Electric Flux Density كثافة الفيض الكهربائي

Michael Faraday had a pair of concentric metallic spheres constructed, the outer one consisting of two hemispheres that could be firmly clamped together. He also prepared shells of insulating material (dielectric material) which would occupy the entire volume between the concentric spheres His experiment, then, consisted essentially of the following steps:

1. With the equipment dismantled, the inner sphere was given a known positive charge.
2. The hemispheres were then clamped together around the charged sphere with about 2 cm of dielectric material between them.
3. The outer sphere was discharged by connecting it momentarily to ground.
4. The outer space was separated carefully, using tools made of insulating material in order not to disturb the induced charge on it, and the negative induced charge on each hemisphere was measured.

Faraday found that the total charge on the outer sphere was equal in magnitude to the original charge placed on the inner sphere and that this was true regardless of the dielectric material separating the two spheres. He concluded that there was some sort of "displacement" from the inner sphere to the outer which was independent of the medium, and we now refer to this flux as displacement, displacement flux, or simply electric flux.

Faraday's experiments also showed, of course, that a larger positive charge on the inner sphere induced a correspondingly larger negative charge on the outer sphere, leading to a direct proportionality between the electric flux and the charge on the inner sphere

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$$\Psi = Q$$

Where Ψ (psi) is electric flux in coulombs C

We can obtain more quantitative information by considering an inner sphere of radius a and an outer sphere of radius b, with charges of Q and -Q, respectively (Fig. 3.6). The paths of

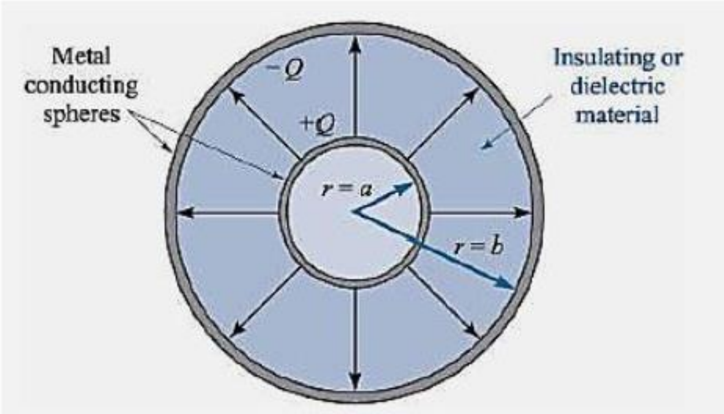

Figure 3.6: The electric flux in the region between a pair of charged concentric sphere

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Electric flux Ψ extending from the inner sphere to the outer sphere is indicated by the symmetrically distributed streamlines drawn radially from one sphere to the other.

At the surface of the inner sphere, Ψ coulombs of electric flux are produced by the charge $Q(=\Psi)$ coulombs distributed uniformly over a surface having an area of $4\pi a^2 m^2$. The density of the flux at this surface is $\Psi/4\pi a^2$ or $Q/4\pi a^2 \ C/m^2$, and this is an important new quantity.

Referring again to Fig. 3.6, the electric flux density is in the radial direction and has a value of

$$\vec{D}(at\ r=a)=\frac{Q}{4\pi a^2}\,\boldsymbol{a}_r \qquad (inner\ shere)$$

$$\vec{D}(at\ r=b)=\frac{Q}{4\pi b^2}\,\boldsymbol{a}_r \qquad (outer\ shere)$$

and at a radial distance r, where $a \le r \le b$

$$\vec{D}=\frac{Q}{4\pi r^2}\,\boldsymbol{a}_r$$

If we now let the inner sphere become smaller and smaller, while still retaining a charge of Q, it becomes a point charge in the limit, but the electric flux density at a point r meter from the point charge is still given by

$$\vec{D}=\frac{Q}{4\pi r^2}\,\boldsymbol{a}_r$$

$$\vec{D}=\varepsilon_0\vec{E} \qquad (in\ free\ space)$$

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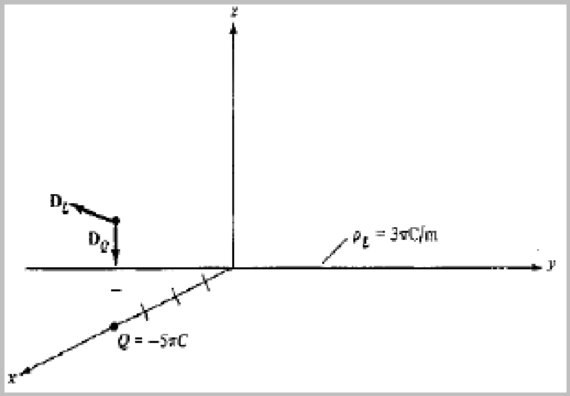
Example: Determine $\overline{D}$ at $(4, 0, 3)$ if there is a point charge $-5\pi\ mC$ at $(4, 0, 0)$ and a line charge $3\pi\ mC/m$ along the $y - axis$.

Solution:



$$\vec{D} = \vec{D}_Q + \vec{D}_L$$

$\vec{D}_Q$ *due to the point charge*

$\vec{D}_L$ *due to the line charge*

$$\vec{D}_Q = \varepsilon_0 \vec{E}$$

$$\vec{D}_Q = \frac{Q}{4\pi|\vec{R}|^2}\,\boldsymbol{a}_R$$

$$\vec{R} = (4, 0, 3) - (4, 0, 0)$$

$$\vec{R} = 3\,\mathbf{a}_z \qquad |\vec{R}| = \sqrt{3^2} = 3$$

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$$\mathbf{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{3\,\boldsymbol{a}_z}{3} = \boldsymbol{a}_z$$

$$\vec{D}_Q = \frac{-5\pi * 10^{-3}}{4\pi * 3^2}\,\boldsymbol{a}_z = -0.138\,\boldsymbol{a}_z \quad mC/m^2$$

$$\vec{D}_L = \frac{\rho_L}{2\pi\rho}\,\mathbf{a}_\rho$$

$$\rho = (4,0,3) - (0,0,0) = 4\,\boldsymbol{a}_x + 3\,\boldsymbol{a}_z$$

$$|\,\vec{\rho}| = \sqrt{4^2+3^2} = 5$$

$$\vec{D}_L = \frac{3\pi}{2\pi * 5}\,\frac{4\,\boldsymbol{a}_x + 3\,\boldsymbol{a}_z}{5} = 0.24\,\boldsymbol{a}_x + 0.18\,\boldsymbol{a}_z \;\; mC/m^2$$

Since

$$\vec{D} = \vec{D}_Q + \vec{D}_L$$

$$\therefore \vec{D} = -0.138\,\boldsymbol{a}_z + 0.24\,\boldsymbol{a}_x + 0.18\,\boldsymbol{a}_z$$

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Gauss's Law

These generalizations of Faraday's experiment lead to the following statement, which is known as Gauss's law:

"The electric flux passing through any closed surface is equal to the total charge enclosed by that surface"

قانون جاوس: الفيض الكهربائي المار خلال اي سلك مغلق يساوي الشحنة الكمية المحتواة بذلك السطح

$$\Psi = Q_{enclosed}$$

$$\Delta\Psi = \vec{D}_S \cdot \Delta S$$

$$\Psi = \oint_S \vec{D}_S \cdot d\vec{S}$$

توضع دائرة صغيرة على علامة التكامل تشير الى ان التكامل مؤدي على سطح مغلق ويسمى هذا السطح ب "سطح جاوس"

- ***The charge enclosed might be several point charges, in which case***

$$Q_{enclosed} = \sum Q_n$$

- *or a line charge*

$$Q_{enclosed} = \int \rho_L \, dL$$

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- *or a surface charge*

$$Q_{enclosed} = \int_S \rho_S \, dS$$

- *or a volume charge*

$$Q_{enclosed} = \int_{vol} \rho_v \, dv$$

The last form is usually used, and we should agree now that it represents any or all of the other forms. With this understanding Gauss's law may be written in terms of the charge distribution as

$$\oint_S \vec{D}_S \cdot d\vec{S} = \int_{vol} \rho_v \, dv$$

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