AL- Mustaqpal University
Science College
Dep. Medical physics

Medical physics
Third Stage

Lec 4

Boltzmann distribution

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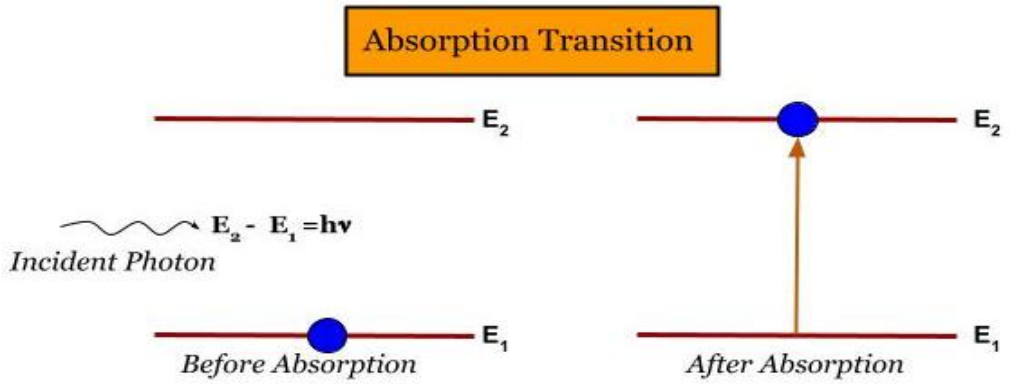
Boltzmann distribution

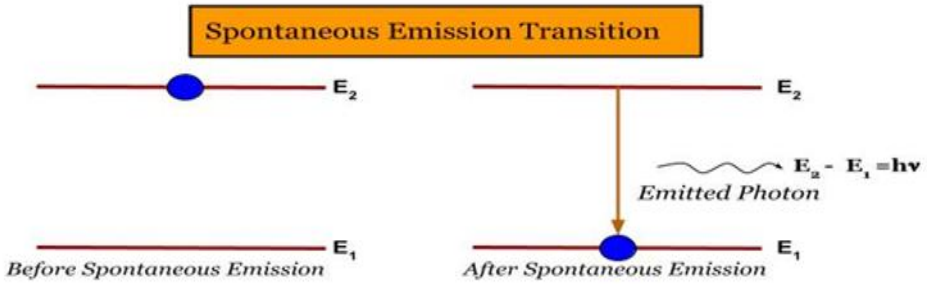
Induced absorption

It is the transition of an atom from a low energy level (E_1) to an excited energy level called (E_2) by absorbing a photon whose energy is equal to the energy difference between these two levels ($hf = E_2 - E_1$)



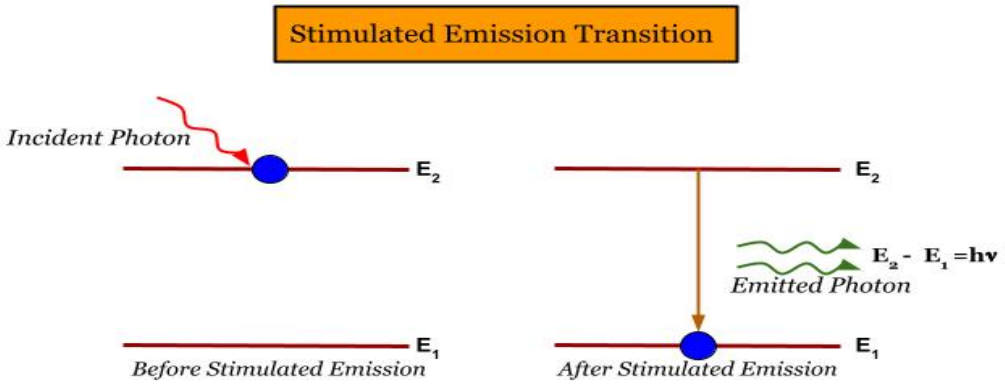
Spontaneous emission

When the atom becomes at the highest energy level (excited), it always tends to a stable state, so it automatically returns after a short period of time to the ground level, and this is accompanied by the emission of a photon... (This transition is called spontaneous emission, and its photons are different in terms of phase and direction).



Stimulated emission

When a photon impacts an excited atom while it is at the energy level (E_2), its energy is exactly equal to the energy difference between (E_1) and (E_2), it stimulates the unstable electron to descend to the level (E_1) and emit a photon similar to the stimulated photon with energy, frequency, phase, and direction (This means that we get two identical photons).



What is Boltzmann distribution ?

The Boltzmann distribution gives the probability that a system will be in a certain state as a function of that state's energy

If we have a system consisting of (molecules, atoms, or ions) in a state of thermal equilibrium, the number of atoms in the lower energy levels will be greater than the number of atoms in the higher (excited) levels, meaning that ($N_1 > N_2$).

But if the atoms gain a certain energy, they become agitated, and the number of atoms in the lower energy levels is less than the number of atoms in the higher (excited) levels, meaning that ($N_1 < N_2$)

What is meant by inverse distribution ?

It is when the number of atoms at high energy levels is greater than their number at low levels... This happens when the atomic system is not thermally balanced (which means that the inverted distribution contradicts the Boltzmann distribution).

The **scientist Boltzmann** established a law regarding this :

$$\frac{N_2}{N_1} = exp\left(\frac{-(E_2 - E_1)}{KT}\right)$$

N_1 is the number of atoms in the lower energy state, N_2 is the number of atoms in the excited state. The process of making $N_2 > N_1$ is called population inversion. **K** is Boltzmann's constant equal to ($1.38064852 \times 10^{-23}\ m^2\ kg\ s^{-2}\ K^{-1}$)

T absolute temperature and is measured in Kelvin

The distribution of the atoms between the levels is given by Boltzmann's Law

This equation means that at thermal equilibrium, $N_1 > N_2$.

To achieve laser operation, one must upset the thermal equilibrium $N_2 > N_1$ in some way so as to produce the nonequilibrium situation of a population inversion.

With a number of energy levels the amount of energy is ($E_1 < E_2$), the energy population ($E_1 < E_2$)

In the Boltzmann Distribution, The Boltzmann equation determines the relation between the population number of a specific energy level and the temperature

Conclusions :

1. The relation between two population numbers (N_2 / N_1) does not depend on the values of the energy levels E_1 and E_2, but only on the difference between them:

   ($E_2 - E_1$)

2. For a certain energy difference, the higher the temperature, the bigger the relative population.

3. If N_2 is greater than N_1, the inverse distribution will be achieved and laser action will occur , If N_2 is smaller than N_1, the inverse distribution will not be achieved and the laser action will not occur.

Example 1

mathematically that the inverse distribution is not achieved when the thermal energy (**KT**) is equal to the energy of the incident photon?

$$\frac{N_2}{N_1} = exp\left(\frac{-(E_2 - E_1)}{KT}\right)$$

$$But \quad E_2 - E_1 = hf \qquad And \qquad KT = hf$$

$$\therefore \frac{N_2}{N_1} = exp - \left(\frac{hf}{hf}\right)$$

$$\therefore \frac{N_2}{N_1} = exp(-1)$$

$$\therefore N_2 = 0.37\, N_1$$

That is, $N_1 > N_2$, and therefore the inverse distribution is not achieved.

Example

Calculate the number of atoms in the highest energy level in thermal equbrium if the number of atoms in the ground energy level is 500 atoms ?

$$\frac{N_2}{N_1} = exp\left(\frac{-(E_2 - E_1)}{KT}\right)$$

In thermal equbrium $KT = (E_2 - E_1)$

$$\therefore \frac{N_2}{N_1} = exp - \left(\frac{(E_2 - E_1)}{(E_2 - E_1)}\right)$$

$$\therefore \frac{N_2}{N_1} = exp - 1$$

$$\therefore \frac{N_2}{N_1} = 0.37$$

$$\therefore \frac{N_2}{500} = 0.37$$

$$\therefore N_2 = 185 \quad Atoms$$

So the inverted distribution does not occur because the number of atoms in the upper level is less than in the ground level.

Discussion

1. What is the Boltzmann distribution?
   A. A probability that a system is in a state as a function of that state's energy
   B. A rule for photon polarization
   C. A law for conservation of charge
   D. A chemical equilibrium constant
   E. A formula for laser cavity length
   Answer: A

2. At thermal equilibrium for two levels E1<E2, the usual relation between populations is:
   A. N1 > N2
   B. N1 = 0
   C. N2 > N1
   D. N1 = N2 always
   E. N2 = 0
   Answer: A

3. The Boltzmann population ratio formula is:
   A. ($N_2 / N_1 = \exp \backslash (E_2 - E_1)/kT$)
   B. ($N_2 / N_1 = (E_2 – E_1) / kT$)
   C. ($N_2 / N_1 =$ exp $(-(E_2 - E_1) / kT$)
   D. ($N_2 / N_1 = kT / (E_2 - E_1)$)
   E. ($N_2 / N_1 =$ ln $(E_2 - E_1) / kT$)
   Answer: C

4. Population inversion means:

   A. $N_2 < N_1$

   B. $N_2 = N_1$

   C. $N_2 > N_1$

   D. All atoms are in ground state

   E. Temperature is zero

   Answer: C

5. For a fixed energy gap $\Delta E = E_2 - E_1$, increasing temperature T will:

   A. Decrease N_2 / N_1

   B. Increase N_2 / N_1

   C. Keep N_2 / N_1 zero

   D. Make N_1 infinite

   E. Make N_2 negative

   Answer: B

6. What condition is required for laser action?

   A. Thermal equilibrium

   B. Complete absence of photons

   C. Population inversion ($N_2 > N_1$)

   D. $E_1 = E_2$

   E. Infinite volume

   Answer: C

7. Which process can produce two identical photons from one excited atom?

   A. Induced absorption

   B. Thermal relaxation

   C. Stimulated emission

   D. Spontaneous ionization

   E. Elastic scattering

   Answer: C

8. If (kT = hf) and ΔE = hf, then (N 2 / N 1) equals:

   A. 0

   B. 0.5

   C. 0.37 (≈ e^{-1})

   D. 1

   E. e

   Answer: C

9. If N1 = 500 and (N 2 / N 1 = 0.37), N2 is approximately:

   A. 370

   B. 185

   C. 500

   D. 135

   E. 1000

   Answer: B

10. Which is the approximate value of Boltzmann's constant (k)?

    A. (1.38 \ 10^{-23}) (SI units)

    B. (9.81)

    C. (6.63 \ 10^{-34})

    D. (3.00 \ 10^{8})

    E. (1.60 \ 10^{-19})

    Answer: A

11. For fixed T, if the energy gap ΔE increases, the ratio (N 2 / N 1) will:

    A. Increase

    B. Decrease

    C. Remain unchanged

    D. Become infinite

    E. Become negative

    Answer: B

12. Which process requires an incoming photon to trigger emission?

A. Spontaneous emission

B. Stimulated emission

C. Thermal conduction

D. Radioactive decay

E. Chemical reaction

Answer: B

13. Which emission happens without any external photon stimulation?

A. Stimulated emission

B. Induced absorption

C. Spontaneous emission

D. Population inversion

E. Optical pumping

Answer: C

14. Induced absorption means atom:

A. Emits

B. Absorbs

C. Reflects

D. Scatters

E. Ionizes

Answer: B

15. Spontaneous emission is:

A. Absorb

B. Forced

C. Automatic

D. Random

E. Thermal

Answer: C

16. Stimulated emission photons are:

A. Random

B. Lower

C. Identical

D. Absorbed

E. Lost

Answer: C

17. Boltzmann law gives:

A. Force

B. Charge

C. Probability

D. Speed

E. Mass

Answer: C

18. At equilibrium:

A. $N1 > N2$

B. $N1 = 0$

C. $N2 > N1$

D. $N1 = N2$

E. $N2 = 0$

Answer: A

19. The correct formula is :

A. $\exp(+\Delta E / kT)$

B. $\Delta E / kT$

C. $\exp(-\Delta E / kT)$

D. $kT / \Delta E$

E. $\ln(\Delta E / kT)$

Answer: C

20. Population inversion:

A. N1 > N2

B. N1 = N2

C. N2 > N1

D. All in E1

E. Zero T

Answer: C

21. Inversion vs Boltzmann:

A. Normal

B. Contradict

C. Low T

D. Always

E. Require

Answer: B

22. Higher T → N2 / N1 :

A. Down

B. Up

C. Zero

D. ∞

E. −1

Answer: B

23. Laser needs:

A. Equil.

B. No photons

C. Inversion

D. E1 = E2

E. ∞ size

Answer: C

24. N2 / N1 depends on:

A. ΔE , T

B. E1 , E2

C. Number

D. Length

E. Mass

Answer: A

25. Spontaneous photon is:

A. Same

B. Higher

C. Random

D. False

E. Polarized

Answer: C

26. Two photons process:

A. Absorb

B. Relax

C. Stimulated

D. Ionize

E. Scatter

Answer: C

27. If kT = hf then :

A. 0

B. 0.5

C. 0.37

D. 1

E. e

Answer: C

28. If N1= 500 , ratio = 0.37 → N2 = ?

A. 370

B. 185

C. 500

D. 135

E. 1000

Answer: B

29. k = ?

A. 1.38×10^{-23}

B. 9.81

C. 6.63×10^{-34}

D. 3×10^{8}

E. 1.6×10^{-19}

Answer: A

30. Stimulated vs spontaneous:

A. No photon

B. Same phase

C. Random

D. Ionize

E. 0K

Answer: B

31. $hf = E_2 - E_1$ means :

A. Equal

B. Zero

C. 0K

D. Auto inversion

E. False

Answer: A

32. Needs incoming photon:

A. Spontaneous

B. Stimulated

C. Heat

D. Decay

E. Chem.

Answer: B

33. Create inversion:

A. Heat

B. Pumping

C. Equil.

D. Remove

E. 0K

Answer: B

34. Level distribution law:

A. Boltzmann

B. Fermi

C. Uniform

D. Free

E. Inverse

Answer: A
