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1.1 introduction

We come across different quantities in the study of physical phenomena such as mass volume of the body time temperature speed and so on. All these quantities are such that they can be expressed is completely by their magnitude by a single number for example mass of a body can be specified by the number of grams and time by minutes and so on. Such a quantities are called scalars. There are certain other quantities which cannot be expressed completely by their magnitude alone like velocity acceleration Force displacement momentum and so on These are the quantities that can be expressed completely by their magnitude and Direction such quantities are called vectors.

1.2 Scalars and vectors

Vector analysis is a mathematical tool with which electromagnetic (EM) concepts are most conveniently expressed and best comprehended.

A **scalar** is a quantity that has only magnitude

Quantities such as time, mass, distance, temperature, electric potential, and population are scalars.

A **vector** is a quantity that has both magnitude and direction

Vector quantities include velocity, force, displacement, and electric field intensity

A **field** is a function that specifies a particular quantity everywhere in a region

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1.2 UNIT VECTOR

Let **A** be a vector $\bar{\mathbf{A}} = A_x\boldsymbol{a}_x + A_y\boldsymbol{a}_y + A_z\boldsymbol{a}_z$ The *magnitude* of **A** is a scalar written as A or $|\vec{A}|$, which is given by

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

A unit vector a_A along A is defined as a vector whose magnitude is unity (i.e., 1) and its direction is along A , that is,

$$\boldsymbol{a}_A = \frac{\mathbf{A}}{|\mathbf{A}|} = \frac{\vec{A}}{|\vec{A}|}$$

or

$$\boldsymbol{a}_A = \frac{A_x\boldsymbol{a}_x + A_y\boldsymbol{a}_y + A_z\boldsymbol{a}_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

Note that $|\boldsymbol{a}_A| = 1$.

1.3 VECTOR ADDITION AND SUBTRACTION

Two vectors **A** and **B** can be added together to give another vector **C**; that is,

$$\mathbf{C} = \mathbf{A} + \mathbf{B}$$

The vector addition is carried out component by component. Thus, if

$$\vec{A} = A_x\boldsymbol{a}_x + A_y\boldsymbol{a}_y + A_z\boldsymbol{a}_z \quad \text{and} \quad \vec{B} = B_x\boldsymbol{a}_x + B_y\boldsymbol{a}_y + B_z\boldsymbol{a}_z.$$

$$\vec{A} + \vec{B} = (A_x + B_x)\boldsymbol{a}_x + (A_y + B_y)\boldsymbol{a}_y + (A_z + B_z)\boldsymbol{a}_z$$

Vector subtraction is similarly carried out as

$$\vec{A} - \vec{B} = (A_x - B_x)\boldsymbol{a}_x + (A_y - B_y)\boldsymbol{a}_y + (A_z - B_z)\boldsymbol{a}_z$$

1.3 POSITION AND DISTANCE VECTORS

The position vector r_P . (or radius vector) of point P is defined as the directed distance from the origin O to P, that is

$$\boldsymbol{r}_p = OP = x\boldsymbol{a}_x + y\boldsymbol{a}_y + z\boldsymbol{a}_z$$

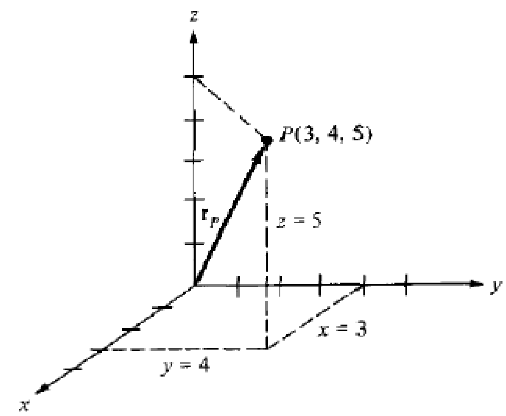

Figure 1.1 Illustration of position vector $r_p = 3a_x + 4a_y + 5a_z$.

The distance vector is the displacement from one point to another

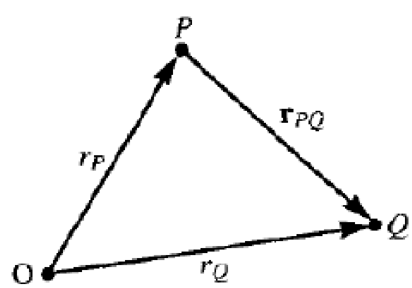

Figure 1.2 Distance vector r_{pQ}.

If two points P and Q are given by (x_P, y_P, z_P) and (x_Q, y_Q, z_Q), the **distance vector** is the displacement from P to Q as shown in Figure 1.2; that is,

$$\boldsymbol{r}_{PQ} = \boldsymbol{r}_Q - \boldsymbol{r}_p$$

$$= (x_Q - x_P)\boldsymbol{a}_x + (y_Q - y_P)\boldsymbol{a}_y + (z_Q - z_P)\boldsymbol{a}_z$$

Example: Find the vector between the points P (1, 4, 2) and Q (3, 1, 6)?

Solution: $\overrightarrow{PQ} = (3,1,6) - (1,4,2)$

$$\overrightarrow{PQ} = (3-1)\boldsymbol{a}_x + (1-4)\boldsymbol{a}_y + (6-2)\boldsymbol{a}_z = 2\boldsymbol{a}_x - 3\boldsymbol{a}_y + 4\boldsymbol{a}_z$$

$$\overrightarrow{QP} = (1,4,2) - (3,1,6)$$

$$\overrightarrow{QP} = (1-3)\boldsymbol{a}_x + (4-1)\boldsymbol{a}_y + (2-6)\boldsymbol{a}_z = -2\boldsymbol{a}_x + 3\boldsymbol{a}_y - 4\boldsymbol{a}_z$$

Example: If $\boldsymbol{A} = 10\boldsymbol{a}_x - 4\boldsymbol{a}_y + 6\boldsymbol{a}_z$ and $\boldsymbol{B} = 2\boldsymbol{a}_x + \boldsymbol{a}_y$ find:

(a) the component of A along $\boldsymbol{a}_y$, (b) the magnitude of $\boldsymbol{A}$,

(c) the magnitude of $\boldsymbol{B}$, (d) the magnitude of $3\boldsymbol{A} - \boldsymbol{B}$,

(e) a unit vector along $\boldsymbol{A}$.

Solution: (a) The component of A along $\boldsymbol{a}_y$ is $A_y = -4$.

(b) $A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{10^2 + 4^2 + 6^2} = 12.32$

(c) $B = \sqrt{2^2 + 1^2} = 2.23$

(d) the magnitude of $3\boldsymbol{A} - \boldsymbol{B}$

$$3\boldsymbol{A} - \boldsymbol{B} = 3(10\boldsymbol{a}_x - 4\boldsymbol{a}_y + 6\boldsymbol{a}_z) - (2\boldsymbol{a}_x + \boldsymbol{a}_y) = 28\boldsymbol{a}_x - 13\boldsymbol{a}_y + 18\boldsymbol{a}_z$$

$$|3\boldsymbol{A} - \boldsymbol{B}| = \sqrt{28^2 + 13^2 + 18^2} = \sqrt{1277} = 35.74$$

(e) $$\boldsymbol{a}_A = \frac{\vec{A}}{|\vec{A}|} = \frac{10\boldsymbol{a}_x - 4\boldsymbol{a}_y + 6\boldsymbol{a}_z}{12.32}$$

$$\boldsymbol{a}_A = 0.812\boldsymbol{a}_x - 0.32\ \boldsymbol{a}_y + 0.49\ \boldsymbol{a}_z$$

$$|\boldsymbol{a}_A| = \sqrt{0.812^2 + 0.32^2 + 0.49^2} = 1$$

Example : Points P and Q are located at (0, 2, 4) and (- 3 , 1, 5). Calculate

(a)The position vector P

(b) The distance vector from P to Q

(c) The distance between P and Q

(d) A vector parallel to PQ with magnitude of 10

Solution:

(a) $\vec{P} = 0\boldsymbol{a}_x + 2\boldsymbol{a}_y + 4\boldsymbol{a}_z = 2\boldsymbol{a}_y + 4\boldsymbol{a}_z$

(b) $\boldsymbol{r}_{PQ} = \boldsymbol{r}_Q - \boldsymbol{r}_P = (-3, 1, 5) - (0, 2, 4)$

$$= -3\boldsymbol{a}_x - \boldsymbol{a}_y + \boldsymbol{a}_z$$

(c) Since $\boldsymbol{r}_{PQ}$ is the distance vector from P to Q, the distance between P and Q is the magnitude of this vector; that is,

$$d = |\boldsymbol{r}_{PQ}| = \sqrt{9 + 1 + 1} = 3.317$$

Alternatively:

$$d = \sqrt{(x_Q - x_P)^2 + (y_Q - y_P)^2 + (z_Q - z_P)^2} = \sqrt{9 + 1 + 1} = 3.317$$

(d) Let the required vector be $\vec{A}$, then

$$\boldsymbol{a}_A = \frac{\vec{A}}{|\vec{A}|} \rightarrow \vec{A} = |\vec{A}|\boldsymbol{a}_A$$

where $|\vec{A}| = 10$ is the magnitude of $\vec{A}$. Since $\vec{A}$ is parallel to PQ, it must have the same unit vector as $\boldsymbol{r}_{PQ}$ or $\boldsymbol{r}_{QP}$. Hence,

$$\boldsymbol{a}_A = \boldsymbol{a}_{PQ} = \frac{\boldsymbol{r}_{PQ}}{|\boldsymbol{r}_{PQ}|} = \frac{-3\boldsymbol{a}_x - \boldsymbol{a}_y + \boldsymbol{a}_z}{3.317}$$

$$\vec{A} = \frac{10(-3\boldsymbol{a}_x - \boldsymbol{a}_y + \boldsymbol{a}_z)}{3.317} = (-9.045\boldsymbol{a}_x - 3.015\boldsymbol{a}_y + 3.015\boldsymbol{a}_z)$$