Case I. *Rock wool insulation not used* :

$$Q_1 = \frac{A(\Delta t)}{\frac{L_A}{k_A} + \frac{L_B}{k_B}} = \frac{A(\Delta t)}{\frac{0.1}{0.7} + \frac{0.04}{0.48}} \quad ...(i)$$

Case II. *Rock wool insulation used* :

$$Q_2 = \frac{A(\Delta t)}{\frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C}} = \frac{A(\Delta t)}{\frac{0.1}{0.7} + \frac{0.04}{0.48} + \frac{x}{0.065}} \quad ...(ii)$$

But, $Q_2 = (1 - 0.8)\, Q_1 = 0.2\, Q_1$ ...(*given*)

$$\therefore \quad \frac{A(\Delta t)}{\frac{0.1}{0.7} + \frac{0.04}{0.48} + \frac{x}{0.065}} = 0.2 \times \frac{A(\Delta t)}{\frac{0.1}{0.7} + \frac{0.04}{0.48}}$$

or,
$$\frac{0.1}{0.7} + \frac{0.04}{0.48} = 0.2\left[\frac{0.1}{0.7} + \frac{0.04}{0.48} + \frac{x}{0.065}\right]$$

or, $0.1428 + 0.0833 = 0.2\,[0.1428 + 0.0833 + 15.385x]$

or, $0.2261 = 0.2\,(0.2261 + 15.385\,x)$

or, $x = 0.0588$ m or 58.8 mm

Thus, the *thickness of rock wool insulation should be* **58.8 mm** **(Ans.)**

Example 2.7. *A furnace wall consists of 200 mm layer of refractory bricks, 6 mm layer of steel plate and a 100 mm layer of insulation bricks. The maximum temperature of the wall is 1150°C on the furnace side and the minimum temperature is 40°C on the outermost side of the wall. An accurate energy balance over the furnace shows that the heat loss from the wall is 400 W/m². It is known that there is a thin layer of air between the layers of refractory bricks and steel plate. Thermal conductivities for the three layers are 1.52, 45 and 0.138 W/m°C respectively. Find :*

(i) To how many millimeters of insulation brick is the air layer equivalent?

(ii) What is the temperature of the outer surface of the steel plate?

(AMIE Winter, 1996)

Solution. Refer Fig. 2.15.

Thickness of refractory bricks,

$L_A = 200$ mm $= 0.2$ m

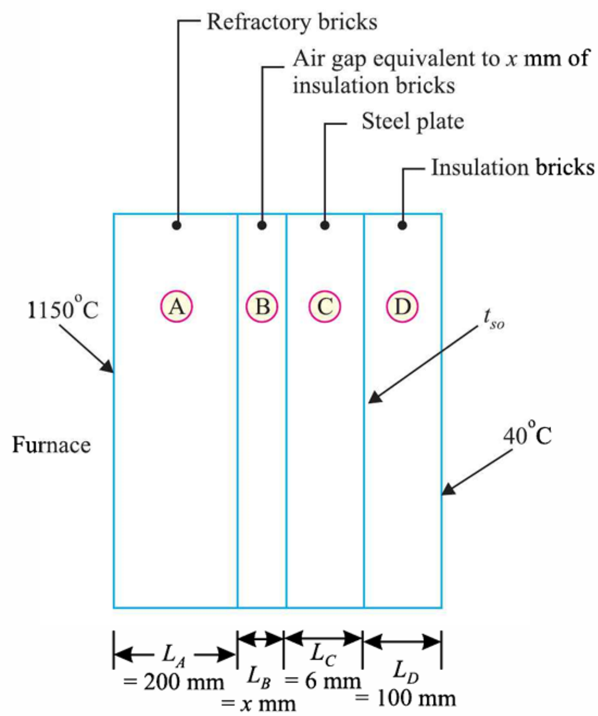


Fig. 2.15.

Thickness of steel plate,

$$L_C = 6 \text{ mm} = 0.006 \text{ m}$$

Thickness of insulation bricks, $L_D = 100 \text{ mm} = 0.1 \text{ m}$

Difference of temperature beetween the innermost and outermost sides of the wall,

$$\Delta t = 1150 - 40 = 1110°\text{C}$$

Thermal conductivities :

$$k_A = 1.52 \text{ W/m°C}; \quad k_B = k_D = 0.138 \text{ W/m°C}; \quad k_C = 45 \text{ W/m°C}$$

Heat loss from the wall, $q = 400 \text{ W/m}^2$

(*i*) **The value of *x* (= L_C) :**

We know,
$$Q = \frac{A \cdot \Delta t}{\Sigma \frac{L}{k}} \quad \text{or} \quad \frac{Q}{A} = q = \frac{\Delta t}{\Sigma \frac{L}{k}}$$

or,
$$400 = \frac{1110}{\frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{L_D}{k_D}}$$

or,
$$400 = \frac{1110}{\frac{0.2}{1.52} + \frac{(x/1000)}{0.138} + \frac{0.006}{45} + \frac{0.1}{0.138}}$$

$$= \frac{1110}{0.1316 + 0.0072x + 0.00013 + 0.7246} = \frac{1110}{0.8563 + 0.0072x}$$

or,
$$0.8563 + 0.0072\,x = \frac{1110}{400} = 2.775$$

or,
$$x = \frac{2.775 - 0.8563}{0.0072} = \mathbf{266.5 \text{ mm}} \quad \textbf{(Ans.)}$$

(*ii*) **Temperature of the outer surface of the steel plate t_{so}:**

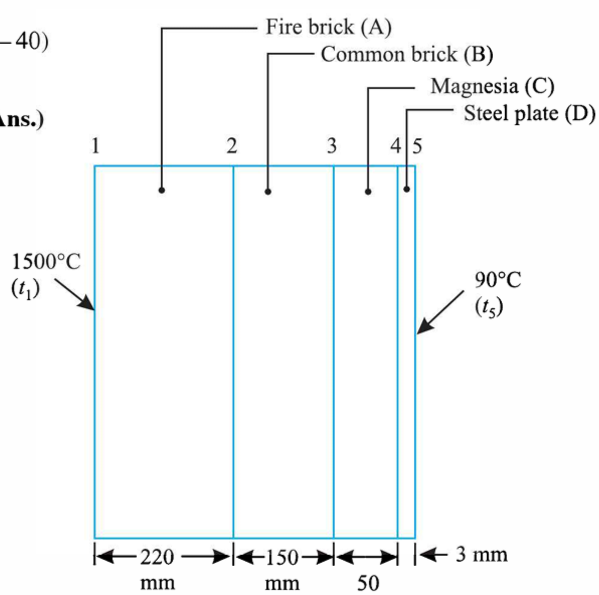
$$q = 400 = \frac{(t_{so} - 40)}{L_D / k_D}$$

or,
$$400 = \frac{(t_{so} - 40)}{(0.1/0.138)} = 1.38\,(t_{so} - 40)$$

or,
$$t_{so} = \frac{400}{1.38} + 40 = \mathbf{329.8°C} \quad \textbf{(Ans.)}$$

Example 2.8. *A furnace wall is composed of 220 mm of fire brick, 150 mm of common brick, 50 mm of 85% magnesia and 3 mm of steel plate on the outside. If the inside surface temperature is 1500°C and outside surface temperature is 90°C, estimate the temperatures between layers and calculate the heat loss in kJ/h- m². Assume, k (for fire brick) = 4kJ/m-h-°C, k(for common brick) = 2.8 kJ/m-h-°C, k (for 85% magnesia) = 0.24 kJ/m-h-°C, and k (steel) = 240 kJ/m-h-°C.*

(AMIE, Winter, 1997)



Solution. *Given* : $L_A = 220$ mm = 0.22 m; $L_B = 150$ mm = 0.15 m; L_C = 50 mm = 0.05m; $L_D = 3$ mm = 0.003 m

$t_1 = 1500°C$, $t_5 = 90°C$;
$k_A = 4$ kJ/mh°C; $k_B = 2.8$ kJ/mh°C
$k_C = 0.24$ kJ/mh°C; $k_D = 240$ kJ/mh°C.

Heat loss in kJ/hm² :

The equivalent thermal resistances of various layers are :

$$R_{th-A} = \frac{L_A}{k_A} = \frac{0.22}{4} = 0.055 \text{ m}^2\text{h°C/kJ}$$

$$R_{th-B} = \frac{L_B}{k_B} = \frac{0.15}{2.8} = 0.05357 \text{ m}^2\text{h°C/kJ}$$

$$R_{th-C} = \frac{L_C}{k_C} = \frac{0.05}{0.24} = 0.2083 \text{ m}^2\text{h°C/kJ}$$

$$R_{th-D} = \frac{L_D}{k_D} = \frac{0.003}{240} = 1.25 \times 10^{-5} \text{ m}^2\text{h°C/kJ}$$

Total thermal resistance,

$$(R_{th})_{total} = 0.055 + 0.05357 + 0.2083 + 1.25 \times 10^{-5} = 0.3169 \text{ m}^2\text{h°C/kJ}$$

Heat loss, $$q = \frac{(t_1 - t_5)}{(R_{th})_{total}} = \frac{(1500 - 90)}{0.3169} = \mathbf{4449.35\ kJ/hm^2} \qquad \textbf{(Ans.)}$$

Example 2.16. *Find the heat flow rate through the composite wall as shown in Fig. 2.24 Assume one dimensional flow.*

$k_A = 150$ *W/m°C,*
$k_B = 30$ *W/m°C,*
$k_C = 65$ *W/m°C, and*
$k_D = 50$ *W/m°C*

(M.U. Winter, 2000)

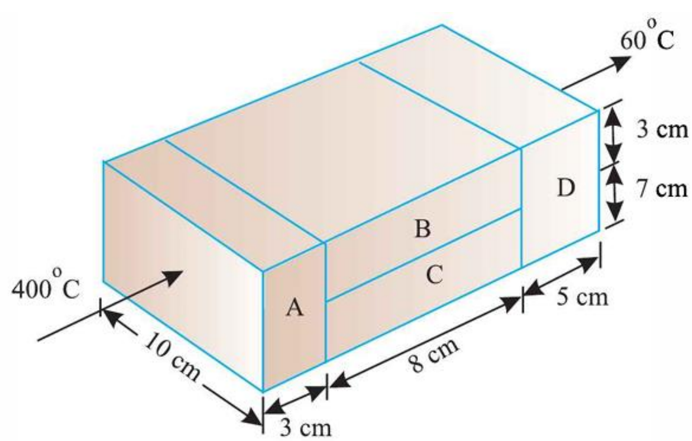


Fig. 2.24.

Solution. The thermal circuit for heat flow in the given composite system (shown in Fig. 2.24) has been illustrated in Fig. 2.25.

Thickness :

$L_A = 3$ cm $= 0.03$ m; $L_B = L_C = 8$ cm $= 0.08$ m; $L_D = 5$ cm $= 0.05$ m

Areas :

$$A_A = 0.1 \times 0.1 = 0.01 \text{m}^2 ; \qquad A_B = 0.1 \times 0.03 = 0.003 \text{m}^2$$
$$A_C = 0.1 \times 0.07 = 0.007 \text{m}^2 ; \qquad A_D = 0.1 \times 0.1 = 0.01 \text{m}^2$$

Heat flow rate, Q :

The thermal resistances are given by,

$$R_{th-A} = \frac{L_A}{k_A A_A} = \frac{0.03}{150 \times 0.01} = 0.02$$
$$R_{th-B} = \frac{L_B}{k_B A_B} = \frac{0.08}{30 \times 0.003} = 0.89$$

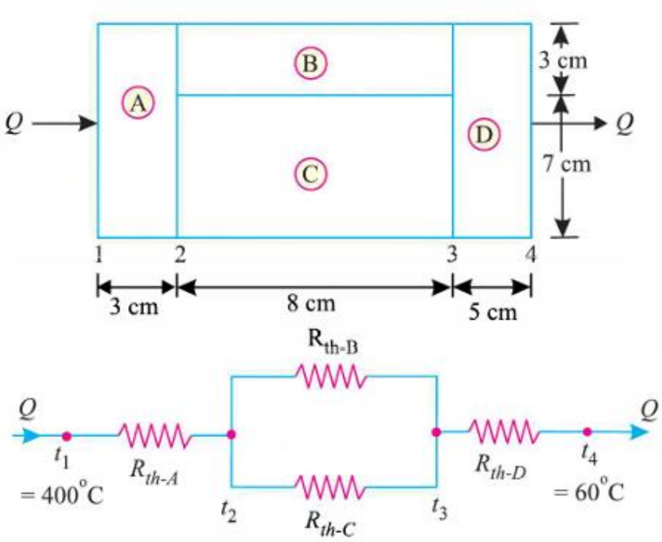


Fig. 2.25. Thermal circuit.

$$R_{th-C} = \frac{L_C}{k_C A_C} = \frac{0.08}{65 \times 0.007} = 0.176$$
$$R_{th-D} = \frac{L_D}{k_D A_D} = \frac{0.05}{50 \times 0.01} = 0.1$$

The equivalent thermal resistance for the parallel thermal resistance R_{th-B} and R_{th-C} is given by

$$\frac{1}{(R_{th})_{eq}} = \frac{1}{R_{th-B}} + \frac{1}{R_{th-C}} = \frac{1}{0.89} + \frac{1}{0.176} = 6.805$$

$$\therefore \qquad (R_{th})_{eq.} = \frac{1}{6.805} = 0.147$$

Now, the total thermal resistance is given by

$$(R_{th})_{\text{total}} = R_{th-A} + (R_{th})_{\text{eq.}} + R_{th-D} = 0.02 + 0.147 + 0.1 = 0.267$$

$$\therefore \qquad Q = \frac{(\Delta t)_{overall}}{(R_{th})_{total}} = \frac{(400 - 60)}{0.267} = \textbf{1273.4 W (Ans.)}$$

Example 2.24. *The interior of a refrigerator having inside dimensions of 0.5 m × 0.5 m base area and 1m height, is to be maintained at 6°C. The walls of the refrigerator are constructed of two mild steel sheets 3mm thick (k = 46.5 W/m°C) with 50 mm of glass wool insulation (k = 0.046 W/m°C) between them. If the average heat transfer coefficients at the outer and inner surfaces are 11.6 W/m²°C and 14.5 W/m²°C respectively, calculate :*

(*i*) *The rate at which heat must be removed from the interior to maintain the specified temperature in the kitchen at 25°C, and*

(*ii*) *The temperature on the outer surface of the metal sheet.*

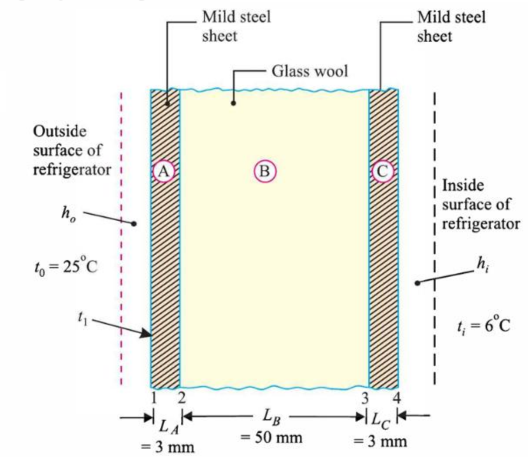


Fig. 2.34.

Solution. Refer to Fig. 2.34.

$$L_A = L_C = 3\text{mm} = 0.003\text{ m}$$
$$L_B = 50\text{ mm} = 0.05\text{ m}$$
$$k_A = k_C = 46.5\text{ W/m°C};$$
$$k_B = 0.046\text{ W/m° C}$$
$$h_0 = 11.6\text{ W/m}^2\text{°C};\ h_i = 14.5\text{ W/m}^2\text{°C}$$
$$t_0 = 25°\text{C};\ t_i = 6°\text{C}.$$

The total area through which heat is coming into the refrigerator,

$$A = 0.5 \times 0.5 \times 2 + 0.5 \times 1 \times 4 = 2.5\text{ m}^2$$

(*i*) The rate of removal of heat, *Q* :

$$Q = \frac{A(t_0 - t_i)}{\frac{1}{h_0} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_i}}$$
$$= \frac{2.5(25-6)}{\frac{1}{11.6} + \frac{0.003}{46.5} + \frac{0.05}{0.046} + \frac{0.003}{46.5} + \frac{1}{14.5}} = \mathbf{38.2\ W} \quad \textbf{(Ans.)}$$

(*ii*) The temperature at the outer surface of the metal sheet, *t* :

$$Q = h_0 A\,(25 - t_1)$$

or, $$38.2 = 11.6 \times 2.5\,(25 - t_1)$$

or, $$t_1 = 25 - \frac{38.2}{11.6 \times 2.5} = \mathbf{23.68°\ C} \quad \textbf{(Ans.)}$$

Example 2.25. *Calculate the rate of heat flow per m^2 through a furnace wall consisting of 200 mm thick inner layer of chrome brick, a centre layer of kaolin brick 100 mm thick and an outer layer of masonry brick 100 mm thick. The unit surface conductance at the inner surface is 74 W/m²°C and the outer surface temperature is 70°C. The temperature of the gases inside the furnace is 1670°C. What temperatures prevail at the inner and outer surfaces of the centre layer ?*

Take : $k_{chrome\ brick} = 1.25$ W/m°C; $k_{kaolin\ brick} = 0.074$ W/m°C; $k_{masonry\ brick} = 0.555$ W/m°C
Assume steady heat flow. **(M.U.)**

Solution. Thickness of chrome bricks, $L_A = 200$ mm $= 0.2$ m

Thickness of kaolin bricks, $L_B = 100$ mm $= 0.1$ m

Thickness of masonry bricks, $L_C = 100$ mm $= 0.1$m

Thermal conductivities : $k_A = 1.25$ W/m°C;

$k_B = 0.074$ W/m°C; $k_C = 0.555$ W/m°C;

The unit surface conductance, $h_{hf} = 74$ W/m²°C

Temperature of hot fluid, $t_{hf} (= t_g) = 1670$°C

Temperature of the outer surface, $t_4 = 70$°C

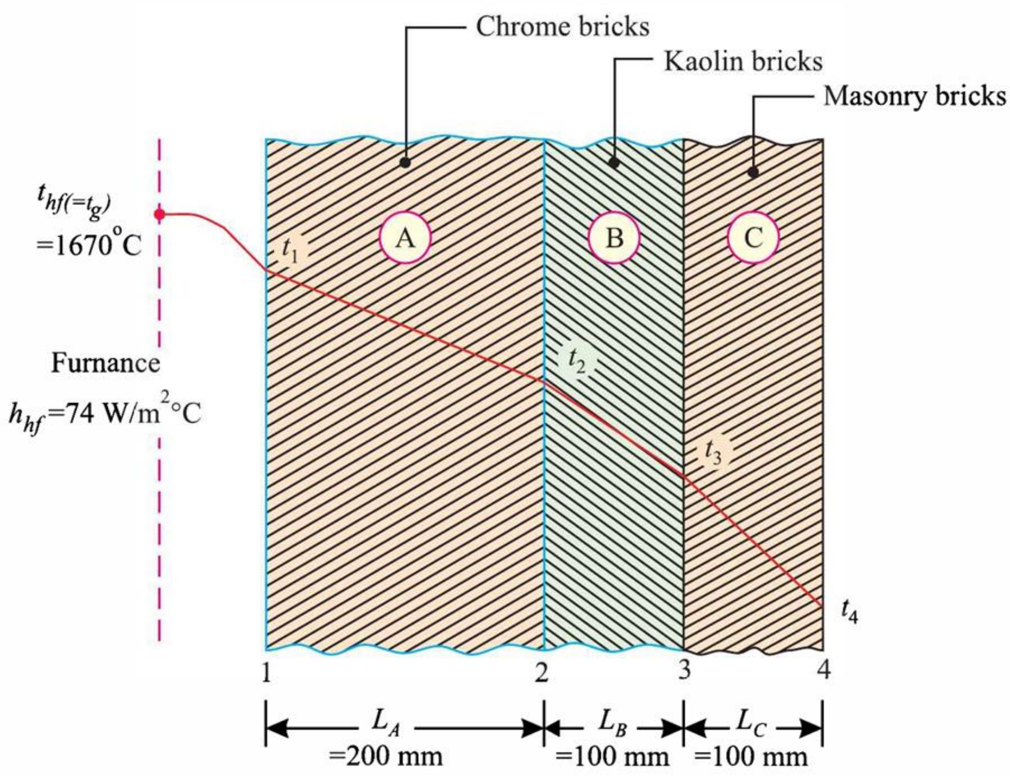


Fig. 2.35

(*i*) Rate of heat flow per m², q :

$$q = \frac{(t_{hf} - t_4)}{\dfrac{1}{h_{hf}} + \dfrac{L_A}{k_A} + \dfrac{L_B}{k_B} + \dfrac{L_C}{k_C}}$$

$$= \frac{(1670 - 70)}{\dfrac{1}{74} + \dfrac{0.2}{1.25} + \dfrac{0.1}{0.074} + \dfrac{0.1}{0.555}}$$

$$= \frac{1600}{0.0135 + 0.16 + 1.351 + 0.1802}$$

$$= \mathbf{938.58\ W/m^2} \qquad \textbf{(Ans.)}$$

(*ii*) Temperatures; t_2, t_3 :

The heat flow is given by

$$q = \frac{(t_{hf} - t_1)}{1/h_{hf}} = \frac{(t_1 - t_2)}{L_A / k_A} = \frac{(t_2 - t_3)}{(L_B / k_B)}$$

$$\therefore \qquad 938.58 = \frac{1670 - t_1}{1/74}$$

or $$t_1 = 1670 - 938.58 \times \frac{1}{74} = 1657.3°C$$

Similarly, $$938.58 = \frac{(1657.3 - t_2)}{0.2/1.25} \quad \text{or} \quad t_2 = 1657.3 - 938.58 \times \frac{0.2}{1.25} = \mathbf{1507.1°C} \quad \textbf{(Ans)}$$

$$938.58 = \frac{(1507.1 - t_3)}{(0.1/0.074)} \quad \text{or} \quad t_3 = 1507.1 - 938.58 \times \frac{0.1}{0.074} = \mathbf{238.7°C} \quad \textbf{(Ans.)}$$

Example 2.26. *A cold storage room has walls made of 220 mm of brick on the outside, 90 mm of plastic foam, and finally 16 mm of wood on the inside. The outside and inside air temperatures are 25°C and –3°C respectively. If the inside and outside heat transfer coefficients are respectively 30 and 11 W/m²°C, and the thermal conductivities of brick, foam and wood are 0.99, 0.022 and 0.17 W/m°C respectively, determine :*

(i) *The rate of heat removal by refrigeration if the total wall area is 85 m²;*

(ii) *The temperature of the inside surface of the brick.*

Solution. Refer Fig. 2.36.

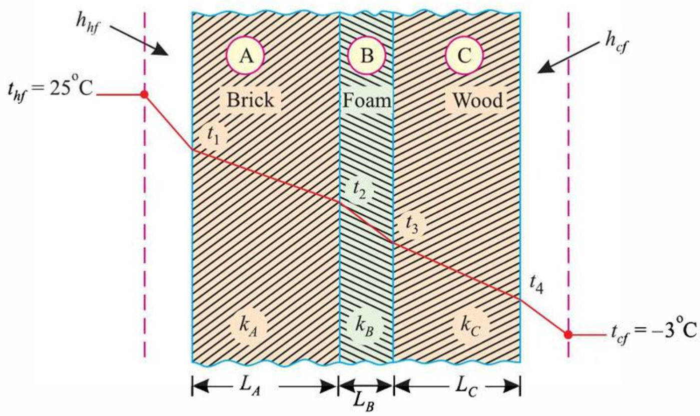


Fig. 2.36.

Thickness of brick wall, L_A = 220 mm = 0.22 m

Thickness of plastic foam, L_B = 90 mm = 0.09 m

Thickness of wood, L_C = 16 mm = 0.016 m

Temperature of hot fluid (air), t_{hf} = 25°C

Temperature cold fluid (air), t_{cf} = –3°C

Heat transfer coefficients :

Hot fluid (air), h_{hf} = 11 W/m²°C

Cold fluid (air), h_{cf} = 30 W/m²°C

Thermal conductivities :

Brick, k_A = 0.99 W/m°C

Foam, k_B = 0.022 W/m°C

Wood, k_C = 0.17 W/m°C

Total wall area, A = 85 m²

(*i*) Rate of heat transfer, *Q* :

$$Q = UA\,(t_{hf} - t_{cf})$$

The overall heat transfer co-efficient (U) may be found from the following relation :

$$\frac{1}{U} = \frac{1}{h_{hf}} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_{cf}}$$

$$= \frac{1}{11} + \frac{0.22}{0.99} + \frac{0.09}{0.022} + \frac{0.06}{0.17} + \frac{1}{30}$$

$$= 0.091 + 0.222 + 4.091 + 0.094 + 0.033 = 4.531$$

$\therefore$ $$U = \frac{1}{4.531} = 0.2207 \text{ W/m}^2\,°\text{C}$$

$\therefore$ $$Q = 0.2207 \times 85\,[25 - (-3)] = \mathbf{525.26\ W} \quad \textbf{(Ans.)}$$

(*ii*) Temperature of inside surface of the brick, t_2 :

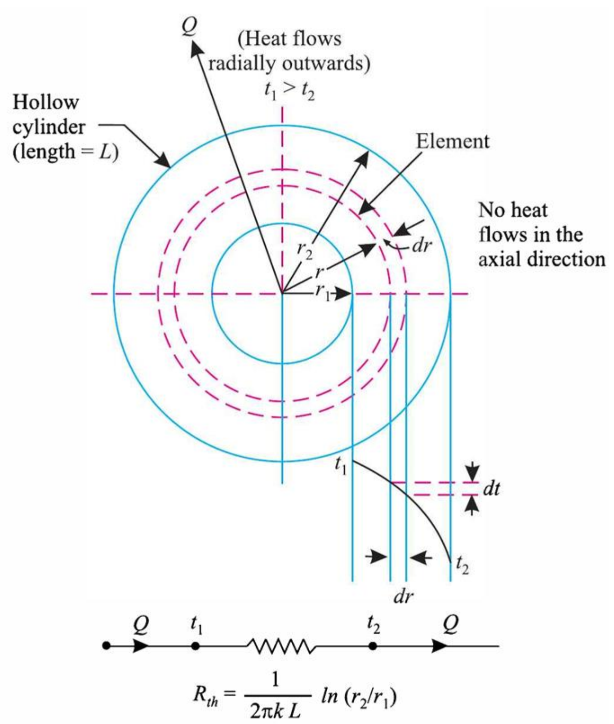
$$Q = \text{U.A}\,(t_{hf} - t_2)$$

or, $$525.26 = \left[\frac{1}{\frac{1}{h_{hf}} + \frac{L_A}{k_A}}\right] A\,(t_{hf} - t_2)$$

$$= \left[\frac{1}{\frac{1}{11} + \frac{0.22}{0.99}}\right] \times 85\,(25 - t_2) = 271.45\,(25 - t_2)$$

$\therefore$ $$t_2 = 25 - \frac{525.26}{271.45} = \mathbf{23.06°C} \quad \textbf{(Ans.)}$$

Heat Conduction through hollow and composite cylinders



$$Q = \frac{(t_1 - t_2)}{\frac{\ln(r_2 / r_1)}{2\pi k L}}$$

Heat Conduction through composite cylinders

Consider flow of heat through a composite cylinder as shown in Fig. 2.47.

Let, t_{hf} = The temperature of the hot fluid flowing inside the cylinder,

t_{cf} = The temperature of the cold fluid (atmospheric air),

k_A = Thermal conductivity of the inside layer *A*,

k_B = Thermal conductivity of the outside layer *B*,

t_1, t_2, t_3 = Temperatures at the points 1, 2, and 3 (see Fig. 2.47)

L = Length of the composite cylinder, and

h_{hf}, h_{cf} = Inside and outside heat transfer coefficients.

The rate of heat transfer is given by

$$Q = h_{hf}\,.2\pi r_1\,.L(t_{hf} - t_1) = \frac{k_A\,.2\pi L(t_1 - t_2)}{\ln(r_2/r_1)}$$

$$= \frac{k_B\,.2\pi L(t_2 - t_3)}{\ln(r_3/r_2)} = h_{cf}\,.2\pi r_3\,.L(t_3 - t_{cf})$$

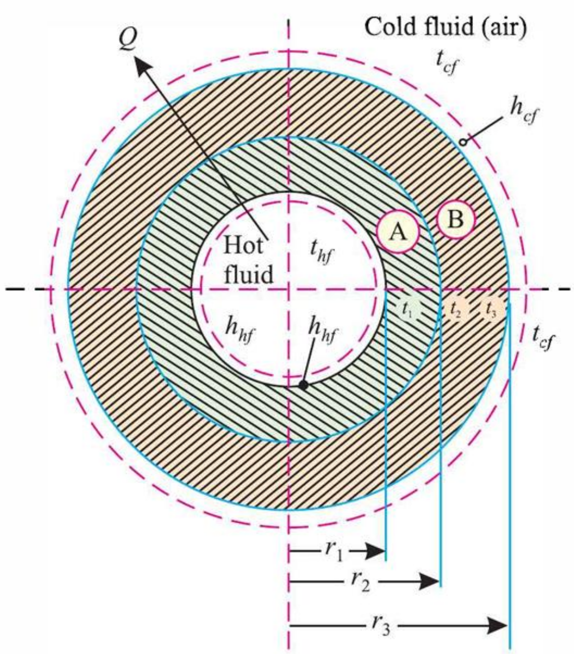


Fig. 2.47.

Rearranging the above expression, we get

$$t_{hf} - t_1 = \frac{Q}{h_{hf} . r_1 . 2\pi L} \quad ...(i)$$

$$t_1 - t_2 = \frac{Q}{\dfrac{k_A . 2\pi L}{\ln(r_2 / r_1)}} \quad ...(ii)$$

$$t_2 - t_3 = \frac{Q}{\dfrac{k_B . 2\pi L}{\ln(r_3 / r_2)}} \quad ...(iii)$$

$$t_3 - t_{cf} = \frac{Q}{h_{cf} . r_3 . 2\pi L} \quad ...(iv)$$

Adding (*i*), (*ii*), (*iii*) and (*iv*), we have

$$\frac{Q}{2\pi L}\left[\frac{1}{h_{hf} . r_1} + \frac{1}{\dfrac{k_A}{\ln(r_2/r_1)}} + \frac{1}{\dfrac{k_B}{\ln(r_3/r_2)}} + \frac{1}{h_{cf} . r_3}\right] = t_{hf} - t_{cf}$$

$\therefore$

$$Q = \frac{2\pi L(t_{hf} - t_{cf})}{\left[\dfrac{1}{h_{hf} . r_1} + \dfrac{1}{\dfrac{k_A}{\ln(r_2/r_1)}} + \dfrac{1}{\dfrac{k_B}{\ln(r_3/r_2)}} + \dfrac{1}{h_{cf} . r_3}\right]}$$

or,

$$Q = \frac{2\pi L(t_{hf} - t_{cf})}{\left[\dfrac{1}{h_{hf} . r_1} + \dfrac{\ln(r_2/r_1)}{k_A} + \dfrac{\ln(r_3/r_2)}{k_B} + \dfrac{1}{h_{cf} . r_3}\right]} \quad ...(2.69)$$

If there are '*n*' concentric cylinders, then

$$Q = \frac{2\pi L(t_{hf} - t_{cf})}{\left[\dfrac{1}{h_{hf} . r_1} + \sum_{n=1}^{n=n} \dfrac{1}{k_n} \ln\left\{r_{(n+1)} / r_n\right\} + \dfrac{1}{h_{cf} . r_{(n+1)}}\right]} \quad ...(2.70)$$

If inside and outside heat transfer coefficients are not considered then the above equation can be written as

$$Q = \frac{2\pi L\left[t_1 - t_{(n+1)}\right]}{\sum_{n=1}^{n=n} \dfrac{1}{k_n} \ln\left[r_{(n+1)} / r_n\right]} \quad ...(2.71)$$

Example 2.35. *A thick walled tube of stainless steel with 20 mm inner diameter and 40 mm outer diameter is convered with a 30 mm layer of asbestos insulation (k = 0.2 W/m°C). If the inside wall temperature of the pipe is maintained at 600°C and the outside insulation at 1000°C, calculate the heat loss per metre of length.* **(AMIE Summer, 1997)**

Solution. Refer to Fig. 2.48.

Given:

$$r_1 = \frac{20}{2} = 10\,\text{mm} = 0.01\text{ m}$$

$$r_2 = \frac{40}{2} = 20\,\text{mm} = 0.02\text{ m}$$

$$r_3 = 20 + 30 = 50\,\text{mm} = 0.05\text{m}$$

$$t_1 = 600°\text{C}$$

$$t_3 = 1000°\text{C}$$

$$k_B = 0.2\,\text{W/m°C}$$

Heat transfer per metre of a length, Q/L:

$$Q = \frac{2\pi L(t_1 - t_3)}{\frac{\ln(r_2/r_1)}{k_A} + \frac{\ln(r_3/r_2)}{k_B}}$$

Since the thermal conductivity of stainless steel is not given, therefore, neglecting the resistance offered by stainless steel to heat transfer across the tube, we have

$$\frac{Q}{L} = \frac{2\pi(t_1 - t_3)}{\frac{\ln(r_3/r_2)}{k_B}} = \frac{2\pi(600 - 1000)}{\frac{\ln(0.05/0.02)}{0.2}}$$

$$= \mathbf{-548.57\ W/m\ \ (Ans.)}$$

Negative sign indicates that the heat transfer takes place *radially inward.*

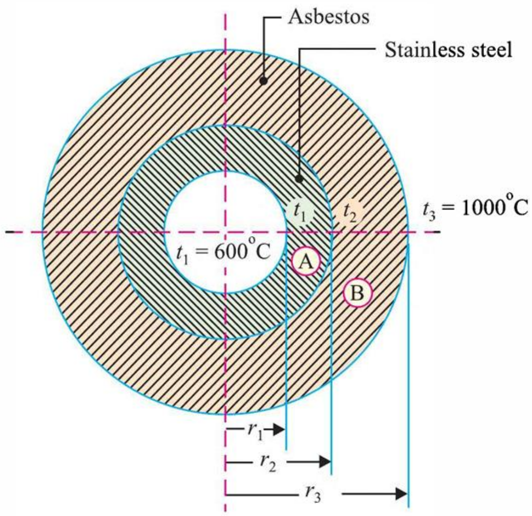


Fig. 2.48.

Example 2.36. *A steel pipe with 50 mm OD is covered with a 6.4 mm asbestos insulation [k = 0.166 W/mK] followed by a 25 mm layer of fiber-glass insulation [k = 0.0485 W/mK]. The pipe wall temperature is 393 K and the outside insulation temperature is 311 K. Calculate the interface temperature between the asbestos and fiber-glass.*

(AMIE Summer, 1998)

Solution.

Given : $r_1 = \frac{50}{2} = 25\,\text{mm} = 0.025\,\text{m};$

$$r_2 = r_1 + 6.4 = 25 + 6.4 = 31.4 \text{ mm or } 0.0314 \text{ m};$$

$$r_3 = r_2 + 25 = 31.4 + 25 = 56.4 \text{ mm} = 0.0564 \text{ m};$$

$$T_1 = 393 \text{ K}; T_3 = 311 \text{ K}$$

$$k_A = 0.166 \text{ W/mK};$$

$$k_B = 0.0485 \text{ W/mK}.$$

Interface temperature between the asbestos and fiber-glass, t_2 :

We know that, $$Q = \frac{2\pi L(T_1 - T_3)}{\frac{\ln(r_2/r_1)}{k_A} + \frac{\ln(r_3/r_2)}{k_B}}$$

$$\frac{Q}{L} = \frac{2\pi(T_1 - T_3)}{\frac{\ln(r_2/r_1)}{k_A} + \frac{\ln(r_3/r_2)}{k_B}}$$

$$= \frac{2\pi(393 - 311)}{\frac{\ln(0.0314/0.025)}{0.166} + \frac{\ln(0.0564/0.0314)}{0.0485}}$$

$$= \frac{515.22}{1.373 + 12.075} = 38.31\,\text{W/m}$$

Also, $$\frac{Q}{L} = \frac{2\pi(T_1 - T_2)}{\frac{\ln(r_2/r_1)}{k_A}}$$

or, $$38.31 = \frac{2\pi(393 - T_2)}{\left[\frac{\ln(0.0314/0.025)}{0.166}\right]}$$

$$38.31 = \frac{2\pi(393 - T_2)}{1.373}$$

$$\therefore \quad T_2 = 393 - \frac{38.31 \times 1.373}{2\pi} = 384.6\,\text{K}$$

or, $t_2 = 384.6 - 273 = $ **111.6° C** **(Ans.)**

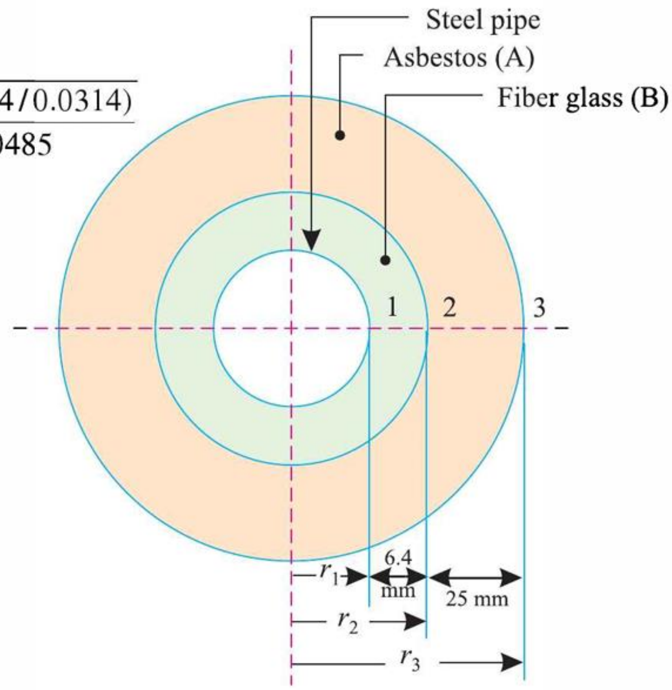


Fig. 2.49.

Example 2.48. *Hot air at a temperature of 65°C is flowing through a steel pipe of 120 mm diameter. The pipe is covered with two layers of different insulating materials of thickness 60 mm and 40 mm, and their corresponding thermal conductivities are 0.24 and 0.4 W/m°C. The inside and outside heat transfer coefficients are 60 W/m°C and 12 W/m°C respectively. The atmosphere is at 20°C. Find the rate of heat loss from 60 m length of pipe.*

Solution. Refer to Fig. 2.61.

Given : $r_1 = \frac{120}{2} = 60\,\text{mm} = 0.06\,\text{m}$

$r_2 = 60 + 60 = 120\,\text{mm} = 0.12\,\text{m}$

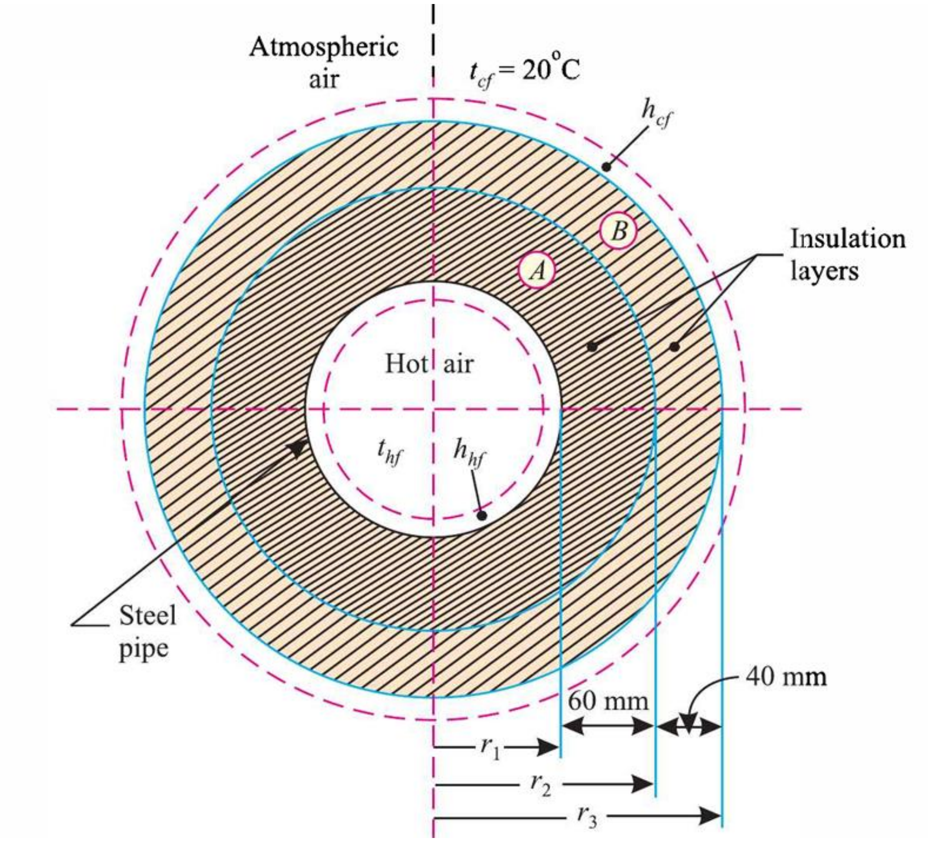
$r_3 = 60 + 60 + 40 = 160\,\text{mm} = 0.16\,\text{m}$

$k_A = 0.24\,\text{W/m°C}$; $k_B = 0.4\,\text{W/m°C}$

$h_{hf} = 60\,\text{W/m}^2\text{°C}$; $h_{cf} = 12\,\text{W/m}^2\text{°C}$

$t_{hf} = 65°\text{C}$; $t_{cf} = 20°\text{C}$

Length of pipe, $L = 60$ m



Rate of heat loss, Q :

Rate of heat loss is given by

$$Q = \frac{2\pi L (t_{hf} - t_{cf})}{\left[\frac{1}{h_{hf} \cdot r_1} + \frac{\ln(r_2/r_1)}{k_A} + \frac{\ln(r_3/r_2)}{k_B} + \frac{1}{h_{cf} \cdot r_3}\right]} \quad \text{[Eqn. (2.69)]}$$

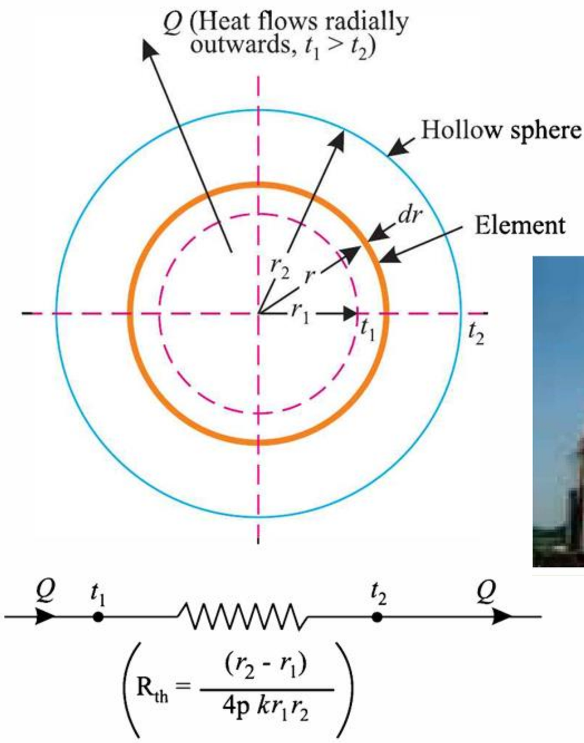
$$= \frac{2\pi \times 60 (65 - 20)}{\left[\frac{1}{60 \times 0.06} + \frac{\ln(0.12/0.06)}{0.24} + \frac{\ln(0.16/0.12)}{0.4} + \frac{1}{12 \times 0.16}\right]}$$

$$= \frac{16964.6}{0.2777 + 2.8881 + 0.7192 + 0.5208} = 3850.5 \text{ W}$$

i.e., Rate of heat loss = **3850.5 W (Ans.)**

Heat conduction through hollow sphere

Refer Fig. 2.76. Consider a hollow sphere made of material having constant thermal conductivity.



Spherical vessels.

Fig. 2.76. Steady state conduction through a hollow sphere.

i.e.

$$Q = \frac{(t_1 - t_2)}{\left[\dfrac{(r_2 - r_1)}{4\pi k\, r_1\, r_2}\right]}\left[= \frac{\Delta t}{R_{th}}\right] \quad ...(2.76)$$

where the term $(r_2 - r_1) / 4\pi\, k\, r_1\, r_2$ is the thermal resistance (R_{th}) for heat conduction through a hollow sphere.

Heat conduction through composite sphere

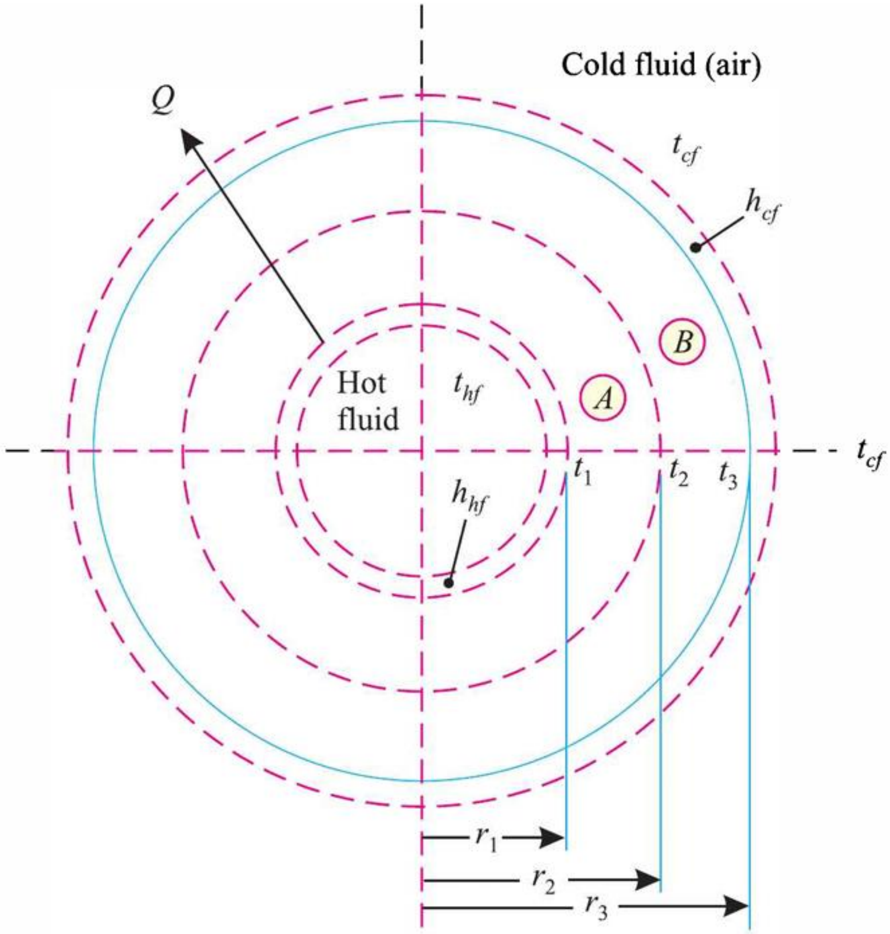


Fig. 2.77. Steady state conduction through a composite sphere.

If there are n concentric spheres then the above equation can be written as follows

$$Q = \frac{4\pi(t_{hf} - t_{cf})}{\left[\frac{1}{h_{hf} \cdot r_1^2} + \sum_{n=1}^{n=n} \left\{\frac{r_{(n+1)} - r_n}{k_n \cdot r_n \cdot r_{(n+1)}}\right\} + \frac{1}{h_{cf} \cdot r_{(n+1)}^2}\right]} \quad \text{...(2.83)}$$

If inside and outside heat transfer coefficients are not considered, then the above equation can be written as follows :

$$Q = \frac{4\pi(t_1 - t_{n+1})}{\sum_{n=1}^{n=n} \left[\frac{r_{(n+1)} - r_n}{k_n \cdot r_n \cdot r_{(n+1)}}\right]} \quad \text{...(2.84)}$$

Example 2.61. *A spherical shaped vessel of 1.4 m diameter is 90 mm thick. Find the rate of heat leakage, if the temperature difference between the inner and outer surfaces is 220°C. Thermal conductivity of the material of the sphere is 0.083 W/m°C.*

Solution. Refer to Fig. 2.78.

$$r_2 = \frac{1.4}{2} = 0.7\,\text{m.}$$

$$r_1 = 0.7 - \frac{90}{1000} = 0.61\,\text{m}$$

$$t_1 - t_2 = 220°\text{C};$$

$$k = 0.083\ \text{W/m°C}$$

The rate of heat transfer/leakage is given by

$$Q = \frac{(t_1 - t_2)}{\left[\frac{(r_2 - r_1)}{4\pi k\, r_1 r_2}\right]} \qquad \text{...Fig. (2.76)}$$

$$= \frac{220}{\left[\frac{(0.7 - 0.61)}{4\pi \times 0.083 \times 0.61 \times 0.7}\right]}$$

$$= 1088.67\ \text{W}$$

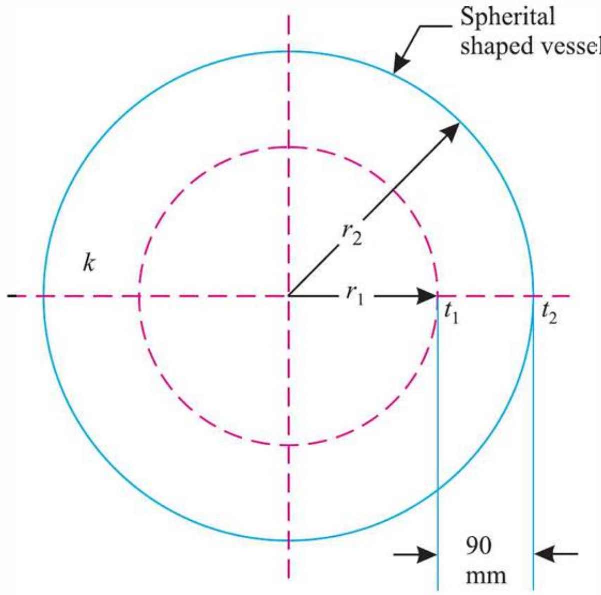


Fig. 2.78.

i.e., Rate of heat leakage **= 1088.67 W** **(Ans.)**