INTERNAL FORCED CONVECTION

Laminar and Turbulent Flow In Tubes

Flow in a tube can be laminar or turbulent, depending on the flow conditions. Fluid flow is streamlined and thus laminar at low velocities, but turns turbulent as the velocity is increased beyond a critical value. Transition from laminar to turbulent flow does not occur suddenly; rather, it occurs over some range of velocity where the flow fluctuates between laminar and turbulent flows before it becomes fully turbulent. Most pipe flows encountered in practice are turbulent. Laminar flow is encountered when highly viscous fluids such as oils flow in small diameter tubes or narrow passages.

For flow in a circular tube, the Reynolds number is defined as

$$\mathrm{Re} = \frac{\rho \mathcal{V}_m D}{\mu} = \frac{\mathcal{V}_m D}{\nu}$$

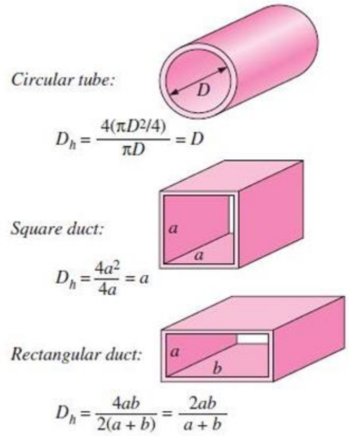
where $\mathcal{V}_m$ is the mean fluid velocity, D is the diameter of the tube, and $\nu = \mu/\rho$ is the kinematic viscosity of the fluid.

For flow through noncircular tubes, the Reynolds number as well as the Nusselt number and the friction factor are based on the **hydraulic diameter** D_h defined as (Fig. 8–4)

$$D_h = \frac{4A_c}{p}$$

where A_c is the cross sectional area of the tube and p is its perimeter. The hydraulic diameter is defined such that it reduces to ordinary diameter D for circular tubes since

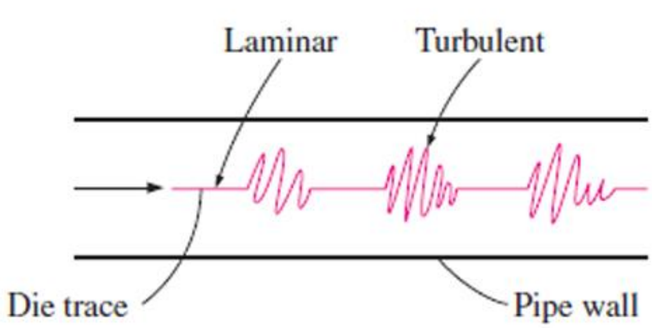
Circular tubes: $$D_h = \frac{4A_c}{p} = \frac{4\pi D^2/4}{\pi D} = D$$



It certainly is desirable to have precise values of Reynolds numbers for laminar, transitional, and turbulent flows, but this is not the case in practice. This is because the transition from laminar to turbulent flow also depends on the degree of disturbance of the flow by *surface roughness, pipe vibrations,* and the *fluctuations in the flow.* Under most practical conditions, the flow in a tube is laminar for Re < 2300, turbulent for Re > **4000** , and transitional in between. That is,

$$\begin{aligned} Re &< 2300 && \text{laminar flow} \\ 2300 \leq Re &\leq 10{,}000 && \text{transitional flow} \\ Re &> 10{,}000 && \text{turbulent flow} \end{aligned}$$

In transitional flow, the flow switches between laminar and turbulent randomly (Fig. 8–5). It should be kept in mind that laminar flow can be maintained at much higher Reynolds numbers in very smooth pipes by avoiding flow disturbances and tube vibrations. In such carefully controlled



8–3 ▪ THE ENTRANCE REGION

Consider a fluid entering a circular tube at a uniform velocity. As in external flow, the fluid particles in the layer in contact with the surface of the tube will come to a complete stop. This layer will also cause the fluid particles in the adjacent layers to slow down gradually as a result of friction. To make up for this velocity reduction, the velocity of the fluid at the midsection of the tube will have to increase to keep the mass flow rate through the tube constant. As a result, a *velocity boundary layer* develops along the tube. The thickness of this boundary layer increases in the flow direction until the boundary layer reaches the tube center and thus fills the entire tube, as shown in Figure 8–6.

The region from the tube inlet to the point at which the boundary layer merges at the centerline is called the **hydrodynamic entrance region,** and the length of this region is called the **hydrodynamic entry length** L_h. Flow in the entrance region is called *hydrodynamically developing flow* since this is the region where the velocity profile develops. The region beyond the entrance region in which the velocity profile is fully developed and remains unchanged is called the **hydrodynamically fully developed region.** The velocity profile in the fully developed region is *parabolic* in laminar flow and somewhat *flatter* in turbulent flow due to eddy motion in radial direction.

The region of flow over which the thermal boundary layer develops and reaches the tube center is called the **thermal entrance region,** and the length of this region is called the **thermal entry length** L_t. Flow in the thermal entrance region is called *thermally developing flow* since this is the region where the temperature profile develops. The region beyond the thermal entrance region in which the dimensionless temperature profile expressed as $(T_s - T)/(T_s - T_m)$ remains unchanged is called the **thermally fully developed region.** The region in which the flow is both hydrodynamically and thermally developed and thus both the velocity and dimensionless temperature profiles remain unchanged is called *fully developed flow.* That is,

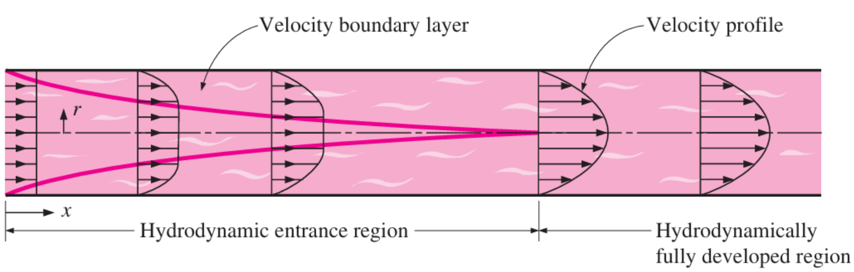


FIGURE 8–6
The development of the velocity boundary layer in a tube. (The developed mean velocity profile will be parabolic in laminar flow, as shown, but somewhat blunt in turbulent flow.)

Entry Lengths

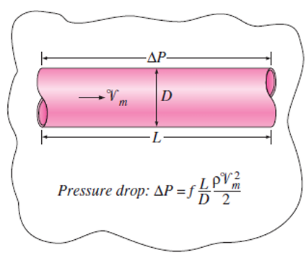
$$L_{h,\ \text{laminar}} \approx 0.05\ \text{Re}\ D$$
$$L_{t,\ \text{laminar}} \approx 0.05\ \text{Re}\ \text{Pr}\ D = \text{Pr}\ L_{h,\ \text{laminar}}$$

$$L_{h,\ \text{turbulent}} = 1.359\ \text{Re}^{1/4}$$

$$L_{h,\ \text{turbulent}} \approx L_{t,\ \text{turbulent}} \approx 10D$$

Pressure Drop

$$\Delta P = f \frac{L}{D} \frac{\mu \mathcal{V}_m^2}{2} \qquad 5$$



Circular tube, laminar:
$$f = \frac{64\mu}{\rho D \mathcal{V}_m} = \frac{64}{\text{Re}}$$

For *smooth* tubes, the friction factor in turbulent flow can be determined from the explicit *first Petukhov equation* [Petukhov (1970), Ref. 21] given as

Smooth tubes: $f = (0.790 \ln \text{Re} - 1.64)^{-2}$ $\quad 10^4 < \text{Re} < 10^6$ $\quad$ 7

$$\dot{W}_{\text{pump}} = \dot{V} \Delta P \qquad 8$$

LAMINAR FLOW IN TUBES

the velocity profile is obtained to be

$$\mathcal{V}(r) = 2\mathcal{V}_m\left(1 - \frac{r^2}{R^2}\right) \qquad 9$$

The maximum velocity occurs at the centerline

$$\mathcal{V}_{\max} = 2\mathcal{V}_m \qquad 10$$

Therefore, *the mean velocity is one-half of the maximum velocity.*

$$\dot{m} = \rho \mathcal{V}_m A_c \qquad 11$$

Nusselt Number

Constant Surface Heat Flux

Circular tube, laminar ($\dot{q}_x$ = constant): $$\text{Nu} = \frac{hD}{k} = 4.36 \qquad 12$$

Constant Surface Temperature

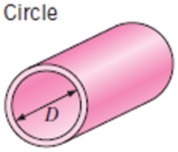
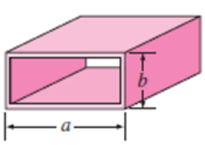
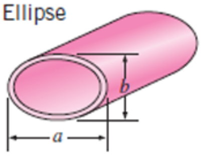
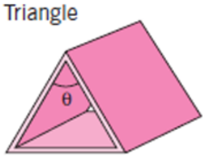
A similar analysis can be performed for fully developed laminar flow in a circular tube for the case of constant surface temperature T_s. The solution procedure in this case is more complex as it requires iterations, but the Nusselt number relation obtained is equally simple (Fig. 8–21):

Circular tube, laminar (T_s = constant): $$\text{Nu} = \frac{hD}{k} = 3.66 \qquad 13$$

Laminar Flow in Noncircular Tubes

The friction factor f and the Nusselt number relations are given in able 8–1 for *fully developed laminar flow* in tubes of various cross sections. The Reynolds and Nusselt numbers for flow in these tubes are based on the hydraulic diameter $D_h = 4A_c/p$, where A_c is the cross sectional area of the tube and p is its perimeter. Once the Nusselt number is available, the convection heat transfer coefficient is determined from $h = k\text{Nu}/D_h$.

Nusselt number and friction factor for fully developed laminar flow in tubes of various cross sections ($D_h = 4A_c/p$, Re $= \mathcal{V}_m D_h/\nu$, and Nu $= hD_h/k$)

Tube Geometry	a/b or θ°	Nusselt Number T_s = Const.	Nusselt Number $\dot{q}_s$ = Const.	Friction Factor f
Circle 	—	3.66	4.36	64.00/Re
Rectangle 	a/b			
	1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re
Ellipse 	a/b			
	1	3.66	4.36	64.00/Re
	2	3.74	4.56	67.28/Re
	4	3.79	4.88	72.96/Re
	8	3.72	5.09	76.60/Re
	16	3.65	5.18	78.16/Re
Triangle 	θ			
	10°	1.61	2.45	50.80/Re
	30°	2.26	2.91	52.28/Re
	60°	2.47	3.11	53.32/Re
	90°	2.34	2.98	52.60/Re
	120°	2.00	2.68	50.96/Re

Developing Laminar Flow in the Entrance Region

For a circular tube of length L subjected to constant surface temperature, the average Nusselt number for the *thermal entrance region* can be determined from

Entry region, laminar: $$\text{Nu} = 3.66 + \frac{0.065\,(D/L)\,\text{Re}\,\text{Pr}}{1 + 0.04[(D/L)\,\text{Re}\,\text{Pr}]^{2/3}} \quad (14)$$

For fully developed laminar flow ($Re_d \Pr \frac{d}{L} > 10$

$$Nu = 1.86(Re_d \Pr)^{\frac{1}{3}} \left(\frac{d}{L}\right)^{1/3} \left(\frac{\mu}{\mu_s}\right)^{0.14} \quad (15)$$

TURBULENT FLOW IN TUBES

which is known as the *Colburn equation*. The accuracy of this equation can be improved by modifying it as

$$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{n} \quad (16)$$

where $n = 0.4$ for *heating* and 0.3 for *cooling* of the fluid flowing through the tube.

for turbulent entrance region $10 < \frac{d}{L} < 400$

$$Nu = 0.036(Re_d)^{0.8}\,(Pr)^{1/3}\left(\frac{d}{L}\right)^{0.055} \quad (17)$$

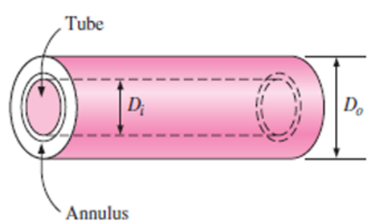
Flow through Tube Annulus

$$D_h = \frac{4A_c}{p} = \frac{4\pi(D_o^2 - D_i^2)/4}{\pi(D_o + D_i)} = D_o - D_i$$

$$\text{Nu}_i = \frac{h_i D_h}{k} \quad \text{and} \quad \text{Nu}_o = \frac{h_o D_h}{k}$$

Nusselt number for fully developed laminar flow in an annulus with one surface isothermal and the other adiabatic (Kays and Perkins, Ref. 14)

D_i/D_o	Nu_i	Nu_o
0	—	3.66
0.05	17.46	4.06
0.10	11.56	4.11
0.25	7.37	4.23
0.50	5.74	4.43
1.00	4.86	4.86



Ex1:An air conditioning duct has a cross section of 0.45 by 0.9 m .Air flow in the duct at a velocity of 7.5 m/s at conditions 0f 100 kpa and 300 k .calculate the heat –transfer coefficient for this system and pressure drop per unit length .
Sol:

At 300 K and 1 atm. $\nu = 15.69\times10^{-6}$ $\rho = 1.1774$ $k = 0.02624$

$\Pr = 0.708$ $D_H = \dfrac{(4)(45)(90)}{(2)(45+90)} = 60 \text{ cm} = 0.6 \text{ m}$

$\text{Re} = \dfrac{(0.6)(7.5)}{15.69\times10^{-6}} = 2.87\times10^5$ $\bar{h} = \dfrac{0.02624}{0.6}(0.023)(2.87\times10^5)^{0.8}(0.708)^{0.4}$

$\bar{h} = 20.35 \ \dfrac{\text{W}}{\text{m}^2\cdot{}^\circ\text{C}}$ $\Delta p = f\dfrac{L}{d}\rho\dfrac{u_m{}^2}{2}$ $f = 0.0145$

$$\Delta p = (0.0145)\left(\frac{1}{0.6}\right)\frac{(1.1774)(7.5)^2}{2} = 0.8 \text{ Pa}$$

Ex2: Engine oil at 40 °C enters a 10 –mm- diameter tube at a flow rate such that Reynolds number at entrance 50 .Calculate the exit oil temperature for a tube length of 80 mm and a constant tube wall temperature of 80 °C.(take ρ =876 kg/m³ , Cp=1.964 kj/kg.°C ,υ=0.00024 m²/s ,k =0.144 w/m.°C ,pr=2870)

At 40°C $\rho = 876 \ \text{kg/m}^3$ $c_p = 1.964 \ \dfrac{\text{kJ}}{\text{kg}\cdot{}^\circ\text{C}}$ $\nu = 0.00024$

$k = 0.144$ $\Pr = 2870$ $\text{Re} = 50 = \dfrac{u(0.01)}{0.00024}$ $u = 1.2 \ \text{m/s}$

$$\dot{m} = \frac{(876)(1.2)\pi(0.01)^2}{4} = 0.0826 \ \text{kg/sec}$$

$$\text{Gz} = (50)(2870)\left(\frac{0.01}{0.08}\right) = 1.794\times10^4$$

at $T_w = 80°\text{C}$ $\nu_w = 0.375\times10^{-4}$

$$\text{Nu}_d = (1.86)(1.794\times10^4)^{1/3}\left(\frac{2.4}{0.375}\right)^{0.15} = 64.3$$

$$h = \frac{(64.3)(0.144)}{0.01} = 926$$

$$q = (926)\pi(0.01)(0.08)\left(80 - 20 - \frac{T_e}{2}\right) = (0.0826)(1964)(T_e - 40)$$

$$T_e = 40.57°\text{C}$$

Ex3: Water flowing at 2 kg/s through a 40-mm-diameter tube is to be heated from 25 to 75ºC by maintaining the tube surface temperature at 100ºC. What is the required tube length for these conditions

PROPERTIES: *Table A.6*, Water ($\overline{T}_m$ = 323 K): c_p = 4181 J/kg·K, $\mu = 547 \times 10^{-6}$ N·s/m², k = 0.643 W/m·K, Pr = 3.56.

ANALYSIS: (a) From Eq. 8.6, the Reynolds number is

$$Re_D = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 2\,kg/s}{\pi\,(0.04\,m)547 \times 10^{-6}\,N \cdot s/m^2} = 1.16 \times 10^5. \quad (1)$$

Hence the flow is turbulent, and assuming fully developed conditions throughout the tube, it follows from the Dittus-Boelter correlation, Eq. 8.60,

$$\overline{h} = \frac{k}{D} 0.023\,Re_D^{4/5}\,Pr^{0.4} = \frac{0.643\,W/m \cdot K}{0.04\,m} 0.023\left(1.16 \times 10^5\right)^{4/5}(3.56)^{0.4} = 6919\,W/m^2 \cdot K \quad (2)$$

From Eq. 8.42a, we then obtain

$$L = \frac{-\dot{m} c_p \ell n\left(\Delta T_o / \Delta T_i\right)}{\pi D \overline{h}} = -\frac{2\,kg/s\,(4181\,J/kg \cdot K)\,\ell n\left(25^\circ C / 75^\circ C\right)}{\pi\,(0.04\,m)\,6919\,W/m^2 \cdot K} = 10.6\,m. \quad <$$

Natural convection heat transfer

Many familiar heat transfer applications involve natural convection as the primary mechanism of heat transfer. Some examples are cooling of electronic equipment such as power transistors, TVs, and VCRs; heat transfer from electric baseboard heaters or steam radiators; heat transfer from the refrigeration coils and power transmission lines; and heat transfer from the bodies of animals and human beings. Natural convection in gases is usually accompanied by radiation of comparable magnitude except for low-emissivity surfaces.

We know that a hot boiled egg (or a hot baked potato) on a plate eventually cools to the surrounding air temperature (Fig. 9–1). The egg is cooled by transferring heat by convection to the air and by radiation to the surrounding surfaces. Disregarding heat transfer by radiation, the physical mechanism of cooling a hot egg (or any hot object) in a cooler environment can be explained as follows:

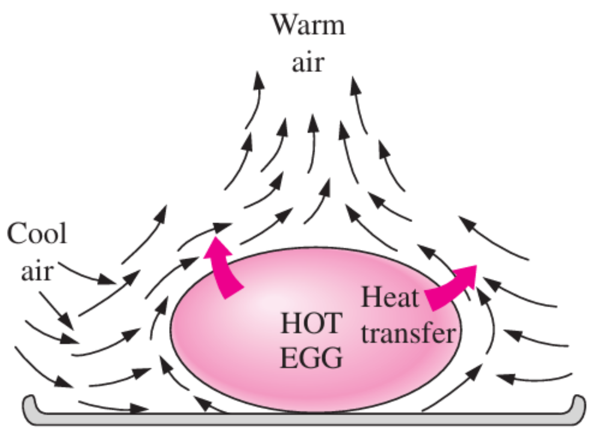


FIGURE 9–1
The cooling of a boiled egg in a cooler environment by natural convection.

As soon as the hot egg is exposed to cooler air, the temperature of the outer surface of the egg shell will drop somewhat, and the temperature of the air adjacent to the shell will rise as a result of heat conduction from the shell to the air. Consequently, the egg will soon be surrounded by a thin layer of warmer air, and heat will then be transferred from this warmer layer to the outer layers of air. The cooling process in this case would be rather slow since the egg would always be blanketed by warm air, and it would have no direct contact with the cooler air farther away. We may not notice any air motion in the vicinity of the egg, but careful measurements indicate otherwise.

The temperature of the air adjacent to the egg is higher, and thus its density is lower, since at constant pressure the density of a gas is inversely proportional to its temperature. Thus, we have a situation in which some low-density or "light" gas is surrounded by a high-density or "heavy" gas, and the natural laws dictate that *the light gas rise.* This is no different than the oil in a vinegar-and-oil salad dressing rising to the top (since $\rho_{oil} < \rho_{vinegar}$). This phenomenon is characterized incorrectly by the phrase "heat rises," which is understood to mean *heated air rises.* The space vacated by the warmer air in the vicinity of the egg is replaced by the cooler air nearby, and the presence of cooler air in the vicinity of the egg speeds up the cooling process. The rise of warmer air and the flow of cooler air into its place continues until the egg is cooled to the temperature of the surrounding air. The motion that results from the continual replacement of the heated air in the vicinity of the egg by the cooler air nearby is called a **natural convection current,** and the heat transfer that is enhanced as a result of this natural convection current is called **natural convection heat transfer.** Note that in the absence of natural convection currents, heat transfer from the egg to the air surrounding it would be by conduction only, and the rate of heat transfer from the egg would be much lower.

Natural convection is just as effective in the heating of cold surfaces in a warmer environment as it is in the cooling of hot surfaces in a cooler environment, as shown in Figure 9–2. Note that the direction of fluid motion is reversed in this case.

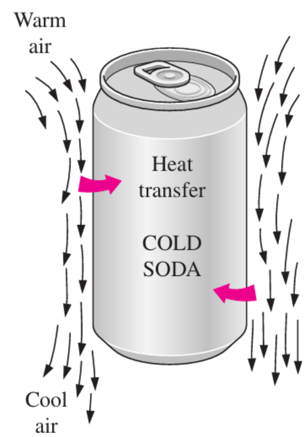


FIGURE 9–2
The warming up of a cold drink in a warmer environment by natural convection.

In heat transfer studies, the primary variable is *temperature,* and it is desirable to express the net buoyancy force (Eq. 9-2) in terms of temperature differences. But this requires expressing the density difference in terms of a temperature difference, which requires a knowledge of a property that represents the *variation of the density of a fluid with temperature at constant pressure. The property that provides that information is the* **volume expansion coefficient** β, defined as (Fig. 9–4)

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \qquad (1/\text{K}) \tag{9-3}$$

In natural convection studies, the condition of the fluid sufficiently far from the hot or cold surface is indicated by the subscript "infinity" to serve as a reminder that this is the value at a distance where the presence of the surface is not felt. In such cases, the volume expansion coefficient can be expressed approximately by replacing differential quantities by differences as

$$\beta \approx -\frac{1}{\rho}\frac{\Delta\rho}{\Delta T} = -\frac{1}{\rho}\frac{\rho_\infty - \rho}{T_\infty - T} \qquad (\text{at constant } P) \tag{9-4}$$

or

$$\rho_\infty - \rho = \rho\beta(T - T_\infty) \qquad (\text{at constant } P) \tag{9-5}$$

where ρ_∞ is the density and T_∞ is the temperature of the quiescent fluid away from the surface.

We can show easily that the volume expansion coefficient β of an *ideal gas* ($P = \rho RT$) at a temperature T is equivalent to the inverse of the temperature:

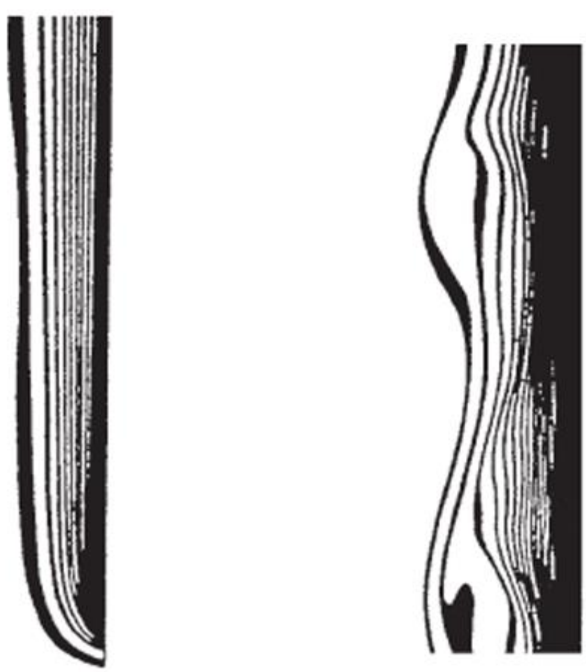
$$\beta_{\text{ideal gas}} = \frac{1}{T} \qquad (1/\text{K}) \tag{9-6}$$

We can show easily that the volume expansion coefficient β of an *ideal gas* ($P = \rho RT$) at a temperature T is equivalent to the inverse of the temperature:

where T is the *absolute* temperature. Note that a large value of β for a fluid means a large change in density with temperature, and that the product $\beta \Delta T$ represents the fraction of volume change of a fluid that corresponds to a temperature change ΔT at constant pressure. Also note that the buoyancy force is proportional to the *density difference,* which is proportional to the *temperature difference* at constant pressure. Therefore, the larger the temperature difference between the fluid adjacent to a hot (or cold) surface and the fluid away from it, the *larger* the buoyancy force and the *stronger* the natural convection currents, and thus the *higher* the heat transfer rate.

Most heat transfer correlations in natural convection are based on experimental measurements. The instrument often used in natural convection

experiments is the *Mach–Zehnder interferometer,* which gives a plot of isotherms in the fluid in the vicinity of a surface. The operation principle of interferometers is based on the fact that at low pressure, the lines of constant temperature for a gas correspond to the lines of constant density, and that the index of refraction of a gas is a function of its density. Therefore, the degree of refraction of light at some point in a gas is a measure of the temperature gradient at that point. An interferometer produces a map of interference fringes, which can be interpreted as lines of *constant temperature* as shown in ıre 9–5. The smooth and parallel lines in (*a*) indicate that the flow is *laminar,* whereas the eddies and irregularities in (*b*) indicate that the flow is *turbulent.* Note that the lines are closest near the surface, indicating a *higher temperature gradient.*



(*a*) Laminar flow (*b*) Turbulent flow

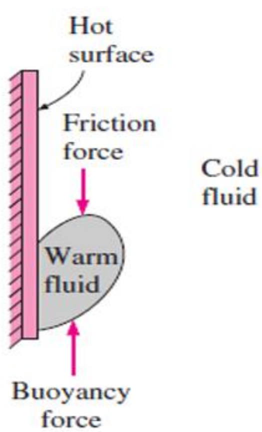
The Grashof Number

The dimensionless parameter in the brackets represents the natural convection effects, and is called the **Grashof number** Gr_L,

$$\mathrm{Gr}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} \qquad 1$$

where

g = gravitational acceleration, m/s^2
β = coefficient of volume expansion, 1/K ($\beta = 1/T$ for ideal gases)
T_s = temperature of the surface, °C
T_∞ = temperature of the fluid sufficiently far from the surface, °C
L_c = characteristic length of the geometry, m
ν = kinematic viscosity of the fluid, m^2/s



We mentioned in the preceding chapters that the flow regime in forced convection is governed by the dimensionless *Reynolds number,* which represents the ratio of inertial forces to viscous forces acting on the fluid. The flow regime in natural convection is governed by the dimensionless *Grashof number,* which represents the ratio of the *buoyancy force* to the *viscous force* acting on the fluid (Fig. 9–8).

NATURAL CONVECTION OVER SURFACES

The simple empirical correlations for the average *Nusselt number* Nu in natural convection are of the form

$$\text{Nu} = \frac{hL_c}{k} = C(\text{Gr}_L \text{Pr})^n = C\,\text{Ra}_L^n$$

where Ra_L is the **Rayleigh number,** which is the product of the Grashof and Prandtl numbers:

$$\text{Ra}_L = \text{Gr}_L \text{Pr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr} \qquad 2$$

Vertical Plates (T_s = constant)

For a vertical flat plate, the characteristic length is the plate height L. In Table 1 we give three relations for the average Nusselt number for an isothermal vertical plate. The first two relations are very simple. Despite its complexity, we suggest using the third one) recommended by Churchill and Chu (1975,) since it is applicable over the entire range of Rayleigh number. This relation is most accurate in the range of $10^{-1} < \text{Ra}_L < 10^9$.

Vertical Cylinders

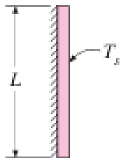
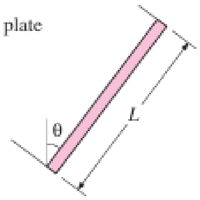
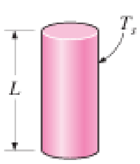
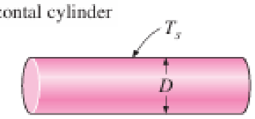
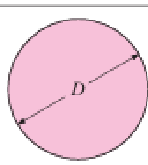
An outer surface of a vertical cylinder can be treated as a vertical plate when the diameter of the cylinder is sufficiently large so that the curvature effects are negligible. This condition is satisfied if

$$D \geq \frac{35L}{\text{Gr}_L^{1/4}}$$

When this criteria is met, the relations for vertical plates can also be used for vertical cylinders. Nusselt number relations for slender cylinders that do not meet this criteria are available in the literature [e.g., Cebeci (1974),

TABLE 9–1

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4–10^9 10^9–10^{13} Entire range	$Nu = 0.59Ra_L^{1/4}$ (9-19) $Nu = 0.1Ra_L^{1/3}$ (9-20) $Nu = \left\{0.825 + \frac{0.387Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2$ (9-21) (complex but more accurate)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g\cos\theta$ for $Ra < 10^9$
Horiontal plate (Surface area A and perimeter p) (*a*) Upper surface of a hot plate (or lower surface of a cold plate)  (*b*) Lower surface of a hot plate (or upper surface of a cold plate) 	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$Nu = 0.54Ra_L^{1/4}$ (9-22) $Nu = 0.15Ra_L^{1/3}$ (9-23) $Nu = 0.27Ra_L^{1/4}$ (9-24)
Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr_L^{1/4}}$
Horizontal cylinder 	D	$Ra_D \leq 10^{12}$	$Nu = \left\{0.6 + \frac{0.387Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}}\right\}^2$ (9-25)
Sphere 	D	$Ra_D \leq 10^{11}$ $(Pr \geq 0.7)$	$Nu = 2 + \frac{0.589Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$ (9-26)

Combined Natural Convection and Radiation

Gases are nearly transparent to radiation, and thus heat transfer through a gas layer is by simultaneous convection (or conduction, if the gas is quiescent) and radiation. Natural convection heat transfer coefficients are typically very low compared to those for forced convection. Therefore, radiation is usually disregarded in forced convection problems, but it must be considered in natural convection problems that involve a gas. This is especially the case for surfaces with high emissivities. For example, about half of the heat transfer through the air space of a double pane window is by radiation. The total rate of heat transfer is determined by adding the convection and radiation components,

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} \qquad \textbf{(9-6:}$$

Ex1: Consider a 0.6m x 0.6m thin square plate in a room at 30°C. One side of the plate is maintained at a temperature of 90°C, while the other side is insulated. Determine the rate of heat transfer from the plate by natural convection if the plate is (*a*) vertical, (*b*) horizontal with hot surface facing up, and (*c*) horizontal with hot surface facing down

Sol:

Properties The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (90 + 30)/2 = 60°C$ and 1 atm are (Table A-15)

$$k = 0.02808 \text{ W/m}\cdot°\text{C} \qquad \text{Pr} = 0.7202$$

$$\nu = 1.896 \times 10^{-5} \text{ m}^2\text{/s} \qquad \beta = \frac{1}{T_f} = \frac{1}{333 \text{ K}}$$

Analysis (*a*) *Vertical.* The characteristic length in this case is the height of the plate, which is $L = 0.6$ m. The Rayleigh number is

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr}$$

$$= \frac{(9.81 \text{ m/s}^2)[1/(333 \text{ K})](90 - 30 \text{ K})(0.6 \text{ m})^3}{(1.896 \times 10^{-5} \text{ m}^2\text{/s})^2}(0.722) = 7.656 \times 10^8$$

Then the natural convection Nusselt number can be determined from Eq. 9-21 to be

$$\text{Nu} = \left\{0.825 + \frac{0.387\,\text{Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2$$

$$= \left\{0.825 + \frac{0.387(7.656 \times 10^8)^{1/6}}{1 + (0.492/0.7202)^{9/16}]^{8/27}}\right\}^2 = 113.4$$

Note that the simpler relation Eq. 9-19 would give $Nu = 0.59\ Ra_L^{1/4} = 98.14$, which is 13 percent lower. Then,

$$h = \frac{k}{L} Nu = \frac{0.02808\ W/m \cdot °C}{0.6\ m}(113.4) = 5.306\ W/m^2 \cdot °C$$

$$A_s = L^2 = (0.6\ m)^2 = 0.36\ m^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.306\ W/m^2 \cdot °C)(0.36\ m^2)(90 - 30)°C = 115\ W$$

(*b*) *Horizontal with hot surface facing up.* The characteristic length and the Rayleigh number in this case are

$$L_c = \frac{A_s}{p} = \frac{L^2}{4L} = \frac{L}{4} = \frac{0.6\ m}{4} = 0.15\ m$$

$$Ra_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} Pr$$

$$= \frac{(9.81\ m/s^2)[1/(333\ K)](90 - 30\ K)(0.15\ m)^3}{(1.896 \times 10^{-5}\ m^2/s)^2}(0.7202) = 1.196 \times 10^7$$

The natural convection Nusselt number can be determined from Eq. 9-22 to be

$$Nu = 0.54\ Ra_L^{1/4} = 0.54(1.196 \times 10^7)^{1/4} = 31.76$$

Then,

$$h = \frac{k}{L_c} Nu = \frac{0.0280\ W/m \cdot °C}{0.15\ m}(31.76) = 5.946\ W/m^2 \cdot °C$$

$$A_s = L^2 = (0.6\ m)^2 = 0.36\ m^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.946\ W/m^2 \cdot °C)(0.36\ m^2)(90 - 30)°C = 128\ W$$

(*c*) *Horizontal with hot surface facing down.* The characteristic length, the heat transfer surface area, and the Rayleigh number in this case are the same as those determined in (*b*). But the natural convection Nusselt number is to be determined from Eq. 9-24,

$$Nu = 0.27\ Ra_L^{1/4} = 0.27(1.196 \times 10^7)^{1/4} = 15.86$$

Then,

$$h = \frac{k}{L_c} Nu = \frac{0.02808\ W/m \cdot °C}{0.15\ m}(15.86) = 2.973\ W/m^2 \cdot °C$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (2.973\ W/m^2 \cdot °C)(0.36\ m^2)(90 - 30)°C = 64.2\ W$$

Ex2 : A horizontal uninsulated steam pipe passes through a large room whose walls and ambient air are at 300 K. The pipe of 150-mm diameter has an emissivity of 0.85 and an outer surface temperature of 400 K. Calculate the heat loss per unit length from the pipe.

Sol:

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2 = 350K$, 1 atm): $\nu = 20.92 \times 10^{-6}\ m^2/s$, $k = 0.030\ W/m{\cdot}K$, $\alpha = 29.9 \times 10^{-6}\ m^2/s$, $Pr = 0.700$, $\beta = 1/T_f = 2.857 \times 10^{-3}\ K^{-1}$.

ANALYSIS: Recognizing that the heat loss from the pipe will be by free convection to the air and by radiation exchange with the surroundings, we can write

$$q' = q'_{conv} + q'_{rad} = \pi D\left[\bar{h}_D\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]. \qquad (1)$$

To estimate $\bar{h}_D$, first find Ra_L, Eq. 9.25, and then use the correlation for a horizontal cylinder, Eq. 9.34,

$$Ra_L = \frac{g\beta\left(T_s - T_\infty\right)D^3}{\nu\alpha} = \frac{9.8m/s^2\left(1/350K\right)\left(400-300\right)K\left(0.150m\right)^3}{20.92\times10^{-6}m^2/s\times29.9\times10^{-6}m^2/s} = 1.511\times10^7$$

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387Ra_L^{1/6}}{\left[1+\left(0.559/Pr\right)^{9/16}\right]^{8/27}}\right\}^2$$

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387\left(1.511\times10^7\right)^{1/6}}{\left[1+\left(0.559/0.700\right)^{9/16}\right]^{8/27}}\right\}^2 = 31.88$$

$$\bar{h}_D = \overline{Nu}_D \cdot k/D = 31.88\times0.030W/m\cdot K/0.15m = 6.38W/m^2\cdot K. \qquad (2)$$

Substituting for h_D from Eq. (2) into Eq. (1), find

$$q' = \pi\left(0.150m\right)\left[6.38W/m^2\cdot K\left(400-300\right)K + 0.85\times5.67\times10^{-8}W/m^2\cdot K^4\left(400^4-300^4\right)K^4\right]$$

$$q' = 301W/m + 397W/m = 698W/m.$$

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