

Lecture (4)

**Composition of Functions: (تركيب الدوال)**

DEFINITION: If f and g are functions, the composite $(f \circ g)$ ((f composed with g)) or $g \circ f$ ((g composed with f)) are defined by:
 $(f \circ g)(x) = f(g(x))$ and $(g \circ f)(x) = g(f(x))$ respectively

Examples 1: Find the formula for $(f \circ g)(x)$ and $(g \circ f)(x)$ if $g(x) = x^2$ and $f(x) = x - 7$,
 then find the value of $f(g(2))$ and $g(f(2))$.

SOL:

A: $(f \circ g)(x) = f(g(x)) = f(x^2) = x^2 - 7$.

$f(g(2)) = 2^2 - 7 = -3$.

B: $(g \circ f)(x) = g(f(x)) = g(x - 7) = (x - 7)^2$.

$g(f(2)) = (2 - 7)^2 = (-5)^2$.

Examples 2: Find the formula for $(f \circ g)(x)$ and $(g \circ f)(x)$ if $f(x) = x^2 + 1$ and $g(x) = \sqrt{x}$,
 then find the value of $f(g(3))$ and $g(f(3))$.

SOL:

A: $(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 + 1 = x + 1$.

$f(g(3)) = 3 + 1 = 4$.

B: $(g \circ f)(x) = g(f(x)) = g(x^2 + 1) = \sqrt{x^2 + 1}$.

$g(f(3)) = \sqrt{3^2 + 1} = \sqrt{10}$.

H.W: Finding formulas for composites If $f(x) = x$ and $g(x) = x + 1$,

Find:

(a) $(f \circ g)(x)$ **(b)** $(g \circ f)(x)$ **(c)** $(f \circ f)(x)$ **(d)** $(g \circ g)(x)$



Even Function, Odd Function: (الدالة الفردية والدالة الزوجية)

A function $y = f(x)$ is an

even function of x if $f(-x) = f(x)$

odd function of x if $f(-x) = -f(x)$

for every x in the function's domain.

Examples: Recognizing Even and Odd functions

1) $f(x) = x^2$

$$f(-x) = (-x)^2 = x^2$$

$$-f(x) = -(x)^2$$

Even function

2) $f(x) = x^2 + 1$

$$f(-x) = (-x)^2 + 1 = x^2 + 1$$

$$-f(x) = -(x)^2 - 1$$

Even function

3) $f(x) = x$

$$f(-x) = -x$$

$$-f(x) = -x$$

odd function

4) $f(x) = x + 1$

$$f(-x) = -x + 1$$

$$-f(x) = -x - 1$$

Not Even function , Not odd function

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5) $f(x) = x - 1$

$$f(-x) = -x - 1$$

$$-f(x) = -x + 1$$

Not Even function , Not odd function

6) $f(x) = x^3$

$$f(-x) = (-x)^3 = -(x)^3$$

$$-f(x) = -(x)^3$$

Odd function

H.W:

1) $f(x) = x^3 - 3.$

2) $f(x) = x^3 + x^2 - 3$

3) $f(x) = x^2 - x$

4) $f(x) = \frac{1}{x}$

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symmetry of the function (تماثل الدالة)

If $f(x,y) = 0$ is any function then:

1. Symmetry about x-axis: If $f(x,-y) = f(x,y)$
2. Symmetry about y-axis: If $f(-x,y) = f(x,y)$ It is called an **even function**.
3. Symmetry about the origin: If $f(-x,-y) = f(x,y)$ It is called an **odd function**

Examples : Check the symmetry of the following curves:

1) $y = x^2$

Sol \ $f(x,y) = x^2 - y = 0$

$f(x,-y) = x^2 - (-y) = x^2 + y \Rightarrow f(x,-y) \neq f(x,y)$ **NOT OK**

$f(-x,y) = (-x)^2 - (y) = x^2 - y \Rightarrow f(-x,y) = f(x,y)$ **OK**

$f(-x,-y) = (-x)^2 - (-y) = x^2 + y \Rightarrow f(-x,-y) \neq f(x,y)$ **NOT OK**

So the function has symmetry only about y-axis. It is called an even function.

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2) $y = x^3$

Sol \ $f(x, y) = x^3 - y = 0$

$f(x, -y) = x^3 - (-y) = x^3 + y \Rightarrow f(x, -y) \neq f(x, y)$ NOT OK

$f(-x, y) = (-x)^3 - (y) = -x^3 - y \Rightarrow f(-x, y) \neq f(x, y)$ NOT OK

$f(-x, -y) = (-x)^3 - (-y) = -x^3 + y = x^3 - y \Rightarrow$

$f(-x, -y) = f(x, y)$ OK

So the function has symmetry only about origin. It is called an odd function.

3) $x^2 = y^2 + 4$

Sol \ $f(x, y) = y^2 - x^2 + 4 = 0$

$f(x, -y) = (-y)^2 - x^2 + 4 = y^2 - x^2 + 4 \Rightarrow f(x, -y) = f(x, y)$ OK

$f(-x, y) = y^2 - (-x)^2 + 4 = y^2 - x^2 + 4 \Rightarrow f(-x, y) = f(x, y)$ OK

$f(-x, -y) = (-y)^2 - (-x)^2 + 4 = y^2 - x^2 + 4 \Rightarrow f(-x, -y) = f(x, y)$ OK

So the function has symmetry about x-axis, y-axis and the origin.

H.W:

1) $y = 3x^2 + 2$.

2) $x^2 + y^2 = 1$