Coulomb's Law – Electric force

Coulomb stated that "The force between two very small objects separated in a vacuum or free space by a distance which is large compared to their size is proportional to the charge on each and inversely proportional to the square of the distance between them".

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}$$

Where:

F: Force in newton (N),

Q1: and Q2 are the positive or negative quantities of charge in Coulomb(C)

R: is the separation in meters (m)

ε_0: is called the permittivity of free space and has the magnitude, measured in farads per meter (F/m)

$$\varepsilon_0 = 8.854 \times 10^{-12} \cong \frac{10^{-9}}{36\pi}\frac{\mathrm{F}}{\mathrm{m}}$$

$$k = \frac{1}{4\pi\varepsilon_0} \cong 9 \times 10^9 \ \mathrm{m/F}$$

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The coulomb is an extremely large unit of charge, for the smallest known quantity of charge is that of the electron (negative) or proton (positive), given in mks units as 1.602×10^{-19} C. Hence a negative charge of one coulomb represents about 6×10^{18}electrons.

If point charges Q_1 and Q_2 are located at points having position vector r_1 and r_2, then the vector force **F_{12}** on Q_2 duo to Q_1, shown in Figure 1, is given by

$$F_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 |R|^2} \mathbf{a}_{R_{12}}$$

where

$$\vec{R}_{12} = \vec{r}_2 - \vec{r}_1$$

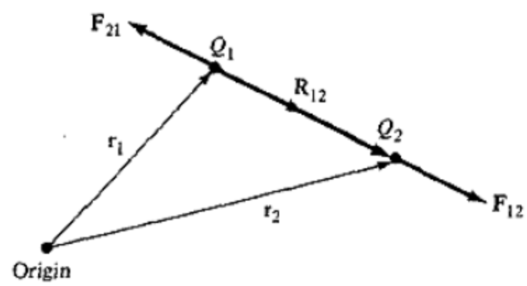

Figure.1 Coulomb vector force on point charges $Q_1$and Q_2

where

$$\vec{R}_{12} = \vec{r}_2 - \vec{r}_1$$

$$R = |\vec{R}_{12}|$$

$$\mathbf{a}_{R_{12}} = \frac{\vec{R}_{12}}{|\vec{R}_{12}|}$$

$$\therefore \vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 |R|^3} \vec{R}_{12}$$

or

$$\vec{F}_{12} = \frac{Q_1 Q_2 (\vec{r}_2 - \vec{r}_1)}{4\pi\varepsilon_0 |\vec{r}_2 - \vec{r}_1|^3}$$

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As shown in Figure .1, the force$\vec{F}21$ on Q1 due to Q2 is given by

$$\vec{F}_{21} = -\vec{F}_{12}$$

Like charges (charges of the same sign) repel each other while unlike charges attract. This is illustrated in Figure .2.

Figure .2 (a), (b) Like charges repel. (c) Unlike charges attract.

From Figure .1 Q_1 located at (x_1, y_1, z_1) and Q_2 at (x_2, y_2, y_3), then

$$\vec{\mathrm{R}}_{12} = (x_2 - x_1)\boldsymbol{a}_x + (y_2 - y_1)\boldsymbol{a}_y + (z_2 - z_1)\boldsymbol{a}_z$$

$$|\vec{\mathrm{R}}_{12}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$\mathbf{a}_{R_{12}} = \frac{\vec{\mathrm{R}}_{12}}{|\vec{\mathrm{R}}_{12}|}$$

$$\mathbf{a}_{R_{12}} = \frac{(x_2 - x_1)^2\boldsymbol{a}_x + (y_2 - y_1)^2\boldsymbol{a}_y +, (z_2 - z_1)^2\boldsymbol{a}_z}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

since

$$\mathbf{F}_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 |R|^2}\mathbf{a}_{R_{12}}$$

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$$= \frac{Q_1Q_2}{4\pi\varepsilon_0[(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2]}\frac{(x_2-x_1)^2\boldsymbol{a}_x+(y_2-y_1)^2\boldsymbol{a}_y+(z_2-z_1)^2\boldsymbol{a}_z}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}$$

$$\mathbf{F}_2 = \frac{Q_1Q_2(x_2-x_1)^2\boldsymbol{a}_x+(y_2-y_1)^2\boldsymbol{a}_y+(z_2-z_1)^2\boldsymbol{a}_z}{4\pi\varepsilon_0[(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2]^{3/2}}$$

If we have more than two-point charges, we can use the principle of superposition to determine the force on a particular charge. The principle states that if there are N charges $\mathbf{Q_1, Q_2, Q_3} \ldots\ldots \mathbf{Q_N}$ located, respectively, at points with position vectors $\mathbf{r_1, r_2} \ldots\ldots\ldots \mathbf{r_N}$ the resultant force **F** on a charge **Q** located at point **r** is the vector sum of the forces exerted on by each of the **Q1, Q2, Q3 ……QN** charges Hence:

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$$\vec{F} = \frac{QQ_1\,(\vec{r}-\vec{r}_1)}{4\pi\varepsilon_0|\vec{r}-\vec{r}_1|^3} + \frac{QQ_2\,(\vec{r}-\vec{r}_2)}{4\pi\varepsilon_0|\vec{r}-\vec{r}_2|^3} + \cdots + \frac{QQ_2\,(\vec{r}-\vec{r}_2)}{4\pi\varepsilon_0|\vec{r}-\vec{r}_2|^3}$$

or

$$\vec{F} = \frac{Q}{4\pi\varepsilon_0}\sum_{k=1}^{N}\frac{Q_k\,(\vec{r}-\vec{r}_k)}{|\vec{r}-\vec{r}_k|^3}$$

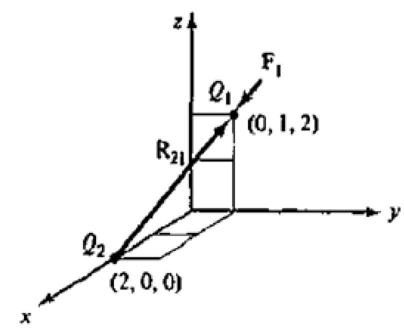
Example: Find the force on Q_1 (20µC) duo to charge Q_2 (-300µC), where Q_1 located at (0, 1, 2) and Q_2 at (2, 0, 0)?

Solution:

$$\vec{R}_{21} = (0,1,2)-(2,0,0)$$
$$= (0-2)\boldsymbol{a}_x + (1-0)\boldsymbol{a}_y + (2-0)\boldsymbol{a}_z$$
$$= -2\boldsymbol{a}_x + \boldsymbol{a}_y + 2\boldsymbol{a}_z$$

$$|\vec{R}_{21}| = \sqrt{(-2)^2+(1)^2+(2)^2} = 3$$

$$\mathbf{F}_{21} = \frac{Q_1Q_2}{4\pi\varepsilon_0 {R_{21}}^2}\mathbf{a}_{R_{21}}$$



$$\vec{F}_1 = \frac{20*10^{-6}*(-300\times10^{-6})}{4\pi*\ 8.854\ *10^{-12}\ *3^2}\left[\frac{-2\boldsymbol{a}_x+\boldsymbol{a}_y+2\boldsymbol{a}_z}{3}\right]$$

$$\vec{F}_1 = 4\boldsymbol{a}_x - 2\boldsymbol{a}_y - 4\boldsymbol{a}_z$$

$$|\vec{F}_1| = \sqrt{(4)^2+(-2)^2+(-4)^2} = 6\text{N}$$

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- **The Electric Field Intensity (E)** شدة المجال الكهربائي

If we now consider one charge fixed in position, say Q_1, and move a second charge slowly around, we note that there exists everywhere a force on this second charge; in other words, this second charge is displaying the existence of a force field. Call this second charge a test charge Qt. The force on it is given by Coulomb's law,

$$\mathbf{F}_t = \frac{Q_1 Q_t}{4\pi\varepsilon_0 |R|^2}\mathbf{a}_{R_{1t}}$$

Writing this force as a force per unit charge gives

$$\frac{\mathbf{F}_t}{Q_t} = \frac{Q_1}{4\pi\varepsilon_0 |R|^2}\mathbf{a}_{R_{1t}}$$

The quantity on the right side of the equation above is a function only of Q_2 and the directed line segment from Q_2 to the position of the test charge. This describes a vector field and is called the electric field intensity.

Using a capital letter E for electric field intensity, we have finally

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$$\mathbf{E} = \frac{\mathbf{F}_t}{Q_t}$$

$$\vec{\mathrm{E}} = \frac{Q_1}{4\pi\varepsilon_0 |R|^2}\mathbf{a}_{R_{1t}}$$

Where $\vec{\mathrm{E}}$ is electric field intensity measured in newtons/coulomb (N/C) or volts/meter (V/m).

The electric field intensity at point r due to a point charge located at r' is readily obtained from eqs.

$$\vec{\mathrm{E}} = \frac{Q(\vec{r} - \vec{r}')}{4\pi\varepsilon_0 |\vec{r} - \vec{r}'|^3}$$

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Example: Find the electric field intensity (E) at (0, 2, 3) due to a point charge Q (0.4μC) located at (2, 0, 4)?

Solution:

$$\vec{R} = (0,2,3) - (2,0,4)$$
$$= (0-2)\boldsymbol{a}_x + (2-0)\boldsymbol{a}_y + (3-4)\boldsymbol{a}_z$$
$$= -2\boldsymbol{a}_x + 2\boldsymbol{a}_y - \boldsymbol{a}_z$$

$$|\vec{R}| = \sqrt{(-2)^2 + (2)^2 + (-1)^2} = 3$$

$$\vec{E} = \frac{Q_1}{4\pi\varepsilon_0 R^2}\mathbf{a}_{R_{1t}}$$

$$\vec{E} = \frac{0.4 \times 10^{-6}}{4\pi \times 8.854 \times 10^{-12} \times 3^2} * \frac{-2\boldsymbol{a}_x + 2\boldsymbol{a}_y - \boldsymbol{a}_z}{3}$$

$$\vec{E} = -266.4\boldsymbol{a}_x + 266.4\boldsymbol{a}_y - 133.2\boldsymbol{a}_z$$

$$|\vec{E}| = \sqrt{(266.4)^2 + (266.4)^2 + (133.2)^2} = 399.6\ V/m$$

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Do This lecture as Assignment

Del Operator - Laplacian Operator - Gradient - Divergence and Curl

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