Quantum Mechanics in Medicine
Lecture 2

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Roadmap

Historical Background

In 1923, Arthur H. Compton discovered that when monochromatic X-rays are scattered by light elements (like carbon or graphite), the scattered radiation contained not only the original wavelength but also radiation of longer wavelength. This wavelength shift was independent of the incident wavelength or target material, depending only on the scattering angle. This became known as the **Compton Effect**.

Experimental Observation

- Incident monochromatic X-rays directed at a graphite target.
- Scattered X-rays analyzed with a spectrometer at different angles θ.
- Observed spectrum showed two peaks:
  1. One at the incident wavelength λ (unmodified radiation).
  2. Another at longer wavelength λ' (modified radiation).

Theoretical Explanation

Compton interpreted scattering as an **elastic collision** between a photon and a free electron initially at rest.

- Photon energy: $E = h\nu = hc/\lambda$, momentum $p = h/\lambda$.
- Electron rest energy: $m_e c^2$.
- After collision, photon has reduced energy $h\nu'$, increased wavelength λ'.
- Recoil electron gains kinetic energy.

Conservation Laws

Apply conservation of energy and momentum:

$$h\nu + m_e c^2 = h\nu' + \gamma m_e c^2,$$
$$\frac{h}{\lambda}\hat{\mathbf{k}} = \frac{h}{\lambda'}\hat{\mathbf{k}}' + \gamma m_e \mathbf{v}, \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}.$$

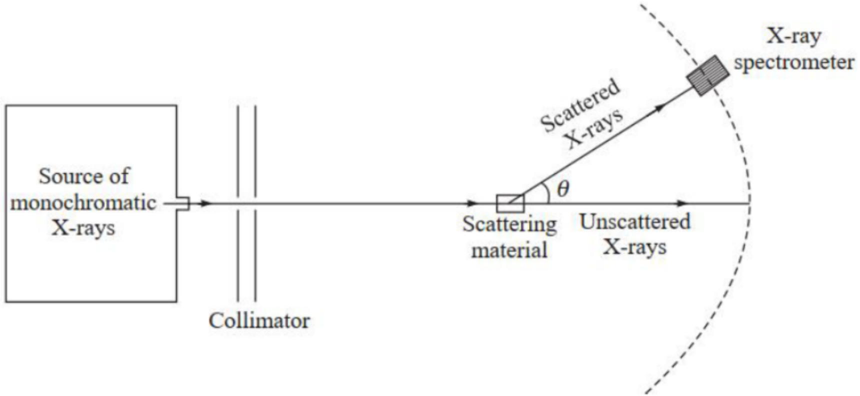
Elimination of electron variables leads to the **Compton shift formula**:

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta).$$

Compton Wavelength and Significance

- The quantity $\frac{h}{m_e c} = 0.02426$ Å is the **Compton wavelength of the electron**.
- The Compton effect demonstrated that photons carry both energy and momentum, confirming the particle nature of light.
- Shift depends only on scattering angle θ, not on incident wavelength or target material.

Compton Scattering Geometry



Worked Problem (Full Text)

Problem: X-rays of wavelength 2.0 Å are scattered from a carbon block. The scattered photons are observed at right angles (90°) to the direction of the incident beam. Calculate (a) the wavelength of the scattered photon, (b) the energy of the recoil electron.

Solution Outline:

1. At $\theta = 90^\circ$: $\Delta\lambda = \lambda_C(1 - \cos 90^\circ) = \lambda_C \approx 0.02426$ Å.
2. $\lambda' = \lambda + \Delta\lambda = 2.0 + 0.02426 = 2.02426$ Å.
3. Photon energies: $E = hc/\lambda$, $E' = hc/\lambda'$.
4. Recoil electron energy $= E - E'$.

Homework (Full Text)

1) In a Compton scattering experiment, the incident radiation has wavelength 2 Å while the wavelength of the radiation scattered through 180° is 2.048 Å. Calculate (a) the wavelength of the radiation scattered at an angle of 60° to the direction of incidence, and (b) the energy of the recoil electron which scatters the radiation through 60°.
2) A photon of energy 0.9 MeV is scattered through 120° by a free electron. Calculate the energy of the scattered photon.

Worked Problem

Problem: X-rays of wavelength 2.0 Å are scattered from a carbon block. The scattered photons are observed at right angles to the direction of the incident beam. Calculate **(a)** the wavelength of the scattered photon, **(b)** the energy of the recoil electron.

Solution Outline: Using $\Delta\lambda = \lambda_C(1 - \cos\theta)$ with $\theta = 90°$ gives $\Delta\lambda = \lambda_C \approx 0.02426$ Å. Hence $\lambda' \approx 2.02426$ Å. The recoil electron energy follows from $E_\gamma - E_{\gamma'}$ with $E_\gamma = hc/\lambda$.

Homework

1) In a Compton scattering experiment, the incident radiation has wavelength 2 Å while the wavelength of the radiation scattered through 180° is 2.048 Å. Calculate **(a)** the wavelength of the radiation scattered at an angle of 60° to the direction of incidence, and **(b)** the energy of the recoil electron which scatters the radiation through 60°. **2)** A photon of energy 0.9 MeV is scattered through 120° by a free electron. Calculate the energy of the scattered photon.

Wave and Particle Aspects

- Interference, diffraction and polarization require **wave** nature.
- Blackbody radiation, photoelectric effect and Compton effect require **particle** (photon) description.
- Radiation therefore has a **dual nature** — exhibiting both wave and particle properties.

Introduction to Atomic Spectra

When atoms are excited (by electric discharge or heating), they emit or absorb radiation at specific wavelengths. Instead of a continuous spectrum, discrete narrow lines are observed. These are known as **line spectra**. Each element has its own characteristic line spectrum, acting as a "fingerprint".

Types of Spectra

1. **Continuous Spectrum:** Produced by hot solids, liquids, or dense gases. All wavelengths present without interruption.
2. **Line Spectrum:** Produced by atoms in the gaseous state. Shows discrete lines at specific wavelengths.
3. **Band Spectrum:** Produced by molecules, appears as groups of closely spaced lines forming bands.

Spectral Series Concept

For hydrogen, lines are grouped into regular patterns called **spectral series**. Each series corresponds to electron transitions ending at a particular lower energy level n_1.

- Lyman series: $n_1 = 1$ (ultraviolet region).
- Balmer series: $n_1 = 2$ (visible region).
- Paschen series: $n_1 = 3$ (infrared region).
- Brackett, Pfund, Humphreys series: infrared and beyond.

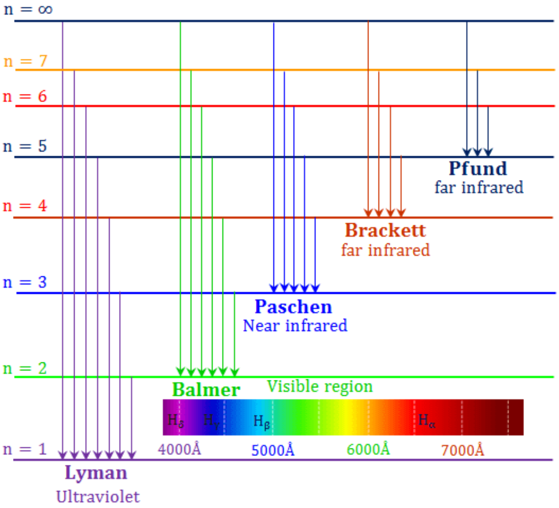
Balmer Formula

Johann Balmer (1885) found that the wavelengths of visible hydrogen lines can be expressed as:

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \qquad n = 3, 4, 5, \ldots$$

where $R = 1.097 \times 10^7\ \mathrm{m}^{-1}$ is the Rydberg constant. This formula accurately predicted all visible hydrogen lines, now called the **Balmer series**.

Hydrogen Spectrum Regions



Rydberg General Formula

The general expression for spectral lines of hydrogen-like atoms:

$$\frac{1}{\lambda} = RZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right), \qquad n_2 > n_1$$

where Z is the atomic number.

- Explains all hydrogen series (Lyman, Balmer, Paschen, etc.).
- For $Z > 1$, explains hydrogen-like ions (He^+, Li^{2+}, etc.).

Significance of Atomic Spectra

- Experimental confirmation of quantized energy levels in atoms.
- Spectroscopy became a tool for identifying elements in stars and distant galaxies.
- Rydberg constant measured very precisely from hydrogen lines.

Homework (Spectral Series)

1) Calculate the wavelength of the first line of the Lyman series of hydrogen ($n_2 = 2 \rightarrow n_1 = 1$).

2) Calculate the wavelength of the Hα line of the Balmer series ($n_2 = 3 \rightarrow n_1 = 2$).

3) Determine the wavelength of the first line of the Paschen series ($n_2 = 4 \rightarrow n_1 = 3$).

Historical Context

In 1913, Niels Bohr proposed that classical electromagnetic theory fails for atomic-scale systems. He combined Rutherford's nuclear model with Planck's and Einstein's quantum ideas, creating a theory for hydrogen-like atoms. This model explained the hydrogen spectrum successfully.

Bohr Postulates

1. Electrons revolve around the nucleus in circular orbits without radiating energy.
2. Only those orbits are allowed for which the angular momentum is quantized:

$$L_n = n\hbar, \quad n = 1, 2, 3, \dots$$

3. Radiation occurs when an electron jumps between two stationary states. The frequency of emitted/absorbed radiation is given by the energy difference:

$$h\nu = E_{n_2} - E_{n_1}.$$

Orbit Radius

The radius of the *n*th Bohr orbit is given by:

$$r_n = \frac{a_0}{Z} n^2,$$

where $a_0 = 0.529$ Å is the Bohr radius for hydrogen and Z is the atomic number.

Thus for hydrogen ($Z = 1$):

$$r_n = a_0 n^2.$$

Electron Velocity

The velocity of the electron in the *n*th orbit is:

$$v_n = \frac{Z\alpha c}{n}, \qquad \alpha \approx \frac{1}{137}.$$

For hydrogen ($Z = 1$), the ground state ($n = 1$) electron velocity is approximately 2.18×10^6 m/s.

Angular Momentum

Bohr quantization condition:

$$L_n = n\hbar = \frac{nh}{2\pi}.$$

For $n = 1$: $L_1 = \hbar$; for $n = 2$: $L_2 = 2\hbar$, and so on.

This discrete angular momentum distinguishes quantum orbits from classical ones.

Energy Levels

The energy of the electron in the nth orbit:

$$E_n = -\frac{13.6\, Z^2}{n^2}\ \text{eV}.$$

- For hydrogen ($Z = 1$), $E_1 = -13.6$ eV (ground state), $E_2 = -3.4$ eV, $E_3 = -1.51$ eV, etc.
- Negative sign indicates bound states.
- Ionization energy: 13.6 eV is required to free the electron from ground state.

Transitions and Spectral Lines

When an electron jumps from n_2 to n_1 ($n_2 > n_1$), radiation of frequency

$$\nu = RcZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

is emitted. Equivalently,

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right).$$

This relation explains the spectral series (Lyman, Balmer, Paschen, etc.) observed in hydrogen.

Worked Problem (Hydrogen)

Problem: The energy of an excited hydrogen atom is -3.4 eV. Determine the angular momentum of the electron according to Bohr theory.

Solution: $E_n = -\frac{13.6}{n^2}$ eV. Setting $E_n = -3.4$ eV gives $n = 2$.

Thus, $L_2 = 2\hbar = \frac{2h}{2\pi}$.

Homework

1) The energy of the ground state of hydrogen atom is -13.6 eV. Find the energy of the photon emitted in the transition from $n = 4$ to $n = 2$.

2) The H line of Balmer series is obtained from the transition $n = 3$ (energy $= -1.5$ eV) to $n = 2$ (energy $= -3.4$ eV). Calculate the wavelength for this line.

Worked Example

The energy of an excited hydrogen atom is -3.4 eV $\Rightarrow n = 2$. The electron's angular momentum is

$$L_2 = 2\hbar = \frac{2h}{2\pi}.$$

Homeworkt

1) The energy of the ground state of hydrogen atom is -13.6 eV. Find the energy of the photon emitted in the transition from $n = 4$ to $n = 2$. **2)** The H line of Balmer series is

obtained from the transition $n = 3$ (energy $= -1.5$ eV) to $n = 2$ (energy $= -3.4$ eV). Calculate the wavelength for this line.

MCQs 1–2

Q1. The Compton effect demonstrates:

1. Wave nature of X-rays
2. Particle nature of photons
3. Dual nature of electrons
4. Continuous energy spectrum

Q2. The Compton wavelength of the electron has a value of:

1. 0.242 Å
2. 0.0242 Å
3. 2.42 Å
4. 24.2 Å

MCQs 3–4

Q3. In Compton scattering, the change in wavelength depends on:

1. Energy of photon
2. Atomic number of target
3. Scattering angle
4. Work function of metal

Q4. X-rays of $\lambda = 2.0$ Å are scattered at 90°. Calculate the wavelength of the scattered photon.

1. 2.048 Å
2. 2.024 Å
3. 2.1 Å
4. 1.98 Å

MCQs 5–6

Q5. Compton explained scattering as:

1. Inelastic collision between photon and nucleus
2. Elastic collision between photon and free electron
3. Refraction of photons
4. Pure wave diffraction

Q6. Dual nature of radiation means:

1. Radiation has only wave property
2. Radiation has only particle property
3. Radiation exhibits both wave and particle properties
4. Radiation has no definite property

MCQs 7–8

Q7. A photon of energy 0.9 MeV is scattered through 120°. What is the approximate energy of the scattered photon?

1. 0.45 MeV
2. 0.75 MeV
3. 0.9 MeV
4. 0.6 MeV

Q8. Atomic spectra of elements consist of:

1. Continuous spectrum
2. Line spectrum
3. Band spectrum
4. Infrared only

MCQs 9–10

Q9. Balmer series of hydrogen lies in which region?

1. Infrared
2. Visible
3. Ultraviolet
4. X-ray

Q10. The Rydberg constant has a value of approximately:

1. $1.097 \times 10^{7}\ \mathrm{m}^{-1}$
2. $3 \times 10^{8}\ \mathrm{m}^{-1}$
3. $6.63 \times 10^{-34}\ \mathrm{m}^{-1}$
4. $9.1 \times 10^{-31}\ \mathrm{m}^{-1}$

MCQs 11–12

Q11. Calculate the wavelength of Hα line of Balmer series for $n_2 = 3$ to $n_1 = 2$ transition.

1. 656 nm
2. 434 nm
3. 486 nm
4. 121 nm

Q12. Bohr model of hydrogen atom combines:

1. Rutherford's model and Planck's quantum idea
2. Newton's laws and Maxwell's theory
3. Relativity and quantum mechanics
4. Wave and matrix mechanics

MCQs 13–14

Q13. According to Bohr, the angular momentum of electron in nth orbit is:

1. nh
2. $nh/2\pi$
3. $h/2\pi$
4. nh^2

Q14. The energy of ground state of hydrogen atom is:

1. -1.5 eV
2. -3.4 eV
3. -13.6 eV
4. 0 eV

MCQs 15–16

Q15. The ionization energy of hydrogen atom is equal to:

1. 3.4 eV
2. 10.2 eV
3. 13.6 eV
4. 1.5 eV

Q16. An excited hydrogen atom has energy -3.4 eV. Calculate the angular momentum of the electron.

1. $h/2\pi$
2. $2h/2\pi$
3. $3h/2\pi$
4. $4h/2\pi$

MCQs 17–18

Q17. Frequency of radiation emitted in transition $n_2 \to n_1$ is given by:

1. E/h
2. $\Delta E/h$
3. hv/c
4. mc^2/h

Q18. The Lyman series of hydrogen lies in which region?

1. Visible
2. Infrared
3. Ultraviolet
4. Microwave

MCQs 19–20

Q19. The Hβ line of Balmer series corresponds to transition $n_2 = 4$ to $n_1 = 2$. Calculate its approximate wavelength.

1. 486 nm
2. 434 nm
3. 656 nm
4. 121 nm

Q20. Bohr's model successfully explains:

1. Spectrum of hydrogen atom
2. Spectrum of helium atom
3. Spectrum of all atoms
4. Spectrum of complex molecules