Sinusoidal Alternating Waveforms

The terminology ac voltage or ac current refers to alternating voltage or current. The term alternating indicates only that the waveform alternates between two prescribed levels in a set time sequence. To be absolutely correct, the term sinusoidal, square wave, or triangular must also be applied

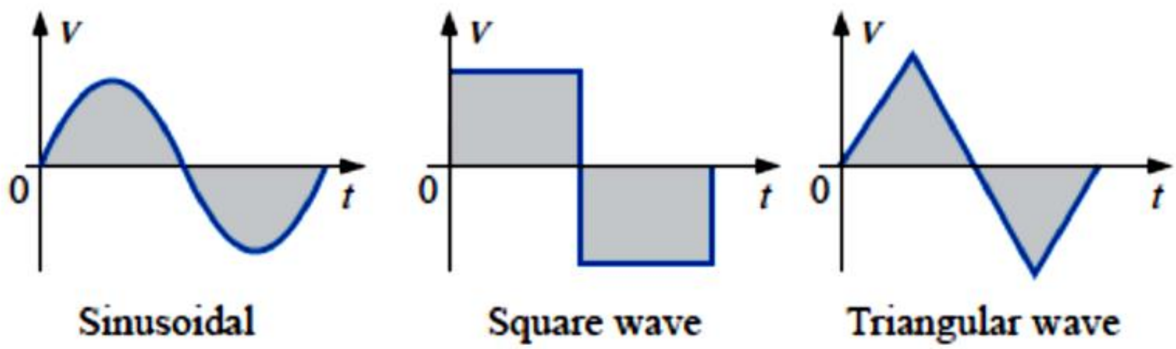


Figure 1: Alternating waveforms.

A sinusoid is a signal that has the form of the sine or cosine function.

A sinusoidal current is usually referred to as alternating current (ac). Such a current reverses at regular time intervals and has alternately positive and negative values. Circuits driven by sinusoidal current or voltage sources are called ac circuits. A sinusoidal signal is easy to generate and transmit. It is the form of voltage generated throughout the world and supplied to homes, factories, laboratories, and so on. It is the dominant form of signal in the communications and electric power industries. Through Fourier analysis, any practical periodic signal can be represented by a sum of sinusoids. Sinusoids, therefore, play an important role in the analysis of periodic signals. Lastly, a sinusoid is easy to handle mathematically. The derivative and integral of a sinusoid are themselves sinusoids. For these and other reasons, the sinusoid is an extremely important function in circuit analysis.

Definitions

- **Periodic waveform**: A waveform that continually repeats itself after the same time interval. The waveform of Fig. 2 is a periodic waveform.
- **Period (T):** The time interval between successive repetitions of a periodic waveform (the period T1 = T2 = T3 in Fig. 2), as long as successive similar points of the periodic waveform are used in determining T.
- **Peak value:** The maximum instantaneous value of a function as measured from the zero-volt level.

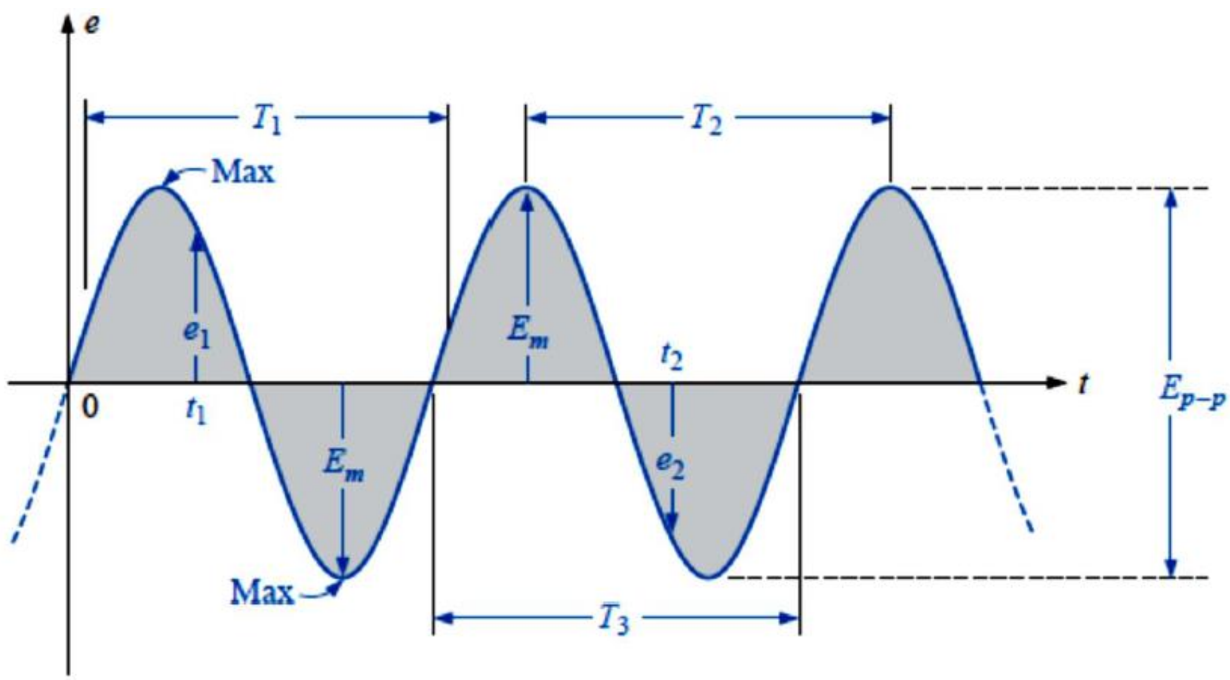
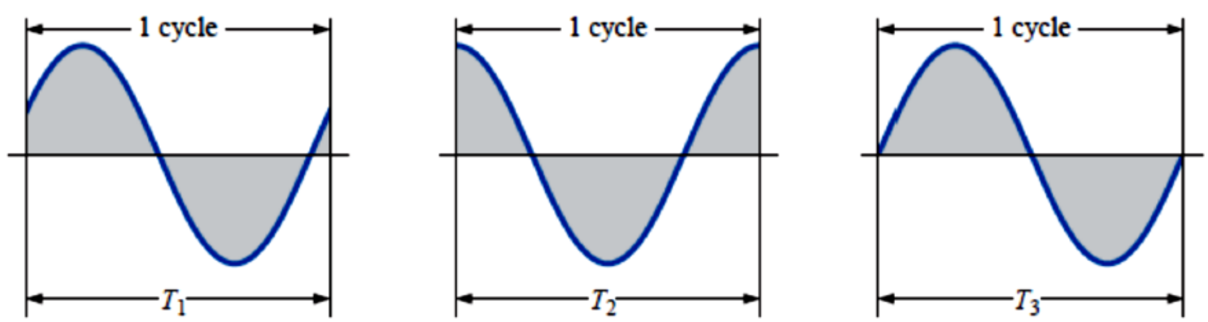


Figure 2: Important parameters for a sinusoidal voltage.

- **Peak-to-peak value:** Denoted by Ep-p or Vp-p, the full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks.
- **Instantaneous value:** The magnitude of a waveform at any instant of time; denoted by lowercase letters (e1, e2).
- **Cycle:** The portion of a waveform contained in one period of time



- **Frequency (f):** The number of cycles that occur in 1 s. The frequency of the waveform of Fig. 4(a) is 1 cycle per second, and for Fig. 4(b), 21⁄2 cycles per second. If a waveform of similar shape had a period of 0.5 s [Fig. 4(c)], the frequency would be 2 cycles per second.

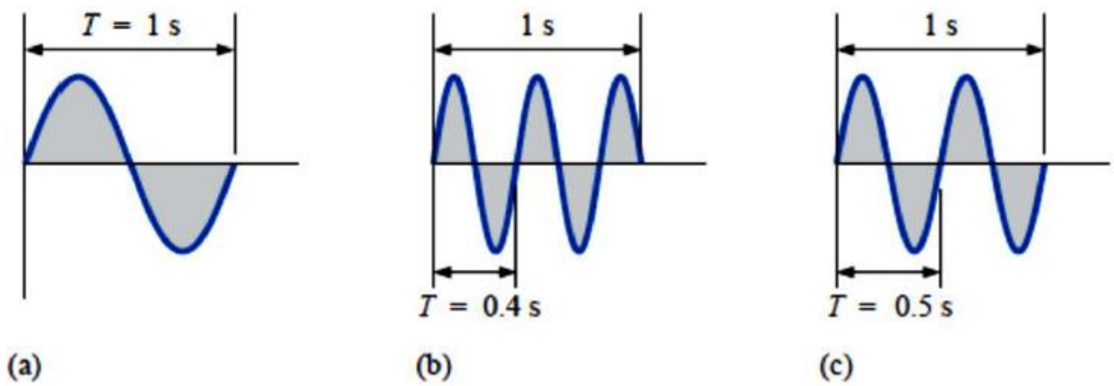


Figure 3: Demonstrating the effect of a changing frequency on the period of a sinusoidal waveform.

The unit of measure for frequency is the hertz (Hz), where:

$$1 \text{ hertz (Hz)} = 1 \text{ cycle per second (c/s)}$$

Since the frequency is inversely related to the period—that is, as one increases, the other decreases by an equal amount—the two can be related by the following equation:

$$f = \frac{1}{T}$$

f = Hz
T = seconds (s)

Example 1: Determine the frequency of the waveform of Fig. 4.

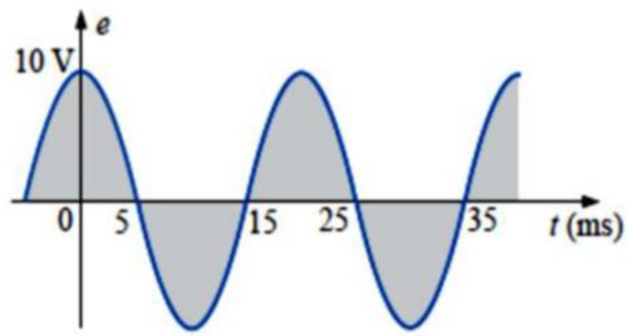


Figure 4: Waveform of Example 1

Solution:

From the figure, T = (25ms - 5ms) = 20ms,

And

$$f = \frac{1}{T} = \frac{1}{20 \times 10^{-3}\ \text{s}} = \mathbf{50\ Hz}$$

To convert from degrees to radians

$$\text{Radians} = \left(\frac{\pi}{180^\circ}\right) \times (\text{degrees})$$

To convert from radian to degree

$$\text{Degrees} = \left(\frac{180^\circ}{\pi}\right) \times (\text{radians})$$

Applying these equations, we find:

$$\mathbf{90^\circ}: \quad \text{Radians} = \frac{\pi}{180^\circ}(90^\circ) = \frac{\pi}{2}\ \mathbf{rad}$$

$$\mathbf{30^\circ}: \quad \text{Radians} = \frac{\pi}{180^\circ}(30^\circ) = \frac{\pi}{6}\ \mathbf{rad}$$

$$\frac{\pi}{3}\ \mathbf{rad}: \quad \text{Degrees} = \frac{180^\circ}{\pi}\left(\frac{\pi}{3}\right) = \mathbf{60^\circ}$$

$$\frac{3\pi}{2}\ \mathbf{rad}: \quad \text{Degrees} = \frac{180^\circ}{\pi}\left(\frac{3\pi}{2}\right) = \mathbf{270^\circ}$$

In general for the sinusoidal voltage or current:

$e = E_m \, Sin\ \theta$, *Where* θ *in degrees*

$e = E_m \, Sin\ \omega t$, *Where* $\theta = \omega t$.

$e = E_m \, Sin\ 2\pi f t$, *Where* $\omega = 2\pi f$.

$e = E_m \, Sin \frac{2\pi}{T} t$, *Where* $f = \frac{1}{T}$

Phase Relations

If the waveform is shifted to the right or left of 0°, the expression becomes:

$$A_m \sin(\omega t \pm \theta)$$

where θ is the angle in degrees or radians that the waveform has been shifted.

If the waveform passes through the horizontal axis with a positive going (increasing with time) slope before 0°, as shown in Fig. 6, the expression is:

$$A_m \sin(\omega t + \theta)$$

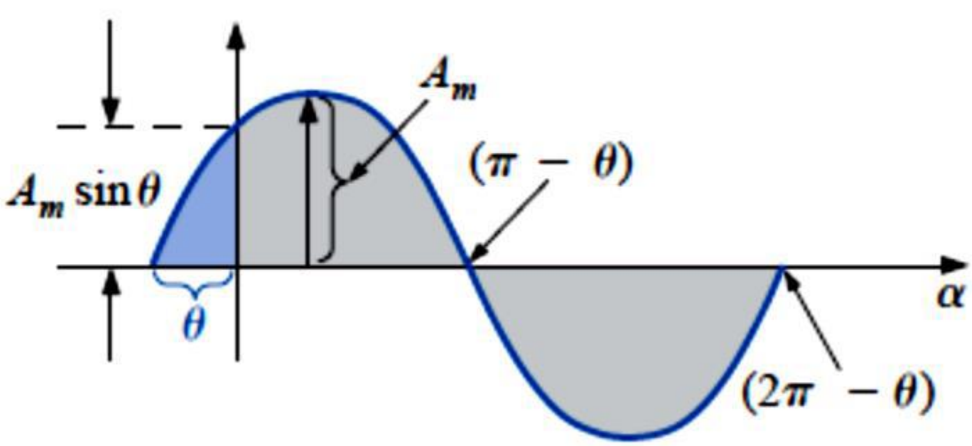


Figure 6:Defining the phase shift for a sinusoidal function that crosses the horizontal axis with a positive slope before 0°. $A_m \sin(\omega t + \theta)$

If the waveform passes through the horizontal axis with a positive-going slope after 0°, as shown in Fig. 7, the expression is:

$$A_m \sin(\omega t - \theta)$$

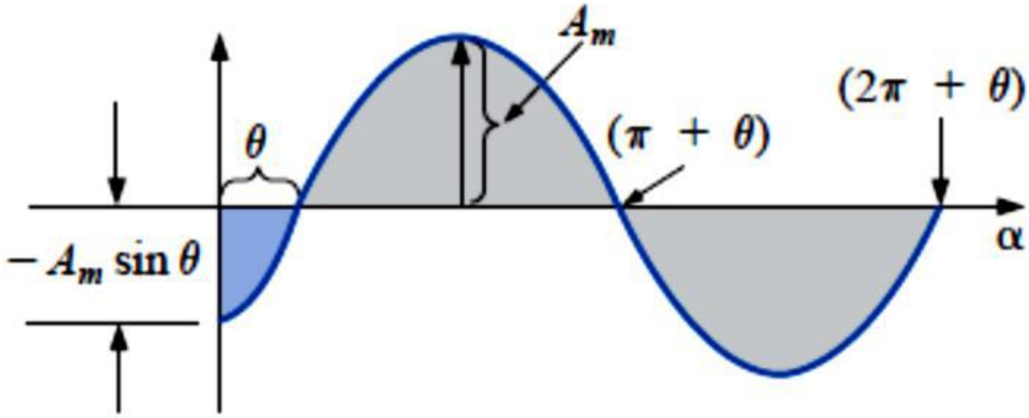


Figure 7:Defining the phase shift for a sinusoidal function that crosses the horizontal axis with a positive slope after 0°. $A_m \sin(wt - \theta)$

If the waveform crosses the horizontal axis with a positive-going slope 90° (π/2) sooner, as shown in Fig. 8, it is called a cosine wave; that is,

$$\sin(\omega t + 90^\circ) = \sin\left(\omega t + \frac{\pi}{2}\right) = \cos \omega t$$

Or

$$\sin \omega t = \cos(\omega t - 90^\circ) = \cos\left(\omega t - \frac{\pi}{2}\right)$$

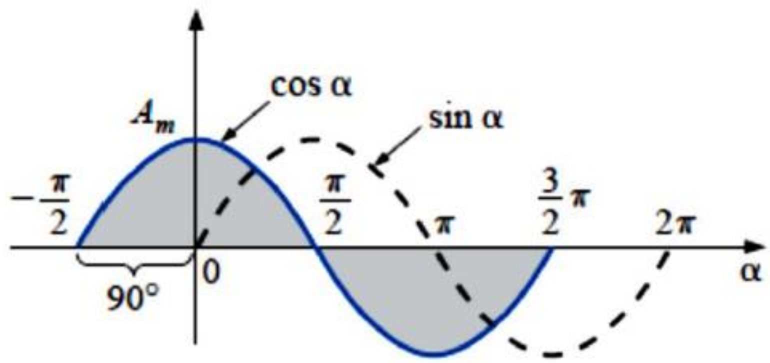


Figure 8: Phase relationship between a sine wave and a cosine wave.

The terms lead and lag are used to indicate the relationship between two sinusoidal waveforms of the same frequency plotted on the same set of axes.

In Fig. 8, the cosine curve is said to lead the sine curve by 90°, and the sine curve is said to lag the cosine curve by 90°. The 90° is referred to as the phase angle between the two waveforms.

Some Useful Relations:

$$\cos\alpha = \sin(\alpha + 90^\circ)$$
$$\sin\alpha = \cos(\alpha - 90^\circ)$$
$$-\sin\alpha = \sin(\alpha \pm 180^\circ)$$
$$-\cos\alpha = \sin(\alpha + 270^\circ) = \sin(\alpha - 90^\circ)$$
etc.

$$\sin(-\alpha) = -\sin\alpha$$
$$\cos(-\alpha) = \cos\alpha$$

The phase relationship between two waveforms indicates which one leads or lags, and by how many degrees or radians.

Example 4: What is the phase relationship between the sinusoidal waveforms of each of the following sets?

a. $v = 10\sin(\omega t + 30^\circ)$
   $i = 5\sin(\omega t + 70^\circ)$

b. $i = 15\sin(\omega t + 60^\circ)$
   $v = 10\sin(\omega t - 20^\circ)$

c. $i = 2\cos(\omega t + 10^\circ)$
   $v = 3\sin(\omega t - 10^\circ)$

d. $i = -\sin(\omega t + 30^\circ)$
   $v = 2\sin(\omega t + 10^\circ)$

e. $i = -2\cos(\omega t - 60^\circ)$
   $v = 3\sin(\omega t - 150^\circ)$

Solution:

a. See Fig. 9.

i leads v by 40°, or v lags i by 40°.

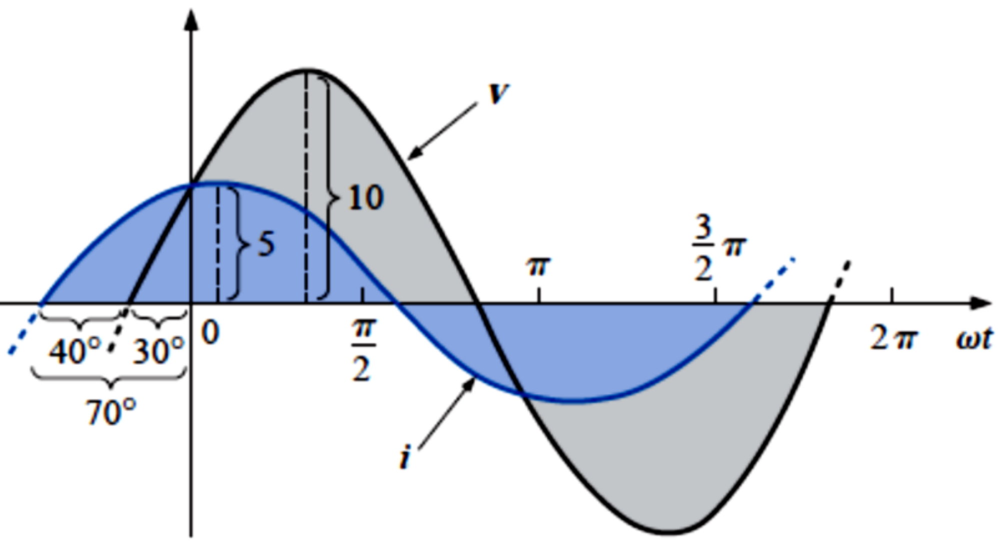
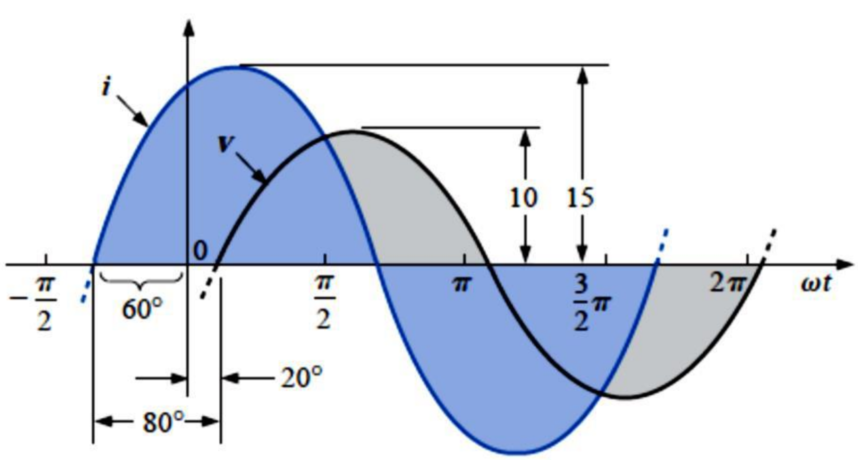


Figure 9: Example 4 (a); i leads v by 40°

b. See Fig. 10.

i leads v by 80°, or v lags i by 80°.



Average Value:

The average value of any current or voltage is the value indicated on a dc meter. In other words, over a complete cycle, the average value is the equivalent dc value. In general the average value of a waveform is given as:

$$G\ (Average\ value) = \frac{Area\ under\ the\ curve}{lemgth\ of\ the\ curve}$$

Example 5: Determine the average value of the waveforms of Fig. 14.

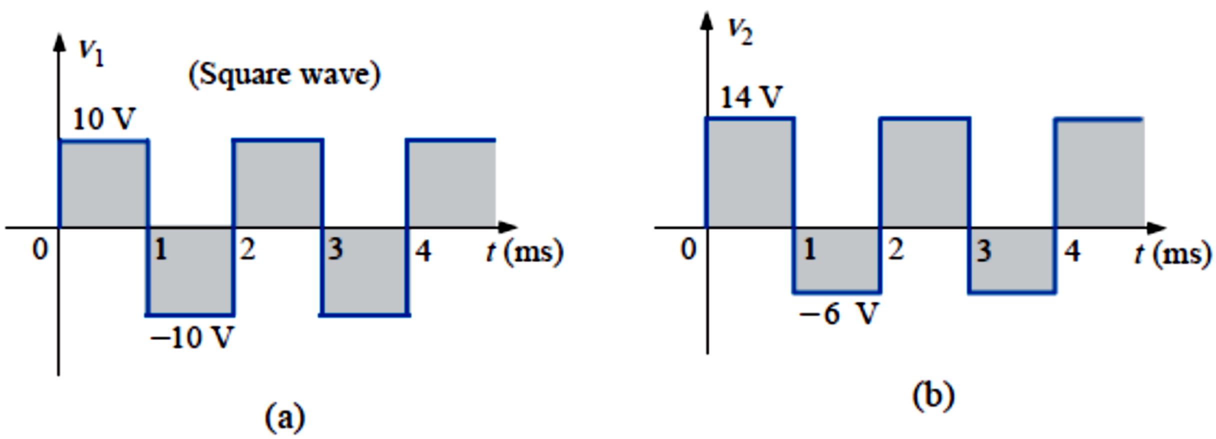


Figure 14: Example 5

Solution:

a. The area above the axis equals the area below over one cycle, resulting in an average value of zero volts. Using the above average equation:

$$G = \frac{(10\text{ V})(1\text{ ms}) - (10\text{ V})(1\text{ ms})}{2\text{ ms}}$$

$$= \frac{0}{2\text{ ms}} = \mathbf{0\ V}$$

b. By using the average equation:

$$G = \frac{(14\text{ V})(1\text{ ms}) - (6\text{ V})(1\text{ ms})}{2\text{ ms}}$$

$$= \frac{14\text{ V} - 6\text{ V}}{2} = \frac{8\text{ V}}{2} = \mathbf{4\ V}$$

In reality, the waveform of Fig. 14(b) is simply the square wave of Fig. 14(a) with a dc shift of 4 V; that is,