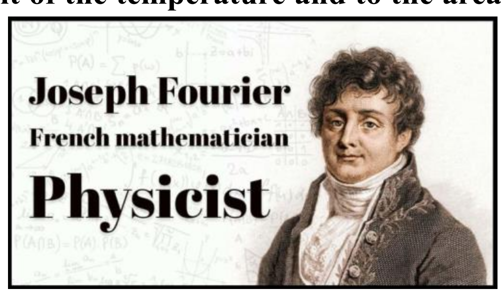
Basic modes of heat transfer

- we defined **heat** as the form of energy that can be transferred from one system to another as a result of temperature difference.
- A **thermodynamic** analysis is concerned with the amount of heat transfer as a system undergoes a process from one equilibrium state to another.
- The science that deals with the determination of the rates of such energy transfers is the **heat transfer**.
- The transfer of energy as heat is always from the higher-temperature medium to the lower-temperature one, and heat transfer stops when the two mediums reach the same temperature.
- Heat can be transferred in **three different modes**: conduction, convection, and radiation.
- All modes of heat transfer require the existence of a temperature difference, and all modes are from the high-temperature medium to a lower-temperature one.
- Below we give a brief description of each mode. A detailed study of these modes is given in later chapters of this text.

CONDUCTION HEAT TRANSFER

- Conduction is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles. Conduction can take place in solids, liquids, or gases.
- **Fourier's Law of Heat Conduction** states that **the rate of heat transfer through a material is proportional to the negative gradient of the temperature and to the area, perpendicular to that gradient, through which the heat flows**. In simpler terms, heat flows from hotter areas to cooler areas, and the rate of flow is dependent on the temperature difference, the material's conductivity, and the area through which it flows.

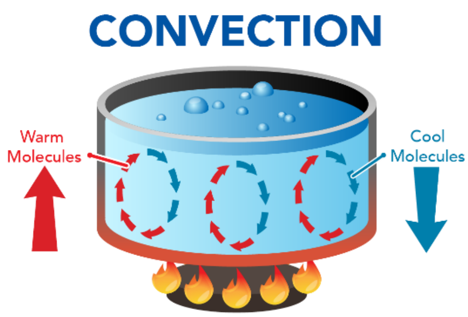


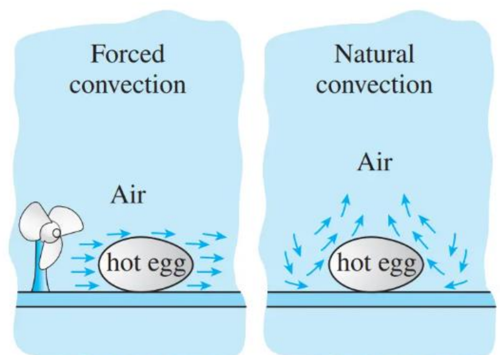
Convection heat transfer

Convection heat transfer is the process of heat transfer through the movement of fluids (liquids or gases) from one location to another.

Free or natural convection. Free or natural convection occurs when the fluid circulates by virtue of the natural differences in densities of hot and cold fluids; the denser portions of the fluid move downward because of the greater force of gravity, as compared with the force on the less dense.

Forced convection. When the work is done to blow or pump the fluid, it is said to be *forced convection*.





Radiation Heat Transfer

"*Radiation*" *is the transfer of heat through space or matter by means other than conduction or convection*.

Radiation heat is thought of as *electromagnetic waves or quanta* (as convenient) an emanation of the same nature as light and radio waves. *All bodies radiate heat*; *so a transfer of heat by radiation occurs because hot body emits more heat than it receives and a cold body receives more heat than it emits*. Radiant *energy* (being electromagnetic radiation) *requires no medium for propagation and will pass through vaccum*.

The properties of radiant heat in general, are similar to those of light. Some of the properties are :

(*i*) It does not require the presence of a material medium for its transmission.

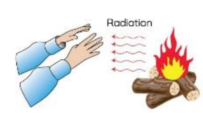
(*ii*) Radiant heat can be reflected from the surfaces and obeys the ordinary laws of reflection.

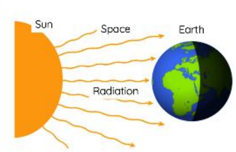
(*iii*) It travels with velocity of light.

(*iv*) Like light, it shows intereference, diffraction and polarisation etc.

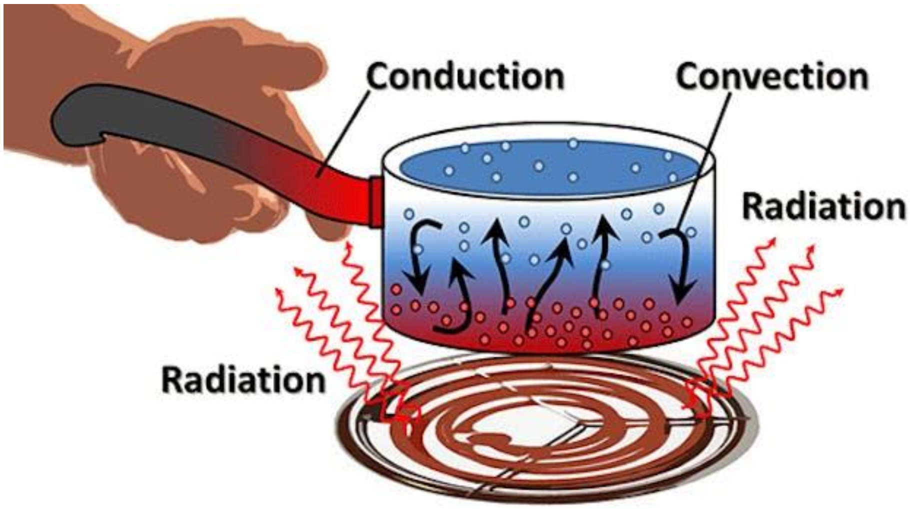
(*v*) It follows the law of inverse square.

The wavelength of heat radiations is longer than that of light waves, hence they are invisible to the eye.





Summary of heat transfer modes



Heat Transfer by conduction

Fourier's law of heat conduction

Fourier's law of heat conduction is an emperical law based on observation and states as follows :

"*The rate of flow of heat through a simple homogeneous solid is directly proportional to the area of the section at right angles to the direction of heat flow, and to change of temperature with respect to the length of the path of the heat flow*".

Mathematically, it can be represented by the equation :

$$Q \propto A \, . \frac{dt}{dx}$$

where, Q = Heat flow through a body per unit time (in watts), W,

A = Surface area of heat flow (*perpendicular to the direction of flow*), m^2,

dt = Temperature difference of the faces of block (homogeneous solid) of thickness 'dx' through which heat flows, °C or K, and

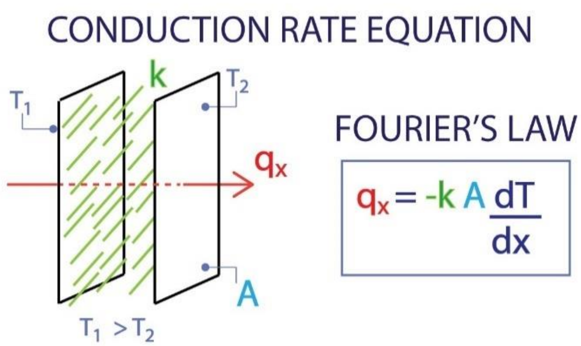
dx = Thickness of body in the direction of flow, m.

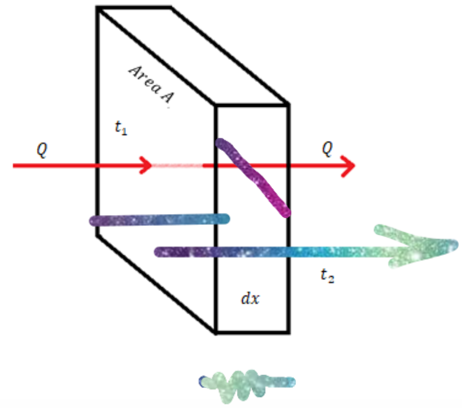
Thus,

$$Q = -\,k \, . \, A \frac{dt}{dx} \qquad \text{...(1.1)}$$

where, k = Constant of proportionality and is known as *thermal conductivity of the body*.

The – ve sign of k [eqn. (1.1)] is to take care of the decreasing temperature alongwith the direction of increasing thickness or the direction of heat flow. The temperature gradient $\frac{dt}{dx}$ is *always negative along positive x direction and, therefore, the value as Q becomes* + ve.





Assumptions :

The following are the assumptions on which Fourier's law is based :

1. Conduction of heat takes place under *steady state conditions*.
2. The heat flow is unidirectional.
3. The temperatures gradient is *constant* and the temperature profile is *linear*.
4. There is no internal heat generation.
5. The bounding surfaces are isothermal in character.
6. The material is homogeneous and isotropic (*i.e.*, the value of thermal conductivity is *constant in all directions*).

Thermal conductivity

From eqn. (1.1), we have

$$k = \frac{Q}{A} \cdot \frac{dx}{dt}$$

The value of $k = 1$ when $Q = 1$, $A = 1$ and $\frac{dt}{dx} = 1$

Now $k = \frac{Q}{1} \cdot \frac{dx}{dt}$ (unit of k : $\text{W} \times \frac{1}{\text{m}^2} \times \frac{\text{m}}{\text{K (or °C)}}$ = W/mK. or W/m°C)

Thus, the **thermal conductivity** *of a material is defined* as follows :

"*The amount of energy conducted through a body of unit area, and unit thickness in unit time when the difference in temperature between the faces causing heat flow is unit temperature difference*".

Material	Thermal conductivity (k) (W/mK)	Material	Thermal conductivity (k) (W/mK)
1. *Silver*	410	8. *Asbestos sheet*	0.17
2. *Copper*	385	9. *Ash*	0.12
3. *Aluminium*	225	10. *Cork, felt*	0.05 – 0.10
4. *Cast iron*	55–65	11. *Saw dust*	0.07
5. *Steel*	20–45	12. *Glass wool*	0.03
6. *Concrete*	1.20	13. *Water*	0.55 – 0.7
7. *Glass (window)*	0.75	14. *Freon*	0.0083

Thermal resistance

When two physical systems are described by similar equations and have similar boundary conditions, these are said to be *analogous*. The heat transfer processes may be compared by *analogy* with the flow of electricity in an electrical resistance. As the flow of electric current in the electrical resistance is directly proportional to potential difference (dV); similarly heat flow rate, Q, is directly proportional to temperature difference (dt), the driving force for heat conduction through a medium.

As per Ohm's law (in elctric-circuit theory), we have

$$\text{Current }(I) = \frac{\text{Potential difference }(dV)}{\text{Electrical resistance }(R)} \qquad \text{...(1.4)}$$

By analogy, the heat flow equation (Fourier's equation) may be written as

$$\text{Heat flow rate }(Q) = \frac{\text{Temperature difference }(dt)}{\left(\dfrac{dx}{kA}\right)} \qquad \text{...(1.5)}$$

By comparing eqns. (1.4) and (1.5), we find that I is analogous to, Q, dV is analogous to dt and R is analogous to the quantity $\left(\frac{dx}{kA}\right)$. The quantity $\frac{dx}{kA}$ is called **thermal conduction resistance** $(R_{th})_{cond.}$ *i.e.*,

$$(R_{th})_{cond.} = \frac{dx}{kA}$$

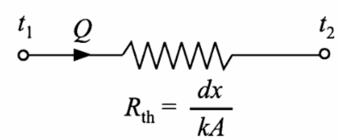


Fig. 1.7.

- The reciprocal of the thermal resistance is called *thermal conductance*.
- It may be noted that *rules for combining electrical resistances in series and parallel apply equally well to thermal resistances*.

The concept of thermal resistance is quite helpful while making calculations for flow of heat.

Example 1.1. *Calculate the rate of heat transfer per unit area through a copper plate 45 mm thick, whose one face is maintained at 350°C and the other face at 50°C. Take thermal conductivity of copper as 370 W/m°C.*

Solution. Temperature difference, $dt\ (= t_2 - t_1) = (50 - 350)$

Thickness of copper plate, $L = 45$ mm $= 0.045$ m

Thermal conductivity of copper, $k = 370$ W/m°C

Rate of heat-transfer per unit area, q :

From Fourier's law

$$Q = -kA\frac{dt}{dx} = -kA\frac{(t_2 - t_1)}{L}$$

...(Eqn. 1.1)

or,

$$q = \frac{Q}{A} = -k\frac{dt}{dx}$$

$$= -370 \times \frac{(50 - 350)}{0.045}$$

$$= 2.466 \times 10^6 \text{ W/m}^2 \text{ or}$$

2.466 MW/m² (Ans.)

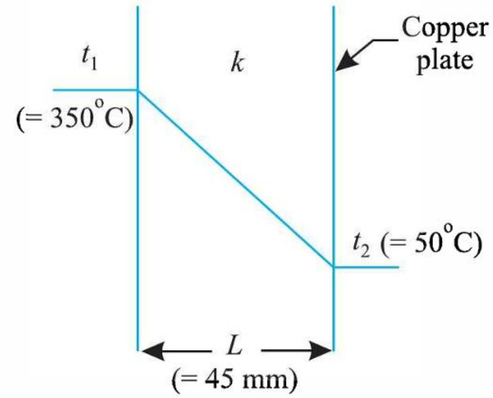


Fig. 1.8.

Example 1.2. *A plane wall is 150 mm thick and its wall area is 4.5 m². If its conductivity is 9.35 W/m°C and surface temperatures are steady at 150°C and 45°C, determine :*

(*i*) *Heat flow across the plane wall;*

(*ii*) *Temperature gradient in the flow direction.*

Solution. Thickness of the plane wall,

L = 150 mm
= 0.15 m

Area of the wall, A = 4.5 m²

Temperature difference, $dt = t_2 - t_1 = 45 - 150 = -105$°C

Thermal conductivity of wall material,

k = 9.35 W/m°C

(*i*) Heat flow across the plane wall, Q :

As per Fourier's law,

$$Q = -kA\frac{dt}{dx} = -kA\frac{(t_2 - t_1)}{L}$$

$$= -9.35 \times 4.5 \times \frac{(-105)}{0.15} = \mathbf{29452.5\ W}$$

(*ii*) Temperature gradient, $\frac{dt}{dx}$:

From Fourier's law, we have

$$\frac{dt}{dx} = -\frac{Q}{kA} = \frac{29452.5}{9.35 \times 4.5} = \mathbf{-700°C/m}$$

Example 1.3. *The following data relate to an oven :*

Thickness of side wall of the oven = *82.5 mm*
Thermal conductivity of wall insulation = *0.044 W/m°C*
Temperature on inside of the wall = *175°C*
Energy dissipated by the electrical coil within the oven = *40.5 W*

Determine the area of wall surface, perpendicular to heat flow, so that temperature on the other side of the wall does not exceed 75°C.

Solution. *Given* : $x = 82.5$ mm $= 0.0825$ *m*; $k = 0.044$ W/m°C; $t_1 = 175$°C; $t_2 = 75$°C; $Q = 40.5$W

Area of the wall surface, A :

Assuming one-dimentional steady state heat conduction,

Rate of electrical energy dissipation in the oven.

= Rate of heat transfer (conduction) across the wall

i.e.
$$Q = -kA\frac{dt}{dx} = -kA\frac{(t_2 - t_1)}{x} = \frac{kA\,(t_1 - t_2)}{x}$$

or,
$$40.5 = \frac{0.044\ A\,(175 - 75)}{0.0825}$$

or,
$$A = \frac{40.5 \times 0.0825}{0.044\,(175 - 75)} = \mathbf{0.759\ m^2}$$

Heat transfer by convection

The rate equation for the convective heat transfer (regardless of particular nature) between a surface and an adjacent fluid is prescribed by *Newton's law of cooling* (*Refer Fig. 1.9*)

$$Q = hA\,(t_s - t_f) \qquad ...(1.6)$$

where,
Q = Rate of conductive heat transfer,
A = Area exposed to heat transfer,
t_s = Surface temperature,
t_f = Fluid temperature, and
h = Co-efficient of convective heat transfer.

The units of h are,

$$h = \frac{Q}{A\,(t_s - t_f)} = \frac{\text{W}}{\text{m}^2\,°\text{C}} \quad \text{or} \quad \text{W/m}^2°\text{C}$$

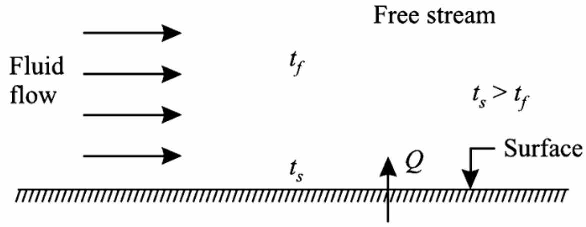
or, $\text{W/m}^2\text{K}$

The coefficient of convective heat transfer 'h' (also known as *film heat transfer coefficient*) may be defined as "*the amount of heat transmitted for a unit temperature difference between the fluid and unit area of surface in unit time.*"

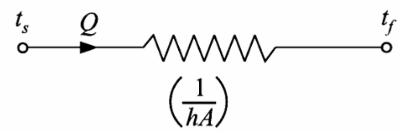
The value of 'h' depends on the following factors :

(*i*) Thermodynamic and transport properties (*e.g.* viscosity, density, specific heat etc.).
(*ii*) Nature of fluid flow.
(*iii*) Geometry of the surface.
(*iv*) Prevailing thermal conditions.

Since 'h' depends upon several factors, it is difficult to frame a single equation to satisfy all the variations, however, by dimensional analysis an equation for the purpose can be obtained.



(*a*) Physical configuration



(*b*) Equivalent circuit

Fig. 1.9. Convective heat-transfer

The *mechanisms of convection* in which phase changes are involved lead to the important *fields of boiling and condensation*. Refer Fig. 1.9 (*b*). The quantity $\frac{1}{hA}\left[Q = \frac{t_s - t_f}{(1/hA)} \text{ ...Eqn (1.6)}\right]$ is called **convection thermal resistance** [$(R_{th})_{conv.}$] to heat flow.

Example 1.4. *A hot plate 1m × 1.5 m is maintained at 300°C. Air at 20°C blows over the plate. If the convective heat transfer coefficient is 20W/m²°C, calculate the rate of heat transfer.*

Solution. Area of the plate exposed to heat transfer, $A = 1 \times 1.5 = 1.5\ m^2$

Plate surface temperature, t_s = 300°C

Temperature of air (fluid), t_f = 20°C

Connvective heat-transfer coefficient, h = 20 W/m² °C

Rate of heat transfer, Q :

From Newton's law of cooling,

$$Q = hA\,(t_s - t_f)$$
$$= 20 \times 1.5\,(300 - 20) = 8400 \text{ W or } \mathbf{8.4\ kW}$$

Example 1.5. *A wire 1.5 mm in diameter and 150 mm long is submerged in water at atmospheric pressure. An electric current is passed through the wire and is increased until the water boils at 100°C. Under the condition if convective heat transfer coefficient is 4500 W/m²°C find how much electric power must be supplied to the wire to maintain the wire surface at 120°C ?*

Solution. Diameter of the wire, d = 1.5 mm = 0.0015 m

Length of the wire, L = 150 mm = 0.15 m

∴ Surface area of the wire (exposed to heat transfer),

$A = \pi\, d\, L = \pi \times 0.0015 \times 0.15 = 7.068 \times 10^{-4}\ \text{m}^2$

Wire surface temperature, t_s = 120°C

Water temperature, t_f = 100°C

Convective heat transfer coefficient, h = 4500 W/m² °C

Electric power to be supplied :

Electric power which must be supplied = Total convection loss (Q)

∴ $Q = hA\,(t_s - t_f) = 4500 \times 7.068 \times 10^{-4}\,(120 - 100) = \mathbf{63.6\ W}$

Heat transfer by radiation

Laws of Radiation :

1. **Wien's law.** It states that *the wavelength λ_m corresponding to the maximum energy is inversely proportional to the absolute temperature T of the hot body.*

   i.e., $\lambda_m \propto \dfrac{1}{T}$ or, $\lambda_m T$ = constant ...(1.7)

2. **Kirchhoff's law.** It states that the *emissivity of the body at a particular temperature is numerically equal to its absorptivity for radiant energy from body at the same temperature.*

3. **The Stefan-Boltzmann law.** The law states that *the emissive power of a black body is directly proportional to fourth power of its absolute temperature.*

   i.e., $Q \propto T^4$ ...(1.8)

Refer Fig. 1.10 (*a*)

$$Q = F\,\sigma\,A\,(T_1^4 - T_2^4) \qquad ...(1.9)$$

where, F = A factor depending on geometry and surface properties,

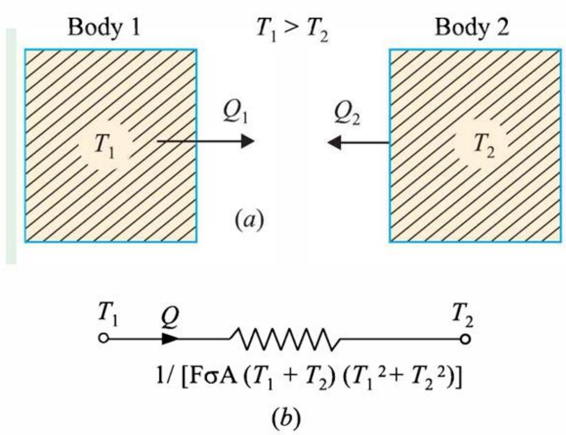


Fig. 1.10. Heat transfer by radiation.

σ = Stefan-Boltzmann constant
= 5.67×10^{-8} W/m^{2}K^4,
A = Area, m^2, and
T_1, T_2 = Temperatures, degrees kelvin (K).

This equation can also be rewritten as :

$$Q = \frac{T_1 - T_2}{1/[F\sigma A(T_1 + T_2)(T_1^2 + T_2^2)]} \quad ...(1.10)$$

vhere denomenator is **radiation thermal resistance**, $(R_{th})_{rad}$. [Fig. 1.10 (*b*)]

i.e., $(R_{th})_{rad} = 1/[F\sigma A(T_1 + T_2)(T_1^2 + T_2^2)]$

The values of F are available for simple configurations in the form of charts and tables.

F = 1 ... for simple cases of black surface enclosed by other surface

F = emissivity (ε) ... for non-black surface enclosed by other surface.

[*Emissivity* (ε) is defined as the ratio of heat radiated by a surface to that of an ideal surface.]

Example 1.6. *A surface having an area of 1.5 m^2 and maintained at 300°C exchanges heat by radiation with another surface at 40°C. The value of factor due to the geometric location and emissivity is 0.52. Determine :*

(*i*) *Heat lost by radiation,*

(*ii*) *The value of thermal resistance, and*

(*iii*) *The value of equivalent convection coefficient.*

Solution. *Given* : $A = 1.5$ m^2; $T_1 = t_1 + 273 = 300 + 273 = 573$K; $T_2 = t_2 + 273 = 40 + 273 = 313$K; $F = 0.52$.

(*i*) Heat lost by radiation, Q :

$$Q = F\sigma A(T_1^4 - T_2^4) \quad ...[\text{Eqn. (1.9)}]$$

(where $\sigma = 5.67 \times 10^{-8}$ W/m^2 K^4)

or, $$Q = 0.52 \times 5.67 \times 10^{-8} \times 1.5\,[(573)^4 - (313)^4]$$

$$= 0.52 \times 5.67 \times 1.5 \left[\left(\frac{573}{100}\right)^4 - \left(\frac{313}{100}\right)^4\right]$$

(Please note this step)

or, Q = **4343 W**

(*ii*) The value of thermal resistance, (Rth)rad :

We know that, $$Q = \frac{(T_1 - T_2)}{(R_{th})_{rad}}$$ ...[Eqn. (1.10)]

$$\therefore \quad (R_{th})_{rad.} = \frac{(T_1 - T_2)}{Q} = \frac{(573 - 313)}{4343} = \mathbf{0.0598\ °C/W}$$

(*iii*) The value of equivalent convection coefficient, hr :

$$Q = h_r A\,(t_1 - t_2)$$

or, $$h_r = \frac{Q}{A\,(t_1 - t_2)} = \frac{4343}{1.5\,(300 - 40)} = \mathbf{11.13\ W/m^2\,°C}$$

[Alternatively, $$h_r = F\,\sigma\,(T_1 + T_2)\,(T_1^2 + T_2^2)$$ ...From eqn. (1.10)

$$= 0.52 \times 5.67 \times 10^{-8}\,(573 + 313)\,(573^2 + 313^2)$$

$$= \mathbf{11.13\ W/m^2\,°C}$$]

Example 1.7. *A carbon steel plate (thermal conductivity = 45 W/m°C) 600 mm × 900 mm × 25 mm is maintained at 310°C. Air at 15°C blows over the hot plate. If convection heat transfer coefficient is 22 W/m² °C and 250 W is lost from the plate surface by radiation, calculate the inside plate temperature.*

Solution. Area of the plate exposed to heat transfer,

$$A = 600\text{ mm} \times 900\text{ mm} = 0.6 \times 0.9 = 0.54\text{ m}^2$$

Thickness of the plate, $L = 25\text{ mm} = 0.025\text{ m}$

Surface temperature of the plate, $t_s = 310°C$

Temperature of air (fluid), $t_f = 15°C$

Convective heat transfer coefficient,

$$h = 22\text{ W/m}^2\,°C$$

Heat lost from the plate surface by radiation,

$$Q_{rad.} = 250\text{W}$$

Thermal conductivity, $k = 45\text{ W/m °C}$

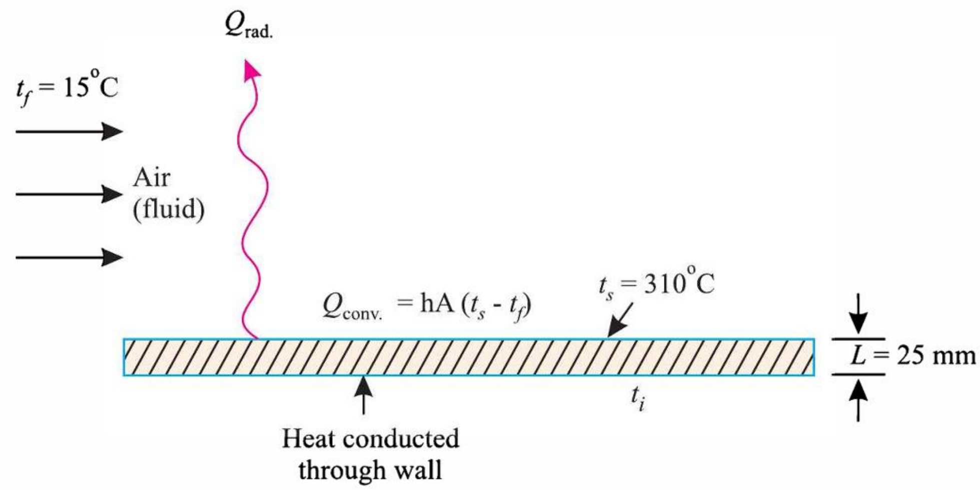


Fig. 1.11. Combination of conduction, convection and radiation heat transfer.

Inside plate temperature, t_i :

In this case the heat conducted through the plate is removed from the plate surface by a *combination of convection and radiation*.

Heat conducted through the plate = Convection heat losses + radiation heat losses.

or, $$Q_{cond.} = Q_{conv.} + Q_{rad.}$$

$$-kA\frac{dt}{dx} = hA(t_s - t_f) + F\sigma A(T_s^4 - T_f^4)$$

or, $$-45 \times 0.54 \times \frac{(t_s - t_i)}{L} = 22 \times 0.54\,(310 - 15) + 250 \text{ (given)}$$

or, $$-45 \times 0.54 \times \frac{(310 - t_i)}{0.025} = 22 \times 0.54 \times 295 + 250$$

or, $$972\,(t_i - 310) = 3754.6$$

or, $$t_i = \frac{3754.6}{972} + 310 = \mathbf{313.86°C}$$

Example 1.8. *A surface at 250°C exposed to the surroundings at 110°C convects and radiates heat to the surroundings. The convection coefficient and radiation factor are 75W/m²°C and unity respectively. If the heat is conducted to the surface through a solid of conductivity 10W/m°C, what is the temperature gradient at the surface in the solid ?*

Solution. Temperature of the surface, $t_s = 250°C$

Temperature of the surroundings, $t_{sur} = 110°C$

The convection co-efficient, $h = 5 W/m^2°C$

Radiation factor, $F = 1$

Boltzmann constant, $\sigma = 5.67 \times 10^{-8}\ W/m^2K^4$

Conductivity of the solid, $k = 10 W/m°C$

Temperature gradient, $\frac{dt}{dx}$:

Heat conducted through the plate = Convection heat losses + radiation heat losses

i.e., $$Q_{cond.} = Q_{conv.} + Q_{rad.} \quad -kA\frac{dt}{dx} = hA(t_s - t_{sur}) + F\sigma A(T_s^4 - T_{sur}^4)$$

Substituting the values, we have

$$-10 \times \frac{dt}{dx} = 75\,(250 - 110) + 1 \times 5.67 \times 10^{-8}\,[(250 + 273)^4 - (110 + 273)^4]$$

$$-10 \times \frac{dt}{dx} = 10500 + 5.67\left[\left(\frac{523}{100}\right)^4 - \left(\frac{383}{100}\right)^4\right]$$

$$= 10500 + 3022.1 = 13522.1$$

$$\therefore \quad \frac{dt}{dx} = -\frac{13522.1}{10} = \mathbf{-1352.21\ °C/m}$$

HIGHLIGHTS

1. The energy in transit is termed *heat*.
2. *Heat transfer* may be defined as "T*he transmission of energy from one region to another as a result of temperature gradient*."
3. The study of heat transfer is carried out for the following *purposes* :
   (*i*) To estimate the rate of flow of energy as heat through the boundary of a system under study (both steady and transient conditions).
   (*ii*) To determine the temperature field under steady and transient conditions.

Thermal resistance (R_{th}) :

Conduction thermal resistance, $(R_{th})_{cond.} = \dfrac{dx}{kA}$

Convection thermal resistance, $(R_{th})_{conv.} = \dfrac{1}{hA}$

Radiation thermal resistance, $(R_{th})_{rad.} = \dfrac{1}{1/[F\sigma A(T_1 + T_2)(T_1^2 + T_2^2)]}$

THEORETICAL QUESTIONS

1. Define the following terms :
   (*i*) Heat (*ii*) Heat transfer (*iii*) Thermodynamics.
2. What is the difference between thermodynamics and heat transfer ?
3. Enumerate the basic laws which govern the heat transfer.
4. Name and explain briefly the various modes of heat transfer.
5. What is conduction heat transfer ? How does it differ from convective heat transfer ?
6. What is the significance of heat transfer ?
7. Enumerate some important areas which are covered under the discipline of heat transfer.
8. What is the difference between the 'natural' and 'forced' convection ?
9. What is 'Fourier's law of conduction'? State also the assumptions on which this law is based.
10. State some essential features of Fourier's law.
11. How is thermal conductivity of a material defined ? What are its units ?
12. What is thermal resistance ?
13. What is 'Newton's law of cooling ?
14. What is Stefan's Boltzmann law ?

HOMEWORK- 01

1-1 If 3 kW is conducted through a section of insulating material 0.6 m^2 in cross section and 2.5 cm thick and the thermal conductivity may be taken as 0.2 W/m · °C, compute the temperature difference across the material.

1-2 A temperature difference of 85°C is impressed across a fiberglass layer of 13 cm thickness. The thermal conductivity of the fiberglass is 0.035 W/m · °C. Compute the heat transferred through the material per hour per unit area.

CHAPTER TWO

CONDUCTION HEAT TRANSFER

General heat conduction equation in cartesian coordinates

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = \frac{\rho.c}{k} \cdot \frac{\partial t}{\partial \tau} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau}$$

where, $\alpha = \frac{k}{\rho.c} = \frac{\text{Thermal conductivity}}{\text{Thermal capacity}}$

The quantity, $\alpha = \frac{k}{\rho.c}$ is known as **thermal diffusivity**.

using Laplacian ∇^2, may be written as :

$$\nabla^2 t + \frac{q_g}{k} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau}$$

Other simplified forms of heat conduction equation in cartesian coordinates :

(*i*) For the case when *no internal source of heat generation is present*, Eqn. (2.8) reduces to

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau}$$ [Unsteady state $\left(\frac{\partial t}{\partial \tau} \neq 0\right)$ heat flow with no internal heat generation]

or, $$\nabla^2 t = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau}$$ (**Fourier's equation**) ...(2.9)

(*ii*) Under the situations when temperature does not depend on time, the conduction then takes place in the steady state $\left(i.e., \frac{\partial t}{\partial \tau} = 0\right)$ and the eqn. (2.8) reduces to

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = 0$$

or, $$\nabla^2 t + \frac{q_g}{k} = 0$$ (**Poisson's equation**) ...(2.10)

In the absence of internal heat generation, Eqn. (2.10) reduces to

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} = 0$$

or, $$\nabla^2 t = 0$$ (**Laplace equation**) ...(2.11)

(*iii*) *Steady state and one-dimensional heat transfer*:

$$\frac{\partial^2 t}{\partial x^2} + \frac{q_g}{k} = 0$$ ...(2.12)

(*iv*) *Steady state, one-dimensional, without internal heat generation*

$$\frac{\partial^2 t}{\partial x^2} = 0$$ ...(2.13)

(*v*) *Steady state, two dimensional, without internal heat generation*

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} = 0$$ ...(2.14)

(*vi*) *Unsteady state, one dimensional, without internal heat generation*

$$\frac{\partial^2 t}{\partial x^2} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau}$$ ...(2.15)