



8. THE IDEAL-GAS EQUATION OF STATE

A gas is made of molecules that move around with random motion. In a perfect gas, the molecules may collide, but they have no tendency to stick together or repel each other. In reality, there is a slight force of attraction between gas molecules, but this is so small that gas laws formulated for an ideal gas work quite well for a real gas.

Any equation that relates the pressure, temperature, and specific volume of a substance is called an equation of state. Property relations that involve other properties of a substance at equilibrium states are also referred to as equations of state. There are several equations of state, some simple and others very complex. The simplest and best-known equation of state for substances in the gas phase is the ideal-gas equation of state. This equation predicts the (P-v-T) behavior of a gas quite accurately within some properly selected region.

Gas and vapor are often used as synonymous words. The vapor phase of a substance is customarily called a gas when it is above the critical temperature. Vapor usually implies a gas that is not far from a state of condensation.

In 1662, Robert Boyle, an Englishman, observed during his experiments with a vacuum chamber that the pressure of gases is inversely proportional to their volume. In 1802, J. Charles and J. Gay-Lussac, Frenchmen, experimentally determined that at low pressures the volume of a gas is proportional to its temperature. That is,

$$PV = mRT \quad \dots\dots\dots(18)$$

$$Pv = RT \quad \dots\dots\dots(19)$$

Where:

R: is called the gas constant and it is different for each gas. Table (1) shows the values of the gas constant for different gases.

P: is the absolute pressure.

T: is the absolute temperature (°K).

V: is the volume (m³)

v: is the specific volume (m³/kg).



Equations (18 and 19) is called the ideal-gas equation of state, or simply the ideal-gas relation, and a gas that obeys this relation is called an ideal gas. $R=R_u/M$

$$\text{and } M=m/N \quad \dots\dots\dots(20)$$

where R_u : is the universal gas constant.

M : is the molar mass (also called molecular weight) of the gas.

The constant (R_u) is the same for all substances, and its value is:

$$R_u = 8.314 \text{ kJ/kmol.}^\circ\text{K} \quad R_u = 1545.37 \text{ ft.lbf/kmol.}^\circ\text{R}$$

Boyle's law states that the pressure of a given mass of an ideal gas is inversely proportional to its volume at a constant temperature. It is expressed as:

$$PV = C \quad \text{or} \quad P_1 V_1 = P_2 V_2 \Rightarrow \quad P_1 / P_2 = V_2 / V_1 \quad \dots\dots\dots(21)$$

Charles's law states that the volume of an ideal gas at constant pressure is directly proportional to the absolute temperature. It is expressed as:

$$V/T = C \quad \text{or} \quad V_1/T_1 = V_2/T_2 \quad \dots\dots\dots(22)$$

Gay-Lussac's law states that, for a given mass and constant volume of an ideal gas, the pressure exerted on the sides of its container is directly proportional to its absolute temperature. It is expressed as:

$$P/T = C \quad \text{or} \quad P_1/T_1 = P_2/T_2 \quad \dots\dots\dots(23)$$



Example (4): Determine the mass of the air in a room whose dimensions are (4 m × 5 m × 6 m) at (100 kPa) and (25°C).

Solution:

From Table (1), the gas constant of air is

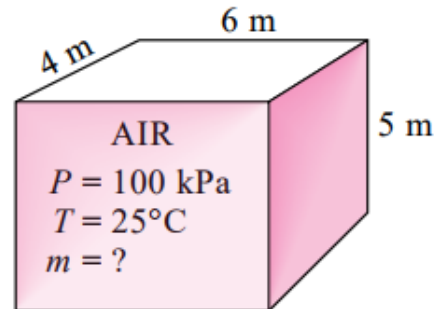
$$R = 0.287 \text{ kJ/kg} \cdot ^\circ\text{K}$$

$$T = 25 + 273 = 298 \text{ } ^\circ\text{K}$$

$$V = 4 \times 5 \times 6 = 120 \text{ m}^3$$

$$PV = mRT \Rightarrow m = PV / RT$$

$$m = PV / RT = 100 \times 120 / 0.287 \times 298 = 140.3 \text{ kg}$$



Example (5): An amount of gas has a pressure of (350 kPa), a volume of (0.03 m³) and a temperature of (35°C). If (R = 0.29 kJ/kg.K), calculate the mass of the gas and the final temperature if the final pressure is (1.05 MPa) and the volume remains constant.

Solution:

The absolute temperature:

$$T_1 = 35 + 273 = 308 \text{ } ^\circ\text{K}$$

$$P_1 V_1 = mRT_1$$

$$m = 350 \times 0.03 / 0.29 \times 308 \Rightarrow m = 0.12 \text{ kg}$$

Applying the equation of state between two conditions at constant volume: $P_1 / T_1 = P_2 / T_2$

$$350 / 308 = 1.05 \times 10^3 / T_2 \Rightarrow T_2 = 1.05 \times 10^3 \times 308 / 350$$

$$T_2 = 924 \text{ } ^\circ\text{K}$$



Example (6): A tank has a volume of (0.5 m^3) and contains (10 kg) of an ideal gas having a molecular weight of (24). The temperature is ($25 \text{ }^\circ\text{C}$). What is the pressure of the gas?

Solution:

The absolute temperature: $T = 25 + 273 = 298 \text{ K}$

$$R = R_u / M = 8.314 / 24 = 0.35 \text{ kJ/kg.}^\circ\text{K}$$

$$PV = mRT$$

$$P \times 0.5 = 10 \times 0.35 \times 298$$

$$P = 10 \times 0.35 \times 298 / 0.5$$

$$P = 2086 \text{ kPa}$$



Home Work (1):

1- A container of (0.2 m^3) contains nitrogen at a pressure of (1.013 bar) and at a temperature of (15°C). (2 kg) of nitrogen was pumped by a special pump to the tank. Calculate the new gas pressure when the tank returns to its initial temperature. Nitrogen was considered an ideal gas, take $R = 296.9 \text{ J / kg. }^\circ\text{K}$.

Ans. (1.87 bar)

2- Air in an internal combustion engine has (227°C), (1000 kPa) with a volume of (0.1 m^3). Now combustion heats it to (1500°K) in a constant volume process. What is the mass of air and how high does the pressure become?

Ans. (0.697 kg, 3000 kPa)

3- A rigid tank of 1 m^3 contains nitrogen gas (molecular weight 28) at 600 kPa, 400°K . By mistake someone lets 0.5 kg flow out. If the final temperature is 375°K what is the final pressure?

Ans. (506.9 kPa)

4- A (1 m^3) rigid tank contains propane (molecular weight 44) at (100 kPa), (300°K) and connected by a valve to another tank of (0.5 m^3) with propane at (250 kPa), (400°K). The valve is opened, and the two tanks come to a uniform state at (325°K). What is the final pressure?

Ans. (139.9 kPa)

5- (0.1) kg of ideal gas occupies a volume of (0.003 m^3) at a pressure of (7 bar) and a temperature (131°C) when the gas was expand to a pressure of (1 bar), its final volume became (0.02 m^3). Calculate the final temperature.

Ans. (384.6°K)

6- Air is at (25°C) and (101.325 kPa). If the gas constant ($R = 287 \text{ J / kg. }^\circ\text{K}$), find the specific volume and the molar mass of this gas, assuming it behaves as an ideal gas.

Ans. ($0.8445 \text{ m}^3/\text{kg}$, 28.97 kg/kmol)



Table (1) Ideal-gas specific heats of various common gases at (300 °K)

Gas	Formula	Gas constant R $\text{kJ}/(\text{kg} \cdot \text{K})$	c_p $\text{kJ}/(\text{kg} \cdot \text{K})$	c_v $\text{kJ}/(\text{kg} \cdot \text{K})$	γ
Air	—	0.2870	1.005	0.718	1.400
Argon	Ar	0.2081	0.5203	0.3122	1.667
Butane	C_4H_{10}	0.1433	1.7164	1.5734	1.091
Carbon dioxide	CO_2	0.1889	0.846	0.657	1.289
Carbon monoxide	CO	0.2968	1.040	0.744	1.400
Ethane	C_2H_6	0.2765	1.7662	1.4897	1.186
Ethylene	C_2H_4	0.2964	1.5482	1.2518	1.237
Helium	He	2.0769	5.1926	3.1156	1.667
Hydrogen	H_2	4.1240	14.307	10.183	1.405
Methane	CH_4	0.5182	2.2537	1.7354	1.299
Neon	Ne	0.4119	1.0299	0.6179	1.667
Nitrogen	N_2	0.2968	1.039	0.743	1.400
Octane	C_8H_{18}	0.0729	1.7113	1.6385	1.044
Oxygen	O_2	0.2598	0.918	0.658	1.395
Propane	C_3H_8	0.1885	1.6794	1.4909	1.126
Steam	H_2O	0.4615	1.8723	1.4108	1.327