# Mechanics — Lecture 5

## Oscillatory Motion: Simple Harmonic, Damped, and Forced Oscillations

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# 1 Oscillatory Motion: Physical Meaning and Scope

Oscillatory motion is a fundamental type of motion in which a physical system executes repetitive movement about a fixed equilibrium position. Such motion arises whenever a system possesses a stable equilibrium and inertia prevents an immediate return to that equilibrium after displacement.

When a particle is displaced slightly from equilibrium, internal forces act to restore it. Because the particle has mass, it overshoots the equilibrium position, leading to sustained back-and-forth motion. This interplay between restoring forces and inertia is the essence of oscillatory motion.

Oscillatory phenomena appear throughout physics and medicine: mechanical vibrations, molecular oscillations, acoustic waves, MRI gradient coils, ultrasound transducers, and biomechanical systems.

## 1.1 Equilibrium and Stability

An equilibrium position $x_0$ is defined by the condition

$$F(x_0) = 0.$$

The equilibrium is:

- **Stable** if small displacements generate restoring forces
- **Unstable** if displacements grow with time

Mathematically, stability is determined by the slope of the force near equilibrium.

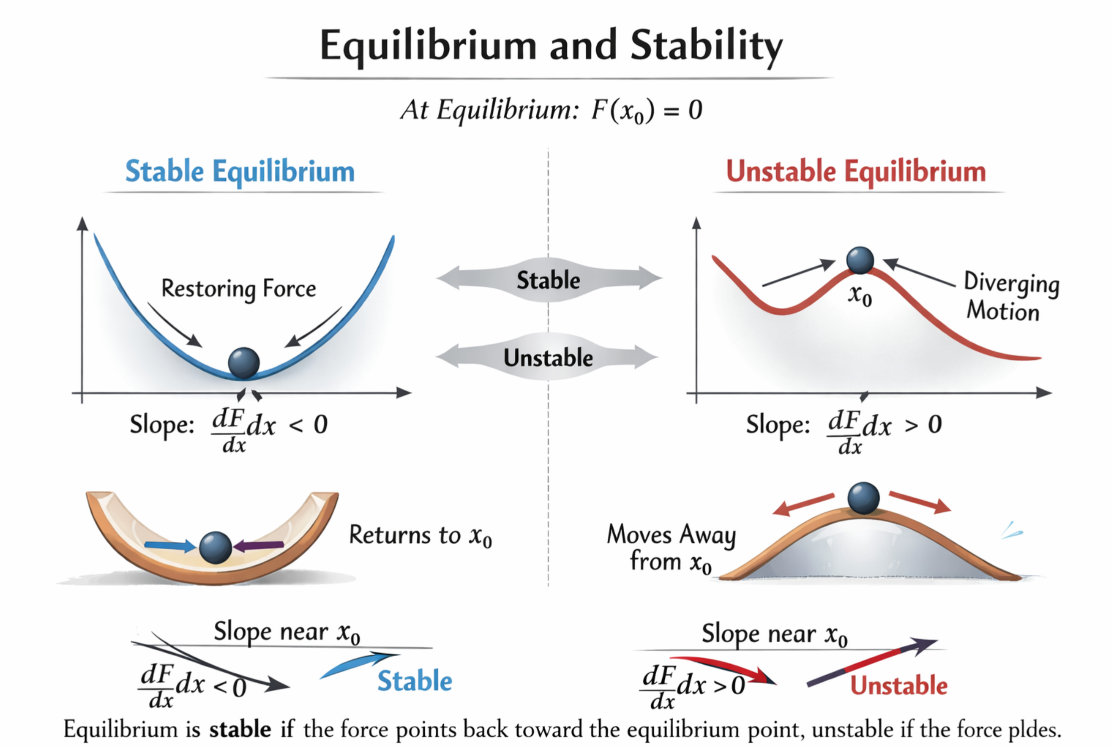



Figure 1: Equilibrium and stability. A stable equilibrium produces restoring forces after small displacements, while an unstable equilibrium causes displacements to grow.

# 2 Hooke's Law: Extension of a Spring

When a force is applied to an elastic spring, the spring undergoes deformation. For sufficiently small deformations, the extension of the spring is directly proportional to the applied force. This linear relationship is known as **Hooke's law**.

## 2.1 Mathematical Statement of Hooke's Law

The restoring force exerted by a spring is given by

$$\boxed{F_{\text{spring}} = -kx}$$

where:

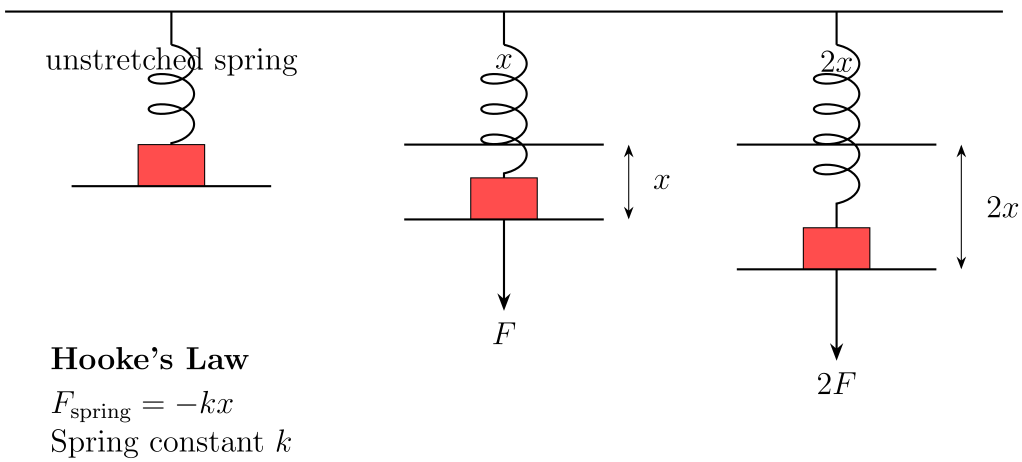
- $k$ is the spring constant (N/m),
- $x$ is the extension or compression from the natural (unstretched) length,
- the negative sign indicates that the force opposes the displacement.

## 2.2 Physical Interpretation

Hooke's law implies that:

- A larger applied force produces a larger extension.
- Doubling the applied force doubles the extension.
- The spring constant $k$ measures the stiffness of the spring.

This linear behavior is valid only within the elastic limit of the spring.



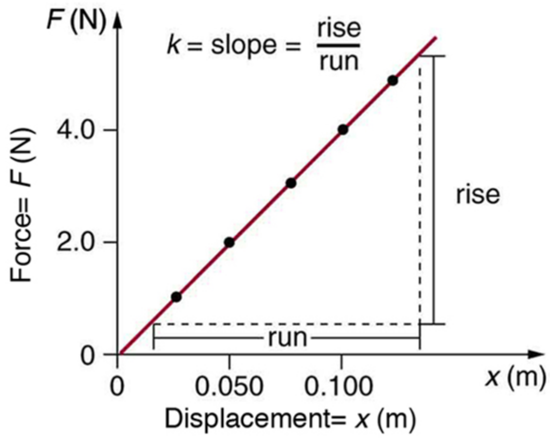

## 2.3 Force–Displacement Diagram (Hooke's Law)

For small displacements from equilibrium, the restoring force exerted by a spring is directly proportional to the displacement and opposite in direction. This linear relationship is known as Hooke's law.

$$F = -kx$$

The linear relationship between the restoring force exerted by a spring and its displacement from equilibrium is known as Hooke's law. Within the elastic limit of the spring, the magnitude of the restoring force is directly proportional to the extension or compression of the spring and acts in the direction opposite to the displacement. This behavior can be demonstrated experimentally by measuring the extension produced by different applied forces.

a)




| m (kg) | w (N) | x (m) |
|---|---|---|
| 0.000 | 0.00 | 0.000 |
| 0.100 | 0.98 | 0.025 |
| 0.200 | 1.96 | 0.050 |
| 0.300 | 2.94 | 0.076 |
| 0.400 | 3.92 | 0.099 |
| 0.500 | 4.90 | 0.127 |

b)

| m (kg) | w (N) | x (m) |
|---|---|---|
| 0.000 | 0.00 | 0.000 |
| 0.100 | 0.98 | 0.025 |
| 0.200 | 1.96 | 0.050 |
| 0.300 | 2.94 | 0.076 |
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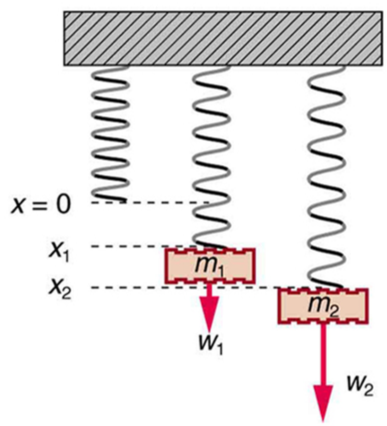



The straight-line graph passing through the origin demonstrates that the restoring force is directly proportional to the displacement. The negative slope indicates that the force always acts in the direction opposite to the displacement, restoring the system toward equilibrium.

## 2.4 Solved Problem 1

A spring has force constant $k = 600\,\mathrm{N/m}$. Find the restoring force when the spring is stretched by $0.04\,\mathrm{m}$.

**Solution**

$$F = -kx = -(600)(0.04) = -24\,\mathrm{N}$$

The negative sign indicates that the force acts toward equilibrium.

## 2.5 Solved Problem 2

Explain why Hooke's law is only an approximation.

**Solution**

For large displacements, higher-order terms in the force expansion become significant. The force is no longer proportional to displacement, and oscillations become anharmonic.

# 3 Simple Harmonic Motion (SHM)

A particle is said to execute simple harmonic motion if its acceleration is proportional to its displacement and directed toward equilibrium.

## 3.1 Equation of Motion

Using Newton's second law with Hooke's law:

$$m\ddot{x} = -kx$$

Dividing by $m$ gives

$$\boxed{\ddot{x} + \omega^2 x = 0} \qquad \omega = \sqrt{\frac{k}{m}}$$

This second-order differential equation defines simple harmonic motion.

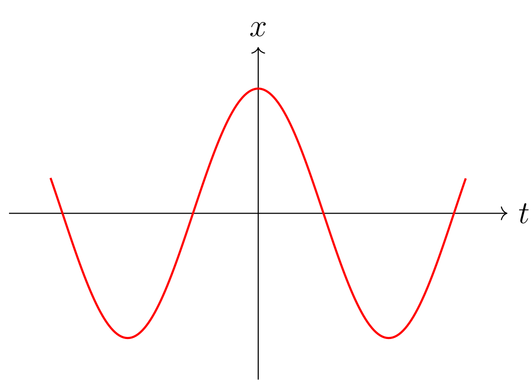
## 3.2 General Solution

The general solution is

$$\boxed{x(t) = A\cos(\omega t + \phi)}$$

where $A$ is the amplitude and $\phi$ is the phase constant.

## 3.3 SHM Trajectory



### 3.4 Solved Problem 1 (Phase Determination)

A particle oscillates with amplitude 0.25 m and angular frequency 4 rad/s. At $t = 0$, its displacement is 0.125 m and it is moving toward equilibrium. Find the equation of motion.

**Solution**

$$0.125 = 0.25 \cos\phi \Rightarrow \phi = \frac{\pi}{3}$$

The motion toward equilibrium requires $\sin\phi > 0$.

$$x(t) = 0.25 \cos(4t + \pi/3)$$

### 3.5 Solved Problem 2 (Turning Points)

Find the times at which the particle reaches maximum displacement.

**Solution**

Maximum displacement occurs when

$$\omega t + \phi = 2\pi n \Rightarrow t = \frac{2\pi n - \phi}{\omega}$$

## 4 Energy in Simple Harmonic Motion

In SHM, mechanical energy is conserved and continuously exchanged between kinetic and potential forms.

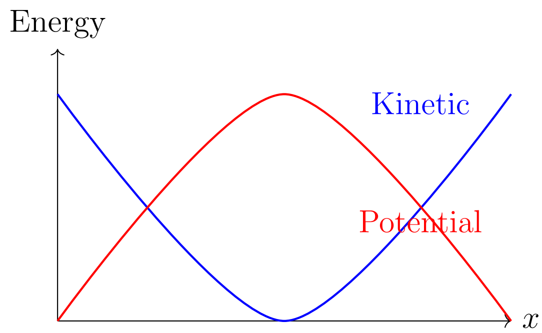
### 4.1 Energy Expressions

$$\boxed{K = \frac{1}{2}mv^2}, \qquad \boxed{U = \frac{1}{2}kx^2}$$

The total energy is

$$\boxed{E = \frac{1}{2}kA^2 = \text{constant}}$$

### 4.2 Energy Exchange Diagram




### 4.3 Solved Problem (Energy Method)

A 2 kg oscillator has total energy 12 J and spring constant 500 N/m.

**(a)** Find the amplitude. **(b)** Find the speed when $x = 0.1$ m.

**Solution**

$$A = \sqrt{\frac{2E}{k}} = 0.22\,\text{m}$$

$$v = \sqrt{\frac{2(E-U)}{m}} = 2.68\,\text{m/s}$$

## 5 Damped Harmonic Motion

In real systems, resistive forces such as friction or air resistance remove energy from oscillatory motion.

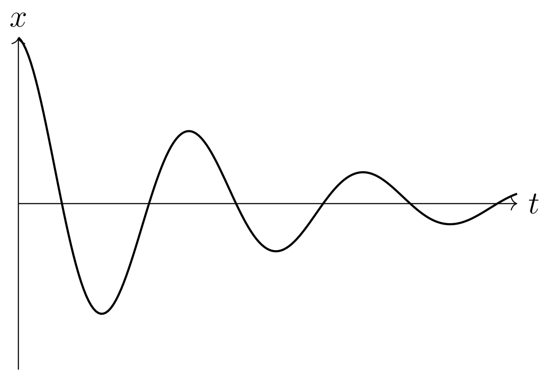
### 5.1 Equation of Motion

$$\boxed{m\ddot{x} + b\dot{x} + kx = 0} \qquad \beta = \frac{b}{2m}$$

For underdamped motion,

$$\boxed{x(t) = Ae^{-\beta t}\cos(\omega t + \phi)}$$

### 5.2 Damped Oscillation




## 6 Forced Oscillations and Resonance

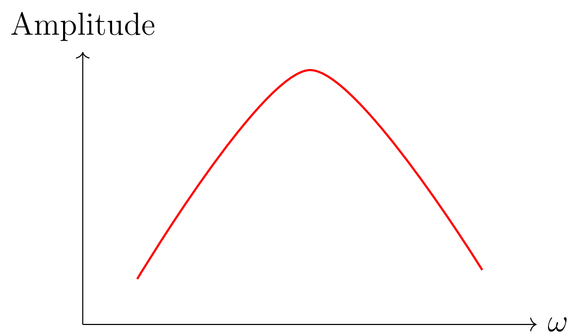
When an external periodic force acts on a damped oscillator, forced oscillations occur.

### 6.1 Driven Oscillator Equation

$$\boxed{m\ddot{x} + b\dot{x} + kx = F_0 \cos(\omega t)}$$

Resonance occurs when the steady-state amplitude is maximized.

### 6.2 Resonance Curve




## Multiple Choice Questions

1. Oscillatory motion is defined as motion that:

   A) is always circular

   B) occurs only under gravity

C) repeats about an equilibrium position

D) occurs at constant speed

E) None of them

2. An equilibrium position is defined by:

A) maximum velocity

B) zero acceleration only

C) zero net force

D) zero displacement

E) None of them

3. A restoring force always acts:

A) away from equilibrium

B) perpendicular to motion

C) toward equilibrium

D) in the direction of velocity

E) None of them

4. Hooke's law is valid when:

A) displacement is large

B) motion is circular

C) deformation is small

D) energy is conserved

E) None of them

5. The mathematical form of Hooke's law is:

A) $F = ma$

B) $F = kx$

C) $F = -kx$

D) $F = mg$

E) None of them

6. The negative sign in $F = -kx$ indicates:

A) force is zero

B) force is constant

C) force is restoring

D) force is maximum

E) None of them

7. The graph of force versus displacement for Hooke's law is:

A) a parabola

B) a circle

C) a straight line through the origin

D) a horizontal line

E) None of them

8. The slope of an $F$–$x$ graph represents:

A) mass

B) acceleration

C) spring constant

D) energy

E) None of them

9. Simple harmonic motion occurs when acceleration is:

A) constant

B) zero

C) proportional to displacement

D) proportional to velocity

E) None of them

10. The defining equation of SHM is:

A) $x = vt$

B) $a = -\omega^2 x$

C) $F = ma$

D) $v = at$

E) None of them

11. The angular frequency $\omega$ depends on:

A) amplitude

B) phase

C) mass and spring constant

D) displacement

E) None of them

12. The SI unit of angular frequency is:

    A) Hz
    B) s
    C) rad/s
    D) m/s
    E) None of them

13. The maximum displacement from equilibrium is called:

    A) period
    B) frequency
    C) amplitude
    D) phase
    E) None of them

14. The velocity of a particle in SHM is maximum at:

    A) maximum displacement
    B) turning points
    C) equilibrium position
    D) random position
    E) None of them

15. The acceleration of a particle in SHM is maximum at:

    A) equilibrium
    B) zero velocity
    C) maximum displacement
    D) zero displacement
    E) None of them

16. The period of SHM is given by:

    A) $T = \omega/2\pi$
    B) $T = 2\pi/\omega$
    C) $T = k/m$
    D) $T = m/k$
    E) None of them

17. The kinetic energy in SHM is maximum at:
    A) amplitude
    B) turning point
    C) equilibrium
    D) maximum force
    E) None of them
18. The potential energy in SHM depends on:
    A) velocity
    B) acceleration
    C) displacement
    D) time
    E) None of them
19. The total mechanical energy in SHM is:
    A) zero
    B) changing
    C) constant
    D) infinite
    E) None of them
20. Damped oscillations occur due to:
    A) restoring force
    B) inertia
    C) resistive force
    D) gravitational force
    E) None of them
21. The damping force is proportional to:
    A) displacement
    B) acceleration
    C) velocity
    D) mass
    E) None of them
22. In damped motion, amplitude:

A) remains constant
B) increases
C) decreases with time
D) becomes zero instantly
E) None of them

23. Forced oscillations require:

A) gravity
B) restoring force only
C) external periodic force
D) constant velocity
E) None of them

24. The steady-state frequency of forced oscillations equals:

A) natural frequency
B) damping frequency
C) driving frequency
D) zero
E) None of them

25. Resonance occurs when:

A) damping is maximum
B) force is zero
C) amplitude is maximum
D) energy is zero
E) None of them

26. Increasing damping causes the resonance peak to:

A) sharpen
B) increase
C) broaden and decrease
D) shift upward
E) None of them

27. The quality factor $Q$ measures:

A) amplitude

B) energy

C) sharpness of resonance

D) displacement

E) None of them

28. A large quality factor implies:

A) strong damping

B) weak damping

C) no oscillation

D) zero energy loss

E) None of them

29. The SI unit of spring constant is:

A) N

B) J

C) kg

D) N/m

E) None of them

30. Oscillatory motion is a result of the balance between:

A) force and velocity

B) inertia and restoring force

C) mass and energy

D) work and power

E) None of them