FM MODULATION

Introduction:

In Frequency Modulation (FM), the frequency of the carrier signal is varied according to the instantaneous value of the modulating signal, while the amplitude and phase remain constant. To generate a frequency-modulated signal, the carrier's frequency is adjusted based on the amplitude of the incoming audio signal. As the voltage of the modulating signal increases, the carrier frequency rises; as the voltage decreases, the carrier frequency lowers.

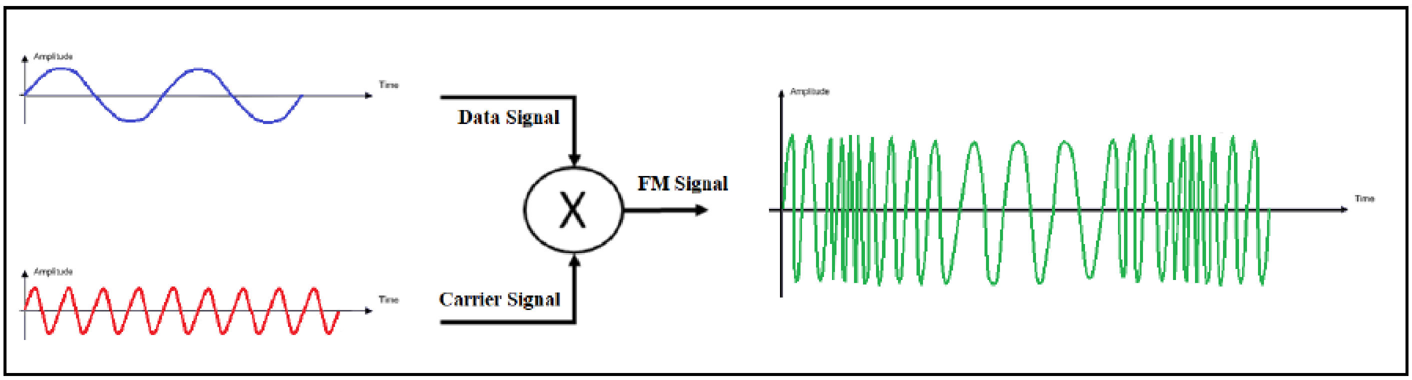
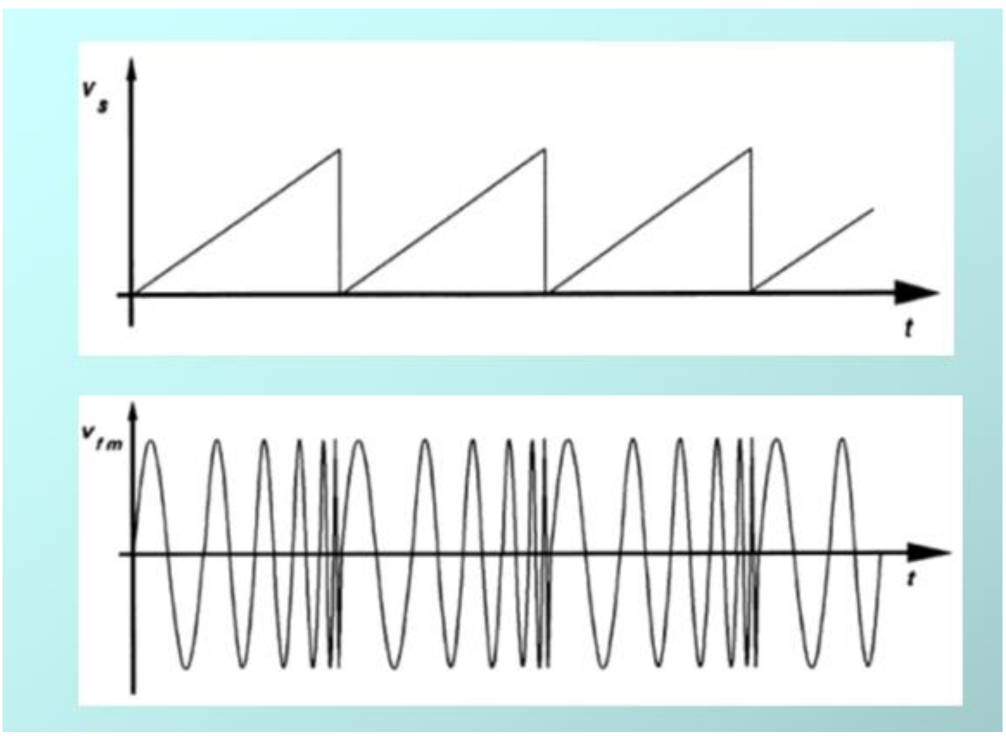


Figure. Frequency Modulation (FM)

When the audio signal is modulated onto the carrier wave, the new carrier frequency oscillates up and down. This change in frequency is called frequency deviation, typically measured in kilohertz (kHz). For instance, if the deviation is ±3 kHz, the carrier frequency shifts up by 3 kHz or down by 3 kHz from its original frequency. We can write an FM wave in the form:

$$E_{FM}(t) = E_c \cos\left(w_c t + \beta \sin(w_m t)\right)$$

In frequency modulated (FM) radio, the frequency of the carrier is varied about a fixed value in accordance with the amplitude of the audio frequency. The amplitude of the carrier is kept constant. The waveform of a sinusoidal carrier modulated by a saw-tooth wave is shown in Figure



FM and PM

The phasor has an angular rate, or frequency

$$\theta(t) = \int_0^t \omega_i(\tau)\,d\tau + \theta_0$$

$$\omega_i(\tau) = \frac{d\theta}{dt}$$

The phase angle $\theta(t)$ is Varied linearity with the input signal $f(t)$

$$\theta(t) = \omega_c t + k_p\, f(t) + \theta_0$$

$$\omega_i = \frac{d\theta}{dt} = \omega_c + k_p \frac{df}{dt}$$

$$\omega_i = \omega_c + k_f\, f(t)$$

Phase angle of this frequency modulation (FM)

$$\theta(t) = \int_0^t \omega_i(\tau)\,d\tau = \omega_c t + \int_0^t k_f\, f(t)\; d\tau + \theta_0$$

FM and PM Explanation

1. Phasor and Angular Frequency

In communication systems, a sinusoidal signal can be represented by a phasor, which is a rotating vector.

- **The angle of the phasor is called the phase angle $\theta(t)$.**
- **The rate at which the phasor rotates is called the instantaneous angular frequency.**

The relationship between them is:

$$\omega_i(t) = \frac{d\theta(t)}{dt}$$

This means:

Instantaneous frequency = derivative of the phase with respect to time.

Also,

$$\theta(t) = \int_0^t \omega_i(\tau) d\tau + \theta_0$$

So:

Phase = integral of the instantaneous frequency.

Phase = integral of the instantaneous frequency.

Where:

- θ_0= **initial phase at** $t = 0$

2. Phase Modulation (PM)

In **Phase Modulation (PM)**, the **phase of the carrier signal changes according to the message signal** $f(t)$.

The phase equation is:

$$\theta(t) = \omega_c t + k_p f(t) + \theta_0$$

Where:

- ω_c= carrier angular frequency
- $f(t)$= message (information) signal
- k_p= phase modulation constant

This means:

The phase varies linearly with the message signal.

Instantaneous Frequency in PM

To find the instantaneous frequency, we differentiate the phase:

$$\omega_i = \frac{d\theta}{dt}$$

$$\omega_i = \omega_c + k_p \frac{df(t)}{dt}$$

This means:

In PM, the instantaneous frequency depends on the derivative of the message signal.

Frequency Modulation (FM)

In **Frequency Modulation (FM)**, the **instantaneous frequency changes according to the message signal**.

The instantaneous frequency is:

$$\omega_i = \omega_c + k_f f(t)$$

Where:

- k_f= frequency modulation constant

This means:

The frequency varies directly with the message signal.

Phase in FM

Since phase is the integral of frequency:

$$\theta(t) = \int_0^t \omega_i\,(\tau)d\tau$$

Substituting the FM equation:

$$\theta(t) = \omega_c t + k_f \int_0^t f(t)dt + \theta_0$$

This shows that:

In FM, the phase depends on the integral of the message signal.

Example

Determine the instantaneous frequency of the signal

$$\phi(t) = A\cos(10\pi t + \pi t^2)$$

Solution :

$$\theta(t) = 10\pi t + \pi t^2$$

$$\omega_i(t) = \frac{d\theta}{dt} = 10\pi + 2\pi t = 2\pi(5 + t)$$

The frequency of $\phi(t)$ is 5 Hz at t = 0 and increases linearity at the rate of 1 Hz per second. This a quadratic phase shift gives a linear frequency dependence

Modulation-index:

In modulation systems, the modulation index represents the extent to which the modulated parameter deviates from its unmodulated state. Specifically, in Frequency Modulation (FM), the modulation index measures the variation in the carrier frequency relative to its unmodulated frequency. It is mathematically defined as:

$$\beta = \frac{Peak\ frequency\ deviation}{modulating\ frequency}$$

$$\beta = \frac{\Delta f}{f_m}$$

The spectrum of FM:

Any modulated signal generates sidebands. While determining sidebands for an amplitude-modulated (AM) signal is straightforward, analyzing sidebands in frequency modulation (FM) is more complex. The sidebands in FM depend not only on the frequency deviation but also on the modulation index (β), which represents the ratio of the deviation to the modulating frequency.

The total spectrum of an FM signal comprises an infinite series of discrete spectral components, described mathematically using Bessel functions of the first kind. Based on Bessel functions, the signal
$s(t)$ can be expressed as a series expansion:

$$s(t) = E_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos[(w_c + nw_m t)]$$

Where $J_n(\beta)$ is the n-th order Bessel function of the first kind. These functions can be computed by the series:

$$J_n(\beta) = \sum_{m=0}^{\infty} (-1)^m \frac{(\frac{1}{2}\beta)^{n+2m}}{m!\,(n+m)!}$$

A summary of the Bessel function of the first kind, for order n and discrete values of the argument β, is provided in Table 5.1. Additionally, a graphical representation of the function is shown in Figure 2. It is important to note that for very small values of β, $J0(\beta)$ approaches unity, while through $J1(\beta)$ to $Jn(\beta)$ approach to zero.

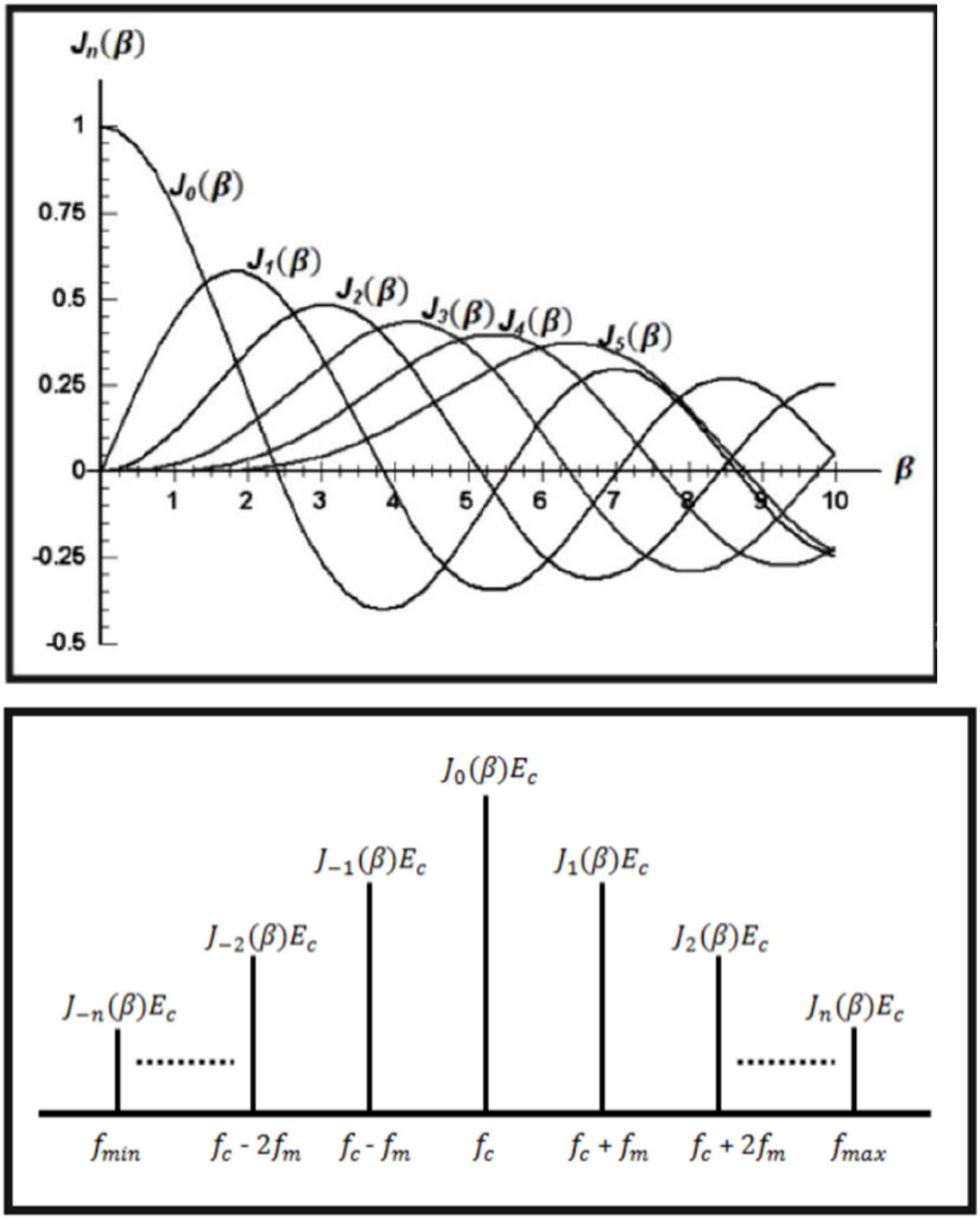


Fig. 3: Frequency Spectrum of FM Signal Showing Carrier and Sidebands at Different Harmonics.

Narrowband FM (NBFM) and Wideband FM (WBFM):

An FM signal with a low modulation index ($\beta \ll 1$ or $\Delta f \ll fm$) is referred to as a narrowband FM (NBFM) signal. For most practical purposes, the contributions of higher-order Bessel functions can be ignored, allowing its spectrum to be approximated as follows:

$$s(t) = E_c\, j_0(\beta)\cos(w_c t) + E_c\, j_1(\beta)\cos\big((w_c + w_m)t\big) - E_c\, j_{-1}(\beta)\, cos\big((w_c - w_m)t\big)$$

Narrowband FM shares similarities with AM in that it has sideband components at frequencies ($f_c \pm f_m$), requiring a transmission bandwidth of $2f_m$. However, the key difference lies in the sideband components of narrowband FM, which are 180 degrees out of phase with each other, unlike in AM.

In contrast, an FM signal with a high modulation index ($\beta \gg 1$ or $\Delta f \gg f_m$)) is known as wideband FM (WBFM). The bandwidth of a wideband FM signal is approximately $2\Delta f$.

By combining the principles of both WBFM and NBFM, we arrive at Carson's Rule, which states that the minimum practical bandwidth required to transmit an FM signal is:

$$BW = 2(f_m + \Delta f)$$

Narrowband FM

$Let \ f(t) = a\cos \omega_m t$

$\because \ \omega_i = \omega_c + k_f f(t)$

$= \omega_c + a\, k_f \, \cos \omega_m t$

Defining a new constant called the peak frequency deviation

$\Delta\omega = a\, k_f$

$\therefore \ \omega_i = \omega_c + \Delta\omega \cos \omega_m t$

The phase of this FM signal (let $\theta_0 = 0$)

$$\theta(t) = \omega_c t + \frac{\Delta\omega}{\omega_m} \sin \omega_m t = \omega_c t + \beta \sin \omega_m t$$

$$\beta = \frac{\Delta\omega}{\omega_m}$$

The resulting FM signal is

$$\phi_{FM}(t) = Ae^{j(\omega_c t + \beta \sin \omega_m t)}$$

$$\text{Re}\{\phi_{FM}(t)\} = A\cos(\omega_c t + \beta \sin \omega_m t)$$

$$\therefore \phi_{FM}(t) = Ae^{j\omega_c t}(1 + j\beta \sin \omega_m t - \frac{1}{2!}\beta^2 \sin^2 \omega_m t - j\frac{1}{3!}\beta^3 \sin^3 \omega_m t + \ \ldots \ )$$

For small values of β only the constant and first-order term are significant and the bandwidth will be 2 w_m

Note that NBFM $\beta < 0.2$

$$\phi_{NBFM}(t) = Ae^{j\omega_c t}(1 + j\beta \sin \omega_m t)$$

It is instructive to compare Eq. above with equivalent expression for an a AM signal

$$\phi_{AM}(t) = Ae^{j\omega_c t}(1 + m\cos\omega_m t)$$

β is called the modulation index of the FM signal

$$\phi_{NBFM}(t) = Ae^{j\omega_c t}\left(1 + \frac{1}{2}\beta e^{j\omega_m t} - \frac{1}{2}\beta e^{-j\omega_m t}\right)$$

$$\phi_{AM}(t) = Ae^{j\omega_c t}\left(1 + \frac{1}{2}me^{j\omega_m t} + \frac{1}{2}me^{-j\omega_m t}\right)$$

Generation of signals using balanced modulator:
(a) AM; (b)NBPM; (c) NBFM

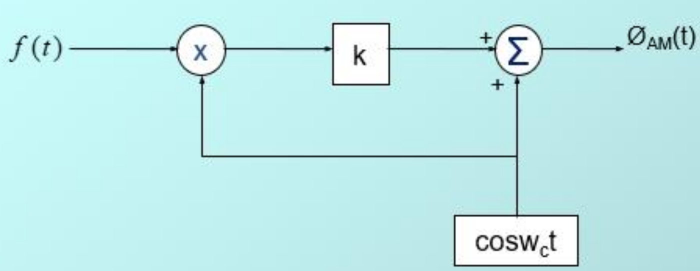


Figure. Generation of signal using balanced modulator: (a) AM

Bandwidth:

represents the range of frequencies required for signal transmission. The bandwidth of an FM signal depends on the modulation index (β), with higher modulation indices requiring a wider system bandwidth. The relationship between FM signal bandwidth and modulation index can be determined using the equations below:

$$Carson\ Law: BW = 2(\beta + 1)f_m$$

$$Bessel\ Law: BW = 2nf_m$$

Power of FM signal:

The total power in the infinite spectrum is:

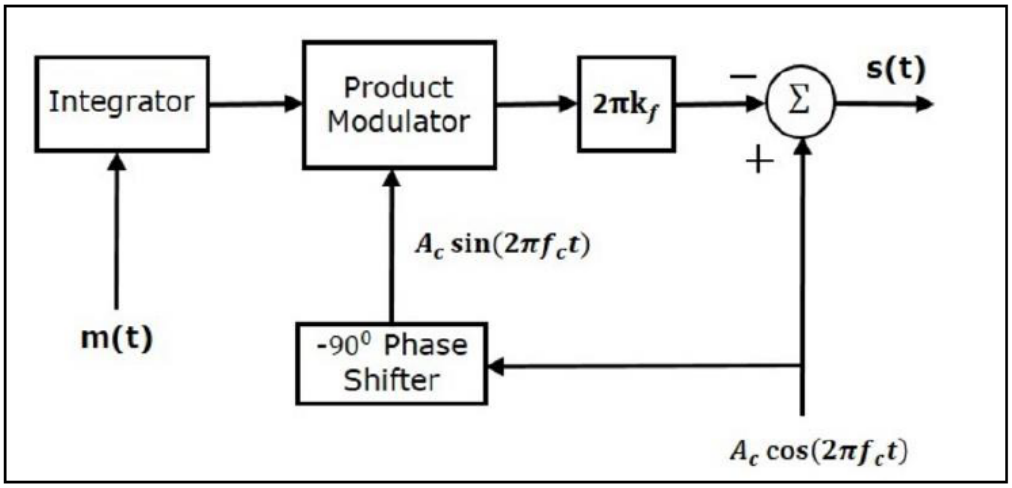
$$P_t = \frac{{E_c}^2}{2R} \sum_{n=-\infty}^{\infty} (J_n(\beta))^2 = \frac{{E_c}^2}{2R}$$

Hence, the carrier power is:

$$P_c = \frac{{E_c}^2}{2R} (J_0(\beta))^2$$

Generation of NBFM:

Narrowband FM (NBFM), similar to DSB-LC, is a type of linear modulation. An NBFM signal can be generated using a phase shifter and a balanced modulator, as illustrated in the figure below.



Common Application:

Frequency Modulation (FM) is widely used in radio and television broadcasting. The FM spectrum is allocated for various applications. Analog television channels (0 to 72) occupy frequencies between 54 MHz and 825 MHz, where FM is used to transmit audio signals.

Additionally, FM radio operates within the 88 MHz to 108 MHz band. Each FM radio station utilizes a 200 kHz bandwidth, with approximately 38 kHz dedicated to audio transmission, while the remaining spectrum is used for stereo and auxiliary signal.