Magnetism

Lecture 2

Vectors

Msc. Ali Jaafar

2nd stage

Department of medical physics Al-Mustaqbal University-College

2024- 2025

This lecture is concerned with vectors. A vector has a magnitude and a direction. The speed of an object is a scalar, whereas its velocity is a vector.

Vector is a quantity having both magnitude and direction such as displacement, velocity, force and acceleration.

A scalar quantity: is a quantity which is completely characterized by its magnitude. Examples of physical quantities that are scalar are **mass, time, temperature, volume, and work**.

A vector quantity: is a quantity which is completely characterized by its magnitude and direction. Force, velocity, displacement, and acceleration are examples of vector quantities. A vector can be represented geometrically by an arrow whose direction is approximately chosen and whose length is proportional to the magnitude of the vector.

Field: If at each point of a region there is a corresponding value of some physical function, the region is called a field. Fields may be classified as either scalar or vector, depending upon the type of function involved.

Vectors: The Basic Concept

- **Magnitude**: The value of the vector (its length).
- **Direction**: Indicates the direction the vector is pointing to.
- **Representation**: A vector is represented by an arrow whose length represents the magnitude and whose direction shows its orientation.

Vector Addition and Subtraction Graphically

- **Start by drawing the first vector**: Draw the first vector on the graph.
- **Add the second vector**: Place the tail of the second vector at the head of the first vector.
- **Resultant vector**: Draw the resultant vector from the tail of the first vector to the head of the second vector.
- **Vector subtraction**: To subtract two vectors, simply reverse the direction of the second vector (find the negative of the vector), then add it using the same method.

Properties of Vector Addition

- **Commutativity** : A+B=B+A
- **Associativity** : A +(B + C) = (A + B) + C
- **Additive Identity**: A + 0 = A   (The zero vector is the identity element).

1. Vector Algebra

Vector: It is the basic building unit of linear algebra. Rather, it is the foundation on which all linear algebra is based.

Laws of vector algebra. If A, B and C are vectors and m and n are scalars, then

A+ B = B + A   **Commutative Law for Addition**

A+ **(B+C) = (A+B) + C**   **Associative Law for Addition .**

Example:-1

Prove: - (5+7) = (-4)+(-8)

Proof:

-(12) = -4-8

-12 = -12

L.H.S = R.H.S

Example:-2

Let us take A = 2, B = 4 and C = 6

L.H.S =A+(B+C) = 2 + (4 + 6)

= 12

R.H.S = (A+B)+C = (2 + 4) + 6

=12

L.H.S =

R.H.S12 =

12

2. Scalar product

The process of multiplying a vector quantity by another vector quantity, the product of which is a non-vector scalar quantity, which has only an amount..

$$\vec{a} \cdot \vec{b} = |a||b|\cos(\theta).$$

$$\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 1$$
$$\hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{i}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = 0$$

- If A . B = **0** **and A and B are not null vectors, then A and B are perpendicular.**

Example:

Calculate the scalar product of vectors **a** and **b**when the modulus of a is 9, modulus of b is 9 and the angle between the two vectors is 60°.

Solution:

To determine the scalar product of vectors a and b, we will use the scalar product formula.

a.b = |a| |b| cosθ

= 9 × 9 cos 60°

= 81 × 1/2

= ……..

Example:

$\vec{A} = 2\hat{\imath} + 3\hat{\jmath}$ $\vec{B} = -\hat{\imath} + 2\hat{\jmath}$

$$\begin{aligned}\mathbf{A}\cdot\mathbf{B} &= (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})\cdot(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) \\ &= -2\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} + 2\hat{\mathbf{i}}\cdot 2\hat{\mathbf{j}} - 3\hat{\mathbf{j}}\cdot\hat{\mathbf{i}} + 3\hat{\mathbf{j}}\cdot 2\hat{\mathbf{j}} \\ &= -2(1) + 4(0) - 3(0) + 6(1) \\ &= -2 + 6 = 4\end{aligned}$$

To calculate the angle between two vectors, we must find the magnitude of each vector.

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(2)^2 + (3)^2} = \sqrt{13}$$
$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}$$

$$\cos\theta = \frac{\mathbf{A}\cdot\mathbf{B}}{AB} = \frac{4}{\sqrt{13}\sqrt{5}} = \frac{4}{\sqrt{65}}$$

$$\theta = \cos^{-1}\frac{4}{8.06} = 60.2°$$

3. Vector products

The process of multiplying a vector quantity by another vector quantity, the product of which is a vector quantity with magnitude and direction.

- The cross product of two vectors is given by the formula $\vec{a} \times \vec{b} = |a||b|\sin(\theta)$.

❖ Cross or vector product

The following laws are valid:

1. A × B = - B × A Commutative Law for Cross ProductsFails
2. A × (B + C) = A × B + A × C Distributive Law
3. m(A × B) = (mA) × B = A × (mB) = (A × B)m , wherem is a scalar.
4. **i × i = j × j = k × k = 0**

i × j = −j × i = k,

j × k = −k × j = i,

k × i = −i × k = j.

5. If A = Al i + A2 j + A3 k and B = Bl i + B2 j + B3 k,then

6. The magnitude of A × B is the same as the area of a parallelogram with sides A and B.

7. If A × B = 0 and A and B are not null vectors, then Aand B are **parallel**.

Example : Find the cross product of two vectors $\vec{a} = (3,4,5)$ and $\vec{b} = (7,8,9)$

Solution:

The cross product is given as,

$$a \times b = \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 5 \\ 7 & 8 & 9 \end{matrix}$$

$= [(4\times9)-(5\times8)]\,\hat{i} - [(3\times9)-(5\times7)]\hat{j} + [(3\times8)-(4\times7)]\,\hat{k}$

$= (36-40)\hat{i} - (27-35)\hat{j} + (24-28)\,\hat{k} = -4\hat{i} + 8\hat{j} - 4\hat{k}$

H.W: Two vectors have their scalar magnitude as $|a|=2\sqrt{3}$ and $|b| = 4$, while the angle between the two vectors is 60∘.Calculate the cross product of two vectors