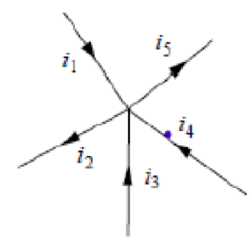
KIRCHHOFF'S LAWS

Ohm's law by itself is not sufficient to analyze circuits. However, when coupled with Kirchhoff's two laws, we have a sufficient, powerful set of tools for analyzing a large variety of electric circuits.

Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node is zero

Mathematically, KCL implies that $\sum_{n=1}^{N} i_n = 0$

Where N is the number of branches connected to the node

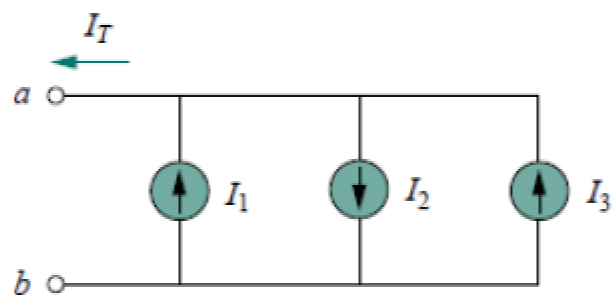


and i_n is the nth current entering (or leaving) the node.

since currents i1, i3, and i4 are entering the node, while currents i2 and i5 are leaving it. By rearranging the terms, we get i1 + i3 + i4 = i2 + i5

The sum of the currents entering a node equals the sum of the currents leaving the node.

$I_T = I_1 - I_2 + I_3$



Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

$$\sum_{m=1}^{M} v_m = 0$$

Where M is the number of voltages in the loop (or the number of branches in the loop) and v_m is the mth voltage.

To illustrate KVL, consider the circuit in Fig. 5.

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

Rearranging terms gives

$$+ v_2 + v_3 + v_5 = v_1 + v_4$$

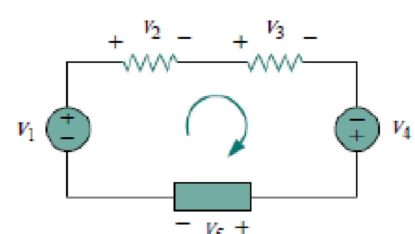


Figure 7

which may be interpreted as

Sum of voltage drops = Sum of voltage rises

Example (3): For the circuit of Fig. 8

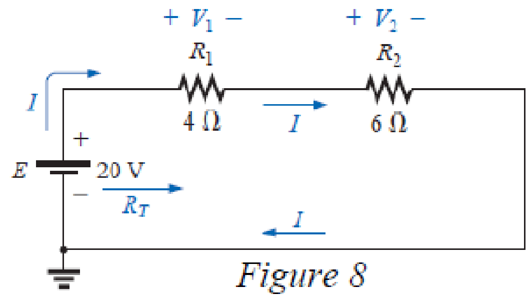


Figure 8

a. Find R_T.

b. Find I.

c. Find V_1 and V_2.

d. Find the power to the 4Ω and 6Ω resistors.

e. Find the power delivered by the battery, and compare it to that dissipated by the 4Ω and 6Ω resistors combined.

f. Verify Kirchhoff's voltage law (clockwise direction).

Solution

a. $RT = R1 + R2 = 4\ \Omega + 6\ \Omega = \mathbf{10\Omega}$

b. $I = \frac{E}{R_T} = \frac{20V}{10\Omega} = 2A$

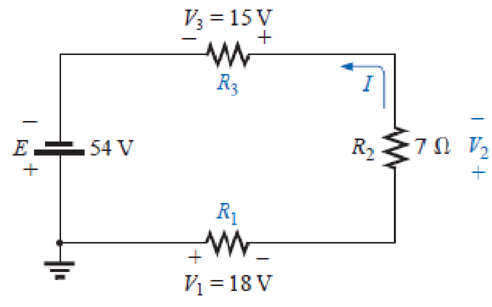
c. $V_1 = IR_1 = 2A \times 4\Omega = 8V$ $\quad$ $V_2 = IR_2 = 2A \times 6\Omega = 12V$

d. $P_1 = \frac{V^2}{R_1} = \frac{8^2}{4} = 16W$ $\quad$ $P_2 = \frac{V^2}{R_2} = \frac{12^2}{6} = 24W$

e. $P_E = P_1 + P_2 = 16 + 24 = 40W$

f. $-E + V_1 + V_2 = 0$ $\quad$ $E = V_1 + V_2$ $\quad$ $20V = 8V + 12V$

Example (4): For the circuit of Fig. 9



a. Determine V_2 using Kirchhoff's voltage law.

b. Determine I.

c. Find $R1$ and $R3$.

Solution

a- Kirchhoff's voltage law (clockwise direction):

$-E + V3 + V2 + V1 = 0$

or $E = V1 + V2 + V3$

and $V2 = E - V1 - V3 = 54\text{ V} - 18\text{ V} - 15\text{ V} = \mathbf{21\text{ V}}$

b- $I = \frac{V_2}{R_2} = \frac{21}{7} = 3A$

c- $R_1 = \frac{V_1}{I} = \frac{18}{3} = 6\Omega$

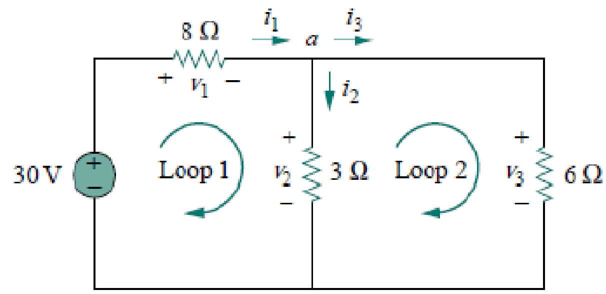
d- $R_3 = \frac{V_3}{I} = \frac{15}{3} = 5\Omega$

Example (5) Find the currents and voltages in the circuit shown in the figure below

Solution

We apply Ohm's law and Kirchhoff's laws. By Ohm's law,

$v1 = 8i_1, \quad v2 = 3i_2, \quad v3 = 6i_3$



At node a, KCL gives

$i_1 - i_2 - i_3 = 0$

Applying KVL to loop 1

$-30 + v_1 + v_2 = 0$

We express this in terms of i_1 and i_2

$-30 + 8i_1 + 3i_2 = 0$

$$i_1 = \frac{(30 - 3i_2)}{8}$$

Applying KVL to loop 2,

$-v_2 + v_3 = 0 \Rightarrow v_3 = v_2$

$$6i_3 = 3i_2 \quad \Rightarrow \quad i_3 = \frac{3i_2}{6} = \frac{i_2}{2} \Rightarrow$$

$i_1 - i_2 - i_3 = 0$

$$\frac{30 - 3i_2}{8} - i_2 - \frac{i_2}{2} = 0$$

$i_2 = 2$ A

$i_1 = 3$ A, $\quad i_3 = 1$ A, $\quad v_1 = 24$ V, $\quad v_2 = 6$ V, $\quad v_3 = 6$ V