# Mechanics


Presented by

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First-year students

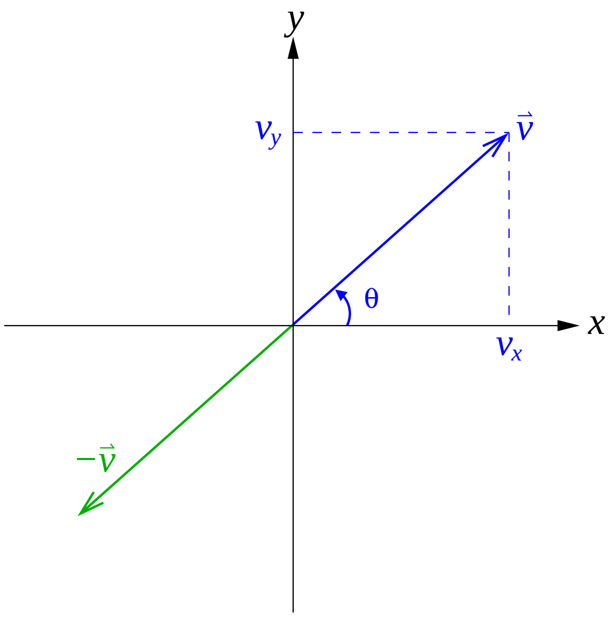
# Lecture 1

# Motion in One Dimension

## 2.1 Vector

a) Definition

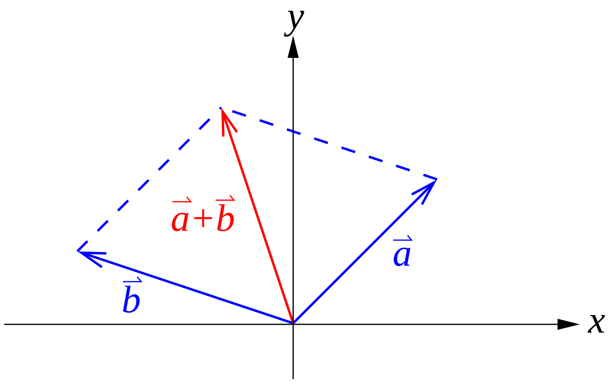
A vector has both magnitude and direction. It is represented by an arrow.




$v_x$ is called the x-component of vector $\vec{v}$ and $v_y$ is called the y-component of vector $\vec{v}$. The magnitude of the vector $\vec{v}$ is denoted by $|\vec{v}|$ or $v$ where

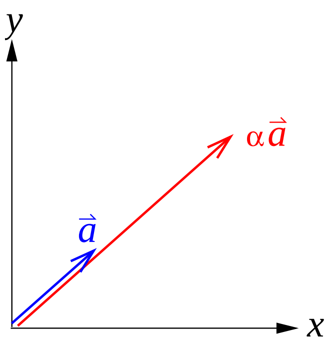
$$|\vec{v}| = v = \sqrt{v_x^2 + v_y^2}$$

b) Addition of vector




c) Multiplication of vector by a scalar

$\alpha\vec{a}$ has the same direction as $\vec{a}$ but has a magnitude equal to $\alpha$ times the magnitude of $\vec{a}$, i. e. $|\alpha\vec{a}| = |\alpha||\vec{a}|$.



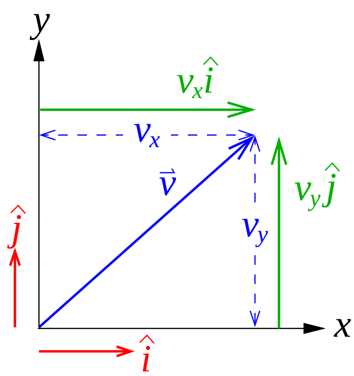

d) Component form of a vector

$\hat{\imath}$ and $\hat{\jmath}$ are called the unit vector of $x$ and $y$ directions which have magnitude of unity, i. e. $|\hat{\imath}| = |\hat{\jmath}| = 1$.
Thus, $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$.
With component form,

$$\vec{u} = u_x\hat{\imath} + u_y\hat{\jmath}, \quad \vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$$

$$\vec{u} + \vec{v} = (u_x + v_x)\hat{\imath} + (u_y + v_y)\hat{\jmath}$$

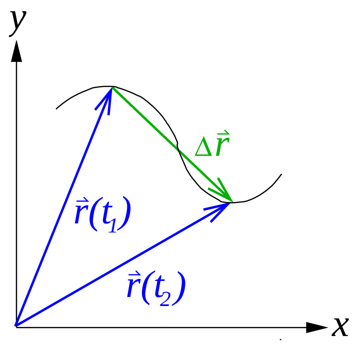
$$\alpha\vec{u} = \alpha u_x\hat{\imath} + \alpha u_y\hat{\jmath}$$




## 2.2 Position, velocity and acceleration vectors

Position vector

- vector is usually used to describe the position of a particle at time $t$.




$$\begin{aligned} \vec{r}(t) &= r_x(t)\hat{\imath} + r_y(t)\hat{\jmath} \\ \Delta\vec{r} &= \vec{r}(t_2) - \vec{r}(t_2) \end{aligned}$$

$\uparrow$
displacement vector

Average velocity in time period $t_1 \to t_2$:

$$\vec{v}_{\text{ave}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1}$$

If the time interval $\Delta t$ is infinitesimal small, i. e. $\Delta t \to 0$, the instantaneous velocity at time $t_1$:

$$\vec{v}_{\text{inst}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}(t)}{dt} \quad \text{(tangential to curve } \vec{r}(t)\text{)}$$

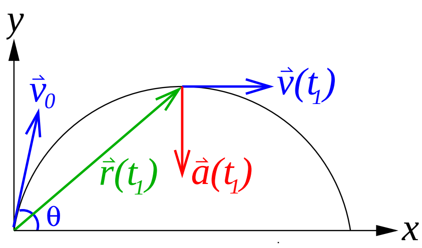
Likewise, the average acceleration in time period $t_1 \to t_2$:

$$\vec{a}_{\text{ave}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}(t_2) - \vec{v}(t_1)}{t_2 - t_1}$$

$$\vec{a}_{\text{inst}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}(t)}{dt} = \frac{d^2\vec{r}(t)}{dt^2}$$

If we talk about velocity or acceleration, we are usually talking about instantaneous velocity and acceleration.

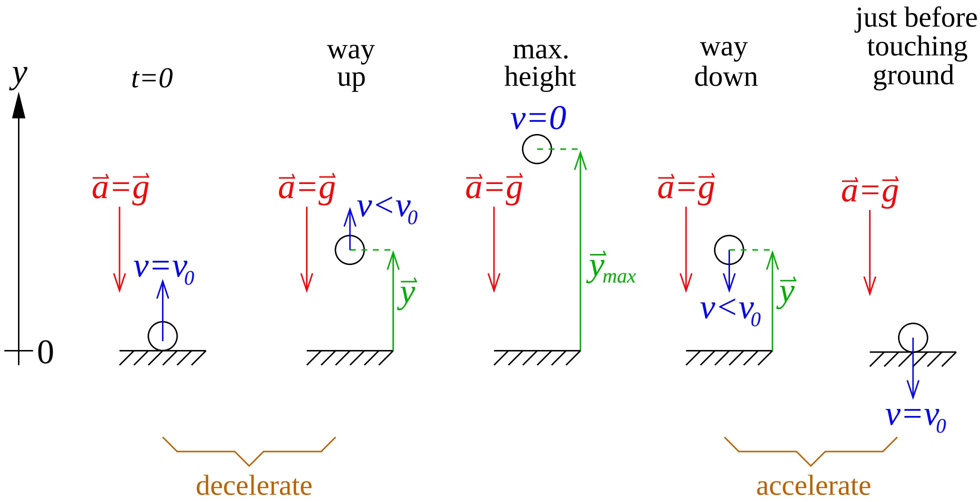
E. g. Projectile motion




$$\begin{aligned} \vec{v}_0 &= v_0\cos\theta\hat{\imath} + v_0\sin\theta\hat{\jmath} \\ \vec{r}(t) &= (v_0\cos\theta)t\hat{\imath} + [(v_0\sin\theta)t - \frac{1}{2}gt^2]\hat{\jmath} \\ \vec{v}(t) &= (v_0\cos\theta)\hat{\imath} + [(v_0\sin\theta) - gt]\hat{\jmath} \\ \vec{a}(t) &= -g\hat{\jmath} \end{aligned}$$

## 2.3 One dimensional motion

An example: Throwing a stone towards the sky vertically with a speed of $v_0$.




Take upward as positive $y$ direction.

$$\therefore \quad a = \frac{dv}{dt} = \frac{d^2y}{dt^2} = -g$$
$$v = \frac{dy}{dt} = -\int gdt = -gt + A, \quad A = \text{constant}$$

To determine A, substitute the initial condition at $t = 0$,

$$\frac{dy}{dt}|_{t=0} = v(t=0) = A = +v_0$$
$$\therefore \quad v = \frac{dy}{dt} = v_0 - gt \tag{2.1}$$
$$y = \int (v_0 - gt)dt = v_o t - \frac{1}{2}gt^2 + B, \quad B = \text{constant}$$

To determine B, substitute $t = 0$ again,

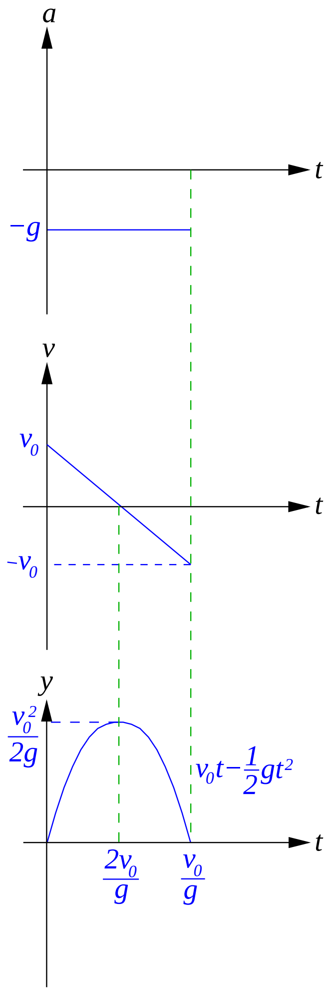
$$y(t=0) = B = 0$$
$$\therefore \quad y = v_0 t - \frac{1}{2}gt^2 \tag{2.2}$$

At maximum height, $\frac{dy}{dt}|_{t=t_{\max}} = 0$, i. e. $v_0 - gt_{\max} = 0 \Rightarrow t_{\max} = \frac{v_0}{g}$.

$$\therefore y_{\max} = y(t_{\max}) = \frac{v_0^2}{g} - \frac{1}{2}g\frac{v_0^2}{g^2} = \frac{1}{2}\frac{v_0^2}{g}$$

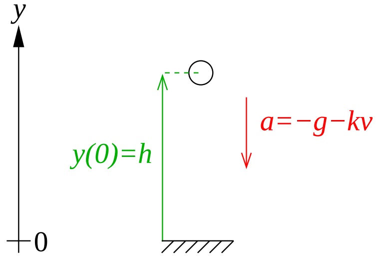
Time for touching the ground again, say $t_0$,

$$y = v_0 t_0 - \frac{1}{2}gt_0^2 = 0 \;\Rightarrow\; \underbrace{t_0 = 0}_{\text{initial at ground}} \quad \text{or} \;\; v_0 = \frac{1}{2}gt_0 \;(\text{i. e. } t_0 = \frac{2v_0}{g})$$




## 2.4 Another example: Non-constant acceleration

Dropping a stone at a height of $h$ with the consideration of the air resistance.
Given: Air drag acceleration $= -kv$, $k =$ positive constant




$g$ is +ve,
$k$ is +ve,
$v$ accelerate

$$a = -g - kv$$
$$\Rightarrow \quad \frac{dv}{dt} = -g - kv$$

$$\Rightarrow \quad \frac{dv}{-g-kv} = dt$$

$$\left(\because \frac{dy(t)}{dt} = f(y) \;\Rightarrow\; \frac{dy(t)}{f(y)} = dt\right)$$

$$\Rightarrow \quad \int \frac{dv}{-g-kv} = \int dt + A, \qquad A = \text{constant}$$

$$\Rightarrow \quad -\frac{1}{k}\ln(-g-kv) = t + A$$

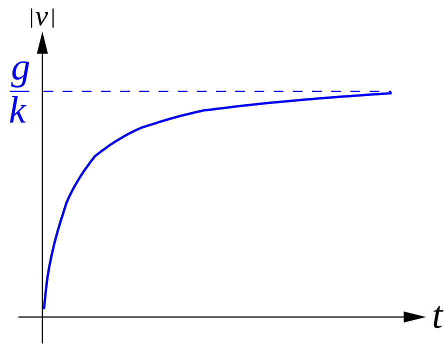
$$\Rightarrow \quad -g-kv = e^{-k(t+A)} = Be^{-kt}, \;\; \text{where } B = e^{-kA}$$

$$\Rightarrow \quad v = \frac{1}{k}(-g - Be^{-kt})$$

$$\text{At } t=0,\; v(0)=0 \;\Rightarrow\; -g-B=0 \;\Rightarrow\; B=-g$$

$$\therefore \quad v = \frac{1}{k}(-g+ge^{-kt}) = -\frac{g}{k}(1-e^{-kt}),$$

$$\text{i. e. } \; \frac{dy}{dt} = -\frac{g}{k}(1-e^{-kt})$$




$$y = -\int \frac{g}{k}(1-e^{-kt})dt + C, \qquad C = \text{constant}$$

$$= -\frac{g}{k}t - \frac{g}{k^2}e^{-kt} + C$$

$$\text{At } t=0,\; y=h \;\Rightarrow\; h = -\frac{g}{k^2} + C \;\Rightarrow\; C = h + \frac{g}{k^2}$$

$$\therefore \; y = -\frac{g}{k}t - \frac{g}{k^2}e^{-kt} + h + \frac{g}{k^2}$$

$$= h - \frac{g}{k}t + \frac{g}{k^2}(1-e^{-kt})$$

# Multiple Choice Questions

- **Q1: What is the formula for calculating velocity in projectile motion?**

  A- $v = v_0 + at$ B- $v = \frac{d}{t}$ C- $v = v_0 - at$ D- $v = v_0 + g$ E- None of them

- **Q2: The equation $y = v_0 t + \frac{1}{2}at^2$ is used to describe which motion?**

  A- Uniform motion B- Projectile motion C- Simple harmonic motion D- Circular motion E- None of them

- **Q3: What does the symbol $\theta$ typically represent in projectile motion?**

  A- Time B- Initial velocity C- Angle of projection D- Acceleration due to gravity E- None of them

- **Q4: In projectile motion, what is the acceleration at the maximum height?**

  A- Zero B- $g$ C- $2g$ D- $v_0$ E- None of them

- **Q5: The horizontal velocity in projectile motion remains constant because:**

  A- There is no horizontal acceleration B- The vertical velocity compensates for horizontal motion C- The gravitational pull only affects vertical motion D- The force of gravity is negligible E- None of them

- **Q6: The range of a projectile is maximized when the angle of projection is:**

  A- 30 degrees B- 45 degrees C- 60 degrees D- 90 degrees E- None of them

- **Q7: In uniform circular motion, the velocity is:**

  A- Constant in magnitude but changing in direction B- Constant in both magnitude and direction C- Changing in magnitude but constant in direction D- Changing in both magnitude and direction E- None of them

- **Q8: The centripetal force acting on an object moving in a circle is always directed:**

  A- Towards the center of the circle B- Away from the center of the circle C- Along the tangent to the circle D- In the direction of motion E- None of them

- **Q9: The acceleration due to gravity is approximately:**

  A- $9.8\,\text{m/s}^2$ B- $10\,\text{m/s}^2$ C- $9.81\,\text{m/s}^2$ D- $9.5\,\text{m/s}^2$ E- None of them

- **Q10: The time of flight for a projectile depends on which of the following?**

  A- The initial velocity only B- The angle of projection only C- Both the initial velocity and angle of projection D- Only the height E- None of them

- **Q11: The horizontal displacement of a projectile is called its:**

  A- Range B- Height C- Velocity D- Acceleration E- None of them

- **Q12: If the initial velocity of a projectile is doubled, its range will:**

  A- Increase by a factor of 4 B- Increase by a factor of 2 C- Remain unchanged D- Decrease by a factor of 2 E- None of them

- **Q13: The formula for centripetal acceleration is:**

  A- $a_c = \frac{v^2}{r}$ B- $a_c = \frac{r^2}{v}$ C- $a_c = \frac{v}{r}$ D- $a_c = \frac{r}{v}$ E- None of them

- **Q14: The total time a projectile is in the air is called:**

  A- Time of flight B- Time interval C- Duration D- Total time E- None of them

- **Q15: Which of the following factors affects the motion of a projectile?**

  A- Air resistance B- Gravity C- Initial velocity D- Angle of projection E- All of the above

- **Q16: The velocity at the highest point in projectile motion is:**

  A- Zero B- $v_0$ C- $v_0/2$ D- $v_0\sqrt{2}$ E- None of them

- **Q17: In projectile motion, what is the relationship between the horizontal and vertical motions?**

  A- They are independent of each other B- The horizontal motion depends on vertical motion C- The vertical motion depends on horizontal motion D- They influence each other equally E- None of them

- **Q18: In circular motion, the period is:**

  A- The time for one complete revolution B- The frequency of the motion C- The time taken for half a revolution D- The velocity of the object E- None of them

- **Q19: The angle of elevation for a projectile determines:**

  A- The range of the projectile B- The height of the projectile C- The velocity of the projectile D- The time of flight E- None of them

- **Q20: In which of the following types of motion is the speed constant?**

  A- Linear motion B- Circular motion C- Projectile motion D- Uniform motion E- None of them