Series Resistors and Voltage Division

The need to combine resistors in series or in parallel occurs so frequently that it warrants special attention. The process of combining the resistors is facilitated by combining two of them at a time. With this in mind, consider the single-loop circuit of Fig. 1. The two resistors

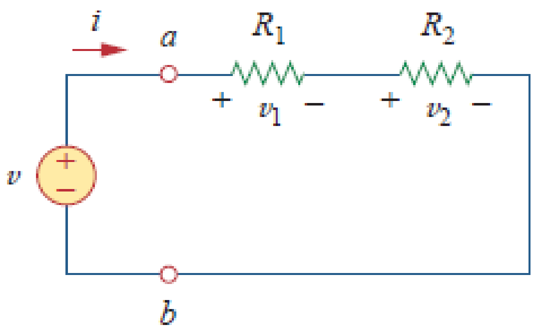

Figure 1. A single-loop circuit with two resistors in series

are in series, since the same current i flows in both of them. Applying Ohm's law to each of the resistors, we obtain

$$v_1 = iR_1, \qquad v_2 = iR_2 \tag{2.24}$$

If we apply KVL to the loop (moving in the clockwise direction), we have

$$-v + v_1 + v_2 = 0 \tag{2.25}$$

Combining Eqs. (2.24) and (2.25), we get

$$v = v_1 + v_2 = i(R_1 + R_2) \tag{2.26}$$

or

$$i = \frac{v}{R_1 + R_2} \tag{2.27}$$

Notice that Eq. (2.26) can be written as

$$v = iR_{eq} \tag{2.28}$$

implying that the two resistors can be replaced by an equivalent resistor R_{eq}; that is,

$$R_{eq} = R_1 + R_2 \tag{2.29}$$

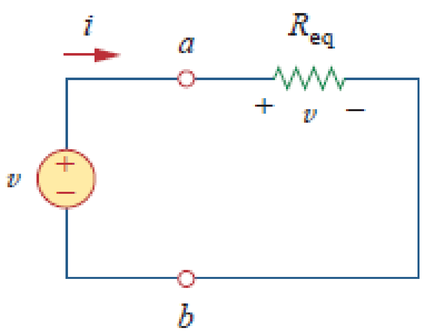

Figure 2. Equivalent circuit of the Fig. 1. circuit.

The **equivalent resistance** of any number of resistors connected in series is the sum of the individual resistances.

For N resistors in series then,

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n \qquad (2.30)$$

Parallel Resistors and Current Division

Consider the circuit in Fig. 3, where two resistors are connected in parallel and therefore have the same voltage across them. From Ohm's law,

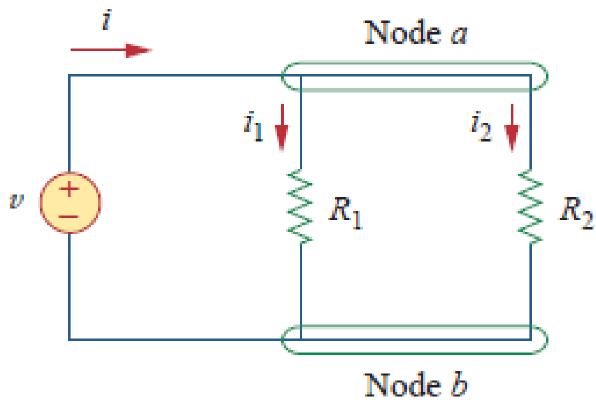

Figure .3. Two resistors in parallel

$$v = i_1 R_1 = i_2 R_2$$

or

$$i_1 = \frac{v}{R_1}, \qquad i_2 = \frac{v}{R_2} \qquad \textbf{(2.33)}$$

Applying KCL at node a gives the total current i as

$$i = i_1 + i_2 \qquad \textbf{(2.34)}$$

Substituting Eq. (2.33) into Eq. (2.34), we get

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}} \qquad \textbf{(2.35)}$$

where R_{eq} is the equivalent resistance of the resistors in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \textbf{(2.36)}$$

or

$$\frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2}$$

or

$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}} \qquad \textbf{(2.37)}$$

The **equivalent resistance** of two parallel resistors is equal to the product of their resistances divided by their sum.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} \qquad (2.38)$$

Note that R_{eq} is always smaller than the resistance of the smallest resistor in the parallel combination. If $R_1 = R_2 = \cdots = R_N = R$, then

$$R_{eq} = \frac{R}{N} \qquad (2.39)$$

Example 2.9

Find R_{eq} for the circuit shown in Fig. 2.34.

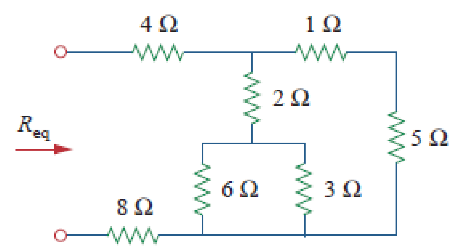

Figure 2.34
For Example 2.9.

Solution:

To get R_{eq}, we combine resistors in series and in parallel. The 6-Ω and 3-Ω resistors are in parallel, so their equivalent resistance is

$$6\,\Omega \parallel 3\Omega = \frac{6 \times 3}{6 + 3} = 2\,\Omega$$

(The symbol ∥ is used to indicate a parallel combination.) Also, the 1-Ω and 5-Ω resistors are in series; hence their equivalent resistance is

$$1\,\Omega + 5\,\Omega = 6\,\Omega$$

Thus the circuit in Fig. 2.34 is reduced to that in Fig. 2.35(a). In Fig. 2.35(a), we notice that the two 2-Ω resistors are in series, so the equivalent resistance is

$$2\,\Omega + 2\,\Omega = 4\,\Omega$$

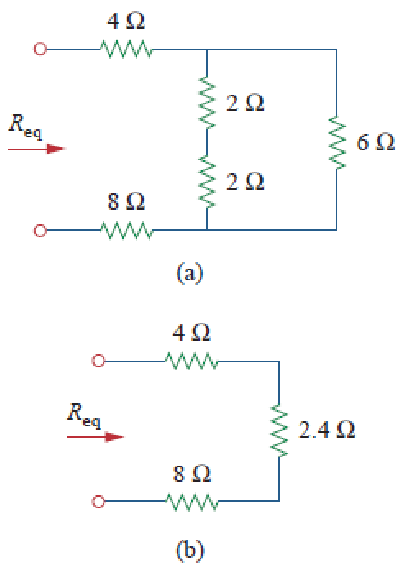

Figure 2.35
Equivalent circuits for Example 2.9.

This 4-Ω resistor is now in parallel with the 6-Ω resistor in Fig. 2.35(a); their equivalent resistance is

$$4\ \Omega \parallel 6\ \Omega = \frac{4 \times 6}{4 + 6} = 2.4\ \Omega$$

The circuit in Fig. 2.35(a) is now replaced with that in Fig. 2.35(b). In Fig. 2.35(b), the three resistors are in series. Hence, the equivalent resistance for the circuit is

$$R_{eq} = 4\ \Omega + 2.4\ \Omega + 8\ \Omega = 14.4\ \Omega$$

Practice Problem 2.9

By combining the resistors in Fig. 2.36, find R_{eq}.

Answer: 6 Ω.

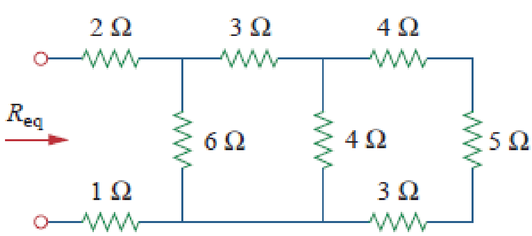

Figure 2.36
For Practice Prob. 2.9.

Calculate the equivalent resistance R_{ab} in the circuit in Fig. 2.37.

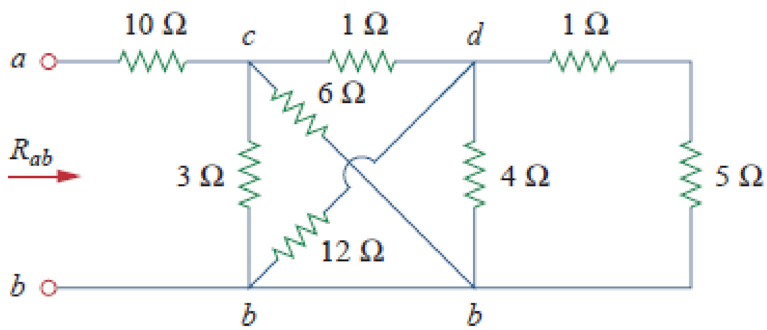

Figure 2.37
For Example 2.10.

Solution:
The 3-Ω and 6-Ω resistors are in parallel because they are connected to the same two nodes *c* and *b*. Their combined resistance is

$$3\ \Omega \parallel 6\ \Omega = \frac{3 \times 6}{3 + 6} = 2\ \Omega \qquad \textbf{(2.10.1)}$$

Similarly, the 12-Ω and 4-Ω resistors are in parallel since they are connected to the same two nodes d and b. Hence

$$12\ \Omega \parallel 4\ \Omega = \frac{12 \times 4}{12 + 4} = 3\ \Omega \qquad \textbf{(2.10.2)}$$

Also the 1-Ω and 5-Ω resistors are in series; hence, their equivalent resistance is

$$1\ \Omega + 5\ \Omega = 6\ \Omega \qquad \textbf{(2.10.3)}$$

With these three combinations, we can replace the circuit in Fig. 2.37 with that in Fig. 2.38(a). In Fig. 2.38(a), 3-Ω in parallel with 6-Ω gives 2-Ω, as calculated in Eq. (2.10.1). This 2-Ω equivalent resistance is now in series with the 1-Ω resistance to give a combined resistance of 1 Ω + 2 Ω = 3 Ω. Thus, we replace the circuit in Fig. 2.38(a) with that in Fig. 2.38(b). In Fig. 2.38(b), we combine the 2-Ω and 3-Ω resistors in parallel to get

$$2\ \Omega \parallel 3\ \Omega = \frac{2 \times 3}{2 + 3} = 1.2\ \Omega$$

This 1.2-Ω resistor is in series with the 10-Ω resistor, so that

$$R_{ab} = 10 + 1.2 = 11.2\ \Omega$$

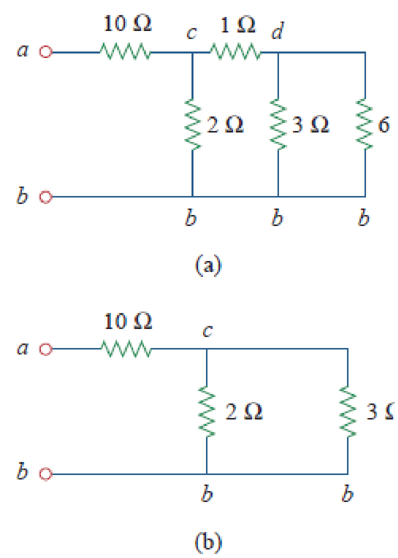

Figure 2.38
Equivalent circuits for Example 2.10.