{\rho} \qquad \text{(7-49)}$$

where $\dot{V} = \mathcal{V}(N_T S_T L)$ is the volume flow rate and $\dot{m} = \rho\dot{V} = \rho\mathcal{V}(N_T S_T L)$ is the mass flow rate of the fluid through the tube bank. Note that the power required to keep a fluid flowing through the tube bank (and thus the operating cost) is proportional to the pressure drop. Therefore, the benefits of enhancing heat transfer in a tube bank via rearrangement should be weighed against the cost of additional power requirements.

In this section we limited our consideration to tube banks with base surfaces (no fins). Tube banks with finned surfaces are also commonly used in practice, especially when the fluid is a gas, and heat transfer and pressure drop correlations can be found in the literature for tube banks with pin fins, plate fins, strip fins, etc.



(*a*) In-line arrangement



(*b*) Staggered arrangement

Ex1: Air flows across a 40 –mm- square cylinder at a velocity of 10 m/s .The surface temperature is maintained at 85 °C .Free –stream air conditions are 20 °C and 60 kpa .Calculate the heat lost from the cylinder per meter length .

$$T_f = \frac{20+85}{2} = 52.5°\text{C} = 325.5 \text{ K} \qquad \mu = 1.96\times10^{-5} \qquad k = 0.0281$$

$$\text{Pr} = 0.7 \qquad \rho = \frac{(0.6)(1.01\times10^5)}{(287)(325.5)} = 0.651 \ \text{kg/m}^3 \qquad C = 0.102$$

$$n = 0.675 \qquad \text{Re} = \frac{(0.651)(12)(0.04)}{1.96\times10^{-5}} = 15{,}937$$

$$h = \frac{0.0281}{0.04}(0.102)(15{,}937)^{0.675}(0.7)^{1/3} = 43.68 \ \frac{\text{W}}{\text{m}^2\cdot°\text{C}}$$

$$\frac{q}{L} = h\pi d(T_w - T_\infty) = (43.68)\pi(0.04)(85-20) = 356.7 \ \text{W/m}$$

Ex2:

A 25 –mm- diameter , high –tension line an electrical resistance of $10^{-4}\Omega/m$ and transmitting a current of 1000 A

(a) If ambient air at 10 °C and 5 m/s in cross flow over the line , what its surface temperature ?

(b) If the made from a solid copper rod (400 w/m.C) What is its centerline temperature?

PROPERTIES: *Table A.4*, Air (T_∞ = 298 K, 1 atm): $\mu = 184 \times 10^{-7}$ N·s/m^2; $\nu = 15.71 \times 10^{-6}$ m^2/s 0.0261 W/m·K, Pr = 0.71; (T_s = 348 K): $\mu = 208 \times 10^{-7}$ N·s/m^2; (T_f = 323 K): $\nu = 18.2 \times 10^{-6}$ m^2/s, 1.085 kg/m^3.

ANALYSIS: (a) Working with properties evaluated at T_f

$$Re_D = \frac{VD}{\nu} = \frac{25\,m/s\,(0.01\,m)}{18.2\times10^{-6}\,m^2/s} = 1.37\times10^4$$

and from Fig. 7.8, find $C_D \approx 0.4$. Hence

$$F_D = C_D\left(\pi D^2/4\right)\left(\rho V^2/2\right) = 0.4(\pi/4)(0.01\,m)^2\,1.085\,kg/m^3\,(25\,m/s)^2/2 = 0.011$$

(b) With

$$Re_D = \frac{VD}{\nu} = \frac{25\,m/s\,(0.01\,m)}{15.71\times10^{-6}\,m^2/s} = 1.59\times10^4$$

it follows from the Whitaker relation that

$$\overline{Nu}_D = 2 + \left[0.4\,Re_D^{1/2} + 0.06\,Re_D^{2/3}\right]Pr^{0.4}\left(\frac{\mu}{\mu_s}\right)^{1/4}$$

$$\overline{Nu}_D = 2 + \left[0.4\left(1.59\times10^4\right)^{1/2} + 0.06\left(1.59\times10^4\right)^{2/3}\right](0.71)^{0.4}\left(\frac{184}{208}\right)^{1/4} = 76.7$$

Hence, the convection coefficient and convection heat rate are

$$\bar{h} = \overline{Nu}_D\,\frac{k}{D} = 76.7\,\frac{0.0261\,W/m\cdot K}{0.01\,m} = 200\,W/m^2\cdot K$$

$$q = \bar{h}\pi D^2\left(T_s - T_\infty\right) = 200\,W/m^2\cdot K\times\pi\,(0.01\,m)^2\,(75-25)^\circ C = 3.14\,W$$