Al-Mustaqbal University / College of Technical Engineering
Department of Fuel and Energy Technical Engineering
Class (Third Year)
Subject (Heat Transfer-2) / Code (UOMU0206062)
Lecturer (Asst. Lect. Sameer Saad Raheem)
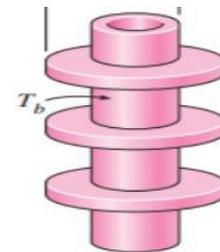**2nd term – Lecture No. 4 & Lecture Name (Overall effectiveness)**

Overall effectiveness

The overall effectiveness for a finned surface is defined as the ratio of the total heat transfer from the finned surface to the heat transfer from the same surface if there were no fins,

$$\varepsilon_{\text{fin, overall}} = \frac{\dot{Q}_{\text{total, fin}}}{\dot{Q}_{\text{total, no fin}}} = \frac{h(A_{\text{unfin}} + \eta_{\text{fin}} A_{\text{fin}})(T_b - T_\infty)}{hA_{\text{no fin}}(T_b - T_\infty)}$$

When determining the rate of heat transfer from a finned surface, we must consider the unfinned portion of the surface as well as the fins. **Therefore, the rate of heat transfer for a surface containing n fins can be expressed as,**

$$\dot{Q}_{\text{total, fin}} = \dot{Q}_{\text{unfin}} + \dot{Q}_{\text{fin}}$$
$$= hA_{\text{unfin}}(T_b - T_\infty) + \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty)$$
$$= h(A_{\text{unfin}} + \eta_{\text{fin}} A_{\text{fin}})(T_b - T_\infty)$$

We can also define an overall effectiveness for a finned surface as the ratio of the total heat transfer from the finned surface to the heat transfer from the same surface if there were no fins,

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where A$_{no\ fin}$ is the area of the surface when there are no fins, A$_{fin}$ is the total surface area of all the fins on the surface, and A$_{unfin}$ is the area of the un-finned portion of the surface. Note that the overall fin effectiveness depends on the fin density (number of fins per unit length) as well as the effectiveness of the individual fins. The overall effectiveness is a better measure of the performance of a finned surface than the effectiveness of the individual fins.

The variation of heat transfer from
a fin relative to that from an
infinitely long fin

aL	$\dfrac{\dot{Q}_{fin}}{\dot{Q}_{long\ fin}} = \tanh aL$
0.1	0.100
0.2	0.197
0.5	0.462
1.0	0.762
1.5	0.905
2.0	0.964
2.5	0.987
3.0	0.995
4.0	0.999
5.0	1.000

$$a = \sqrt{hp/kA_c},$$

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Proper Length of a Fin

To get a sense of the proper length of a fin, we compare heat transfer from a fin of finite length to heat transfer from an infinitely long fin under the same conditions. The ratio of these two heat transfers is,
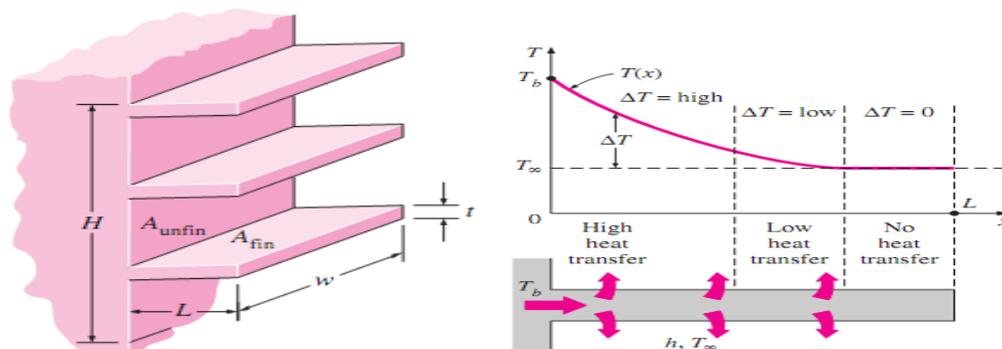
Heat transfer ratio:
$$\frac{\dot{Q}_{fin}}{\dot{Q}_{long\ fin}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)\tanh aL}{\sqrt{hpkA_c}\,(T_b - T_\infty)} = \tanh aL$$

Because of the gradual temperature drop along the fin, **the region near the fin tip makes little or no contribution to heat transfer.**

Using a hand calculator, the values of **tanh aL** are evaluated for some values of aL and the results are given in the Table. There for a fin whose length is **L= a/5** can be considered to be **an infinitely long fin**. We also observe that reducing the **fin length by half** in that case (from **aL=5 to aL=2.5**) causes a **drop of just 1 percent** in heat transfer. We certainly would not hesitate sacrificing **1 percent** in heat transfer performance in return for **50 percent** reduction in the size and possibly the cost of the fin. In practice, a fin length that corresponds to about **aL=1** will transfer **76.2 percent** of the heat that can be transferred by an infinitely long fin.

$$Q_{.} = h\,A_{fin}\,(T_b - T\infty) = h\,A_{fin}\,\Delta T$$

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Example:1 Heat Transfer from Finned Steam Pipe

Steam in a heating system flows through tubes whose outer diameter is D_1 = 3 cm and whose walls are maintained at a temperature of 120°C. Circular aluminum fins (k = 180 W/m · °C) of outer diameter D_2 = 6 cm and constant thickness t = 2 mm are attached to the tube, as shown in Fig. 3–48. The space between the fins is 3 mm, and thus there are 200 fins per meter length of the tube. Heat is transferred to the surrounding air at T_∞ = 25°C, with a combined heat transfer coefficient of h = 60 W/m² · °C. Determine the increase in heat transfer from the tube per meter of its length as a result of adding fins.

Solution:

For finned tube of Length 1m = 1000mm

Number of fins per 1m = $\dfrac{1000}{(2+3)}$ = 200 fins
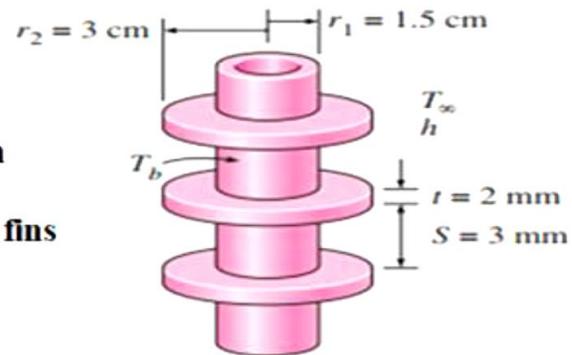
Analysis In the case of no fins, heat transfer from the tube per meter of its length is determined from Newton's law of cooling to be

$$A_{\text{no fin}} = \pi D_1 L = \pi(0.03 \text{ m})(1 \text{ m}) = 0.0942 \text{ m}^2$$

$$\dot{Q}_{\text{no fin}} = hA_{\text{no fin}}(T_b - T_\infty)$$

$$= (60 \text{ W/m}^2 \cdot \text{°C})(0.0942 \text{ m}^2)(120 - 25)\text{°C}$$

$$= 537 \text{ W}$$

The efficiency of the circular fins attached to a circular tube is plotted in Fig. 3–43. Noting that $L = \frac{1}{2}(D_2 - D_1) = \frac{1}{2}(0.06 - 0.03) = 0.015$ m in this case, we have

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$$\left. \begin{array}{l} \dfrac{r_2 + \frac{1}{2}t}{r_1} = \dfrac{(0.03 + \frac{1}{2} \times 0.002)\ \text{m}}{0.015\ \text{m}} = 2.07 \\[2ex] (L + \frac{1}{2}t)\ \sqrt{\dfrac{h}{kt}} = (\ 0.015 + \frac{1}{2} \times 0.002)\ \text{m} \times \sqrt{\dfrac{60\ \text{W/m}^2 \cdot {}^\circ\text{C}}{(180\ \text{W/m} \cdot {}^\circ\text{C})(0.002\ \text{m})}} = 0.207 \end{array} \right\} \eta_{\text{fin}} = 0.95$$

$$\begin{aligned} A_{\text{fin}} &= 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t \\ &= 2\pi[(0.03\ \text{m})^2 - (0.015\ \text{m})^2] + 2\pi(0.03\ \text{m})(0.002\ \text{m}) \\ &= 0.00462\ \text{m}^2 \end{aligned}$$

$$\begin{aligned} \dot{Q}_{\text{fin}} &= \eta_{\text{fin}}\dot{Q}_{\text{fin, max}} = \eta_{\text{fin}}hA_{\text{fin}}\,(T_b - T_\infty) \\ &= 0.95(60\ \text{W/m}^2 \cdot {}^\circ\text{C})(0.00462\ \text{m}^2)(120 - 25){}^\circ\text{C} \\ &= 25.0\ \text{W} \end{aligned}$$

Heat transfer from the unfinned portion of the tube is

$$\begin{aligned} A_{\text{unfin}} &= \pi D_1 S = \pi(0.03\ \text{m})(0.003\ \text{m}) = 0.000283\ \text{m}^2 \\ \dot{Q}_{\text{unfin}} &= hA_{\text{unfin}}(T_b - T_\infty) \\ &= (60\ \text{W/m}^2 \cdot {}^\circ\text{C})(0.000283\ \text{m}^2)(120 - 25){}^\circ\text{C} \\ &= 1.60\ \text{W} \end{aligned}$$

Noting that there are 200 fins and thus 200 interfin spacings per meter length of the tube, the total heat transfer from the finned tube becomes

$$\dot{Q}_{\text{total, fin}} = n(\dot{Q}_{\text{fin}} + \dot{Q}_{\text{unfin}}) = 200(25.0 + 1.6)\ \text{W} = 5320\ \text{W}$$

Therefore, the increase in heat transfer from the tube per meter of its length as a result of the addition of fins is

$$\dot{Q}_{\text{increase}} = \dot{Q}_{\text{total, fin}} - \dot{Q}_{\text{no fin}} = 5320 - 537 = \textbf{4783 W} \qquad \text{(per m tube length)}$$

Discussion The overall effectiveness of the finned tube is

$$\varepsilon_{\text{fin, overall}} = \frac{\dot{Q}_{\text{total, fin}}}{\dot{Q}_{\text{total, no fin}}} = \frac{5320\ \text{W}}{537\ \text{W}} = 9.9$$

That is, the rate of heat transfer from the steam tube increases by a factor of almost 10 as a result of adding fins. This explains the widespread use of finned surfaces.

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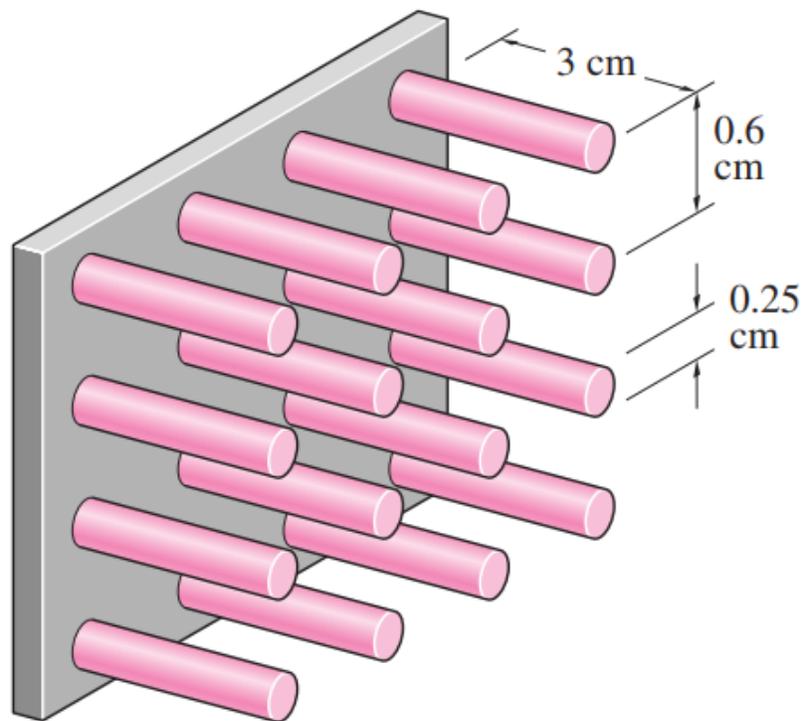**Department of Fuel and Energy Technical Engineering**
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H.W

Q: A hot surface at 100°C is to be cooled by attach-ing 3-cm-long, 0.25-cm-diameter aluminum pin fins (k = 237 W/m · °C) to it, with a center-to-center distance of 0.6 cm. The temperature of the surrounding medium is 30°C, and the heat transfer coefficient on the surfaces is 35 W/m² · °C. Determine the rate of heat transfer from the surface for a 1-m × 1-m section of the plate. Also determine the overall effectiveness of the fins. *Ans. Q= 17.4 kW , ε_{fin} = 7.1*

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Primary References

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Additional Reference

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