

## Lecture (1)



### PREREQUISITES FOR CALCULUS (المتطلبات الأساسية للفيزياء والتكامل)

#### Sets and Intervals (المجموعات والفترات)

##### **DEFINITIONS:**

**Set:** is a collection of things under certain conditions.

**Example 1:**

$$A = \{1, 3, 5, 8, 10\};$$

A is a set, 1,3,5,8,10 are elements.

**Real Numbers (R):** is a set of all rational and irrational numbers.  $R = \{-\infty, +\infty\}$ ,

$$-\infty \leftarrow \qquad \qquad \qquad 0 \qquad \qquad \qquad \rightarrow +\infty$$

**Integer Numbers (I):** a set of all irrational numbers.

$I = \{-\infty, \dots, -3, -2, -1, 0, 1, 2, 3, \dots, +\infty\}$  negative and positive numbers only.

**Natural Numbers (N):** consist of zero and positive integer numbers only.

$$N = \{0, 1, 2, 3, \dots, +\infty\}$$

**Intervals:** is a set of all real numbers between two points on the real number line. (it is a subset of real numbers)

1. **Open interval:** is a set of all real numbers between A & B excluded (A & B are not elements in the set).  $\{x: A < x < B\}$  or  $(A, B)$ .

$$-\infty \leftarrow \qquad A \left( \begin{array}{c} | \\ X \end{array} \right) B \qquad \rightarrow +\infty$$

2. **Closed interval:** is a set of all real numbers between A & B included (A & B are elements in the set).  $\{x: A \leq x \leq B\}$  or  $[A, B]$ .

$$-\infty \leftarrow \qquad A \left[ \begin{array}{c} | \\ X \end{array} \right] B \qquad \rightarrow +\infty$$

3. **Half-Open interval (Half-Close):** is a set of all real numbers between A & B with one of the end-points as an element in the set.

a)  $(A, B] = \{x: A < x \leq B\}$   $-\infty \leftarrow \qquad A \left( \begin{array}{c} | \\ X \end{array} \right] B \qquad \rightarrow +\infty$

b)  $[A, B) = \{x: A \leq x < B\}$   $-\infty \leftarrow \qquad A \left[ \begin{array}{c} | \\ X \end{array} \right) B \qquad \rightarrow +\infty$

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TABLE 1.1 Types of intervals

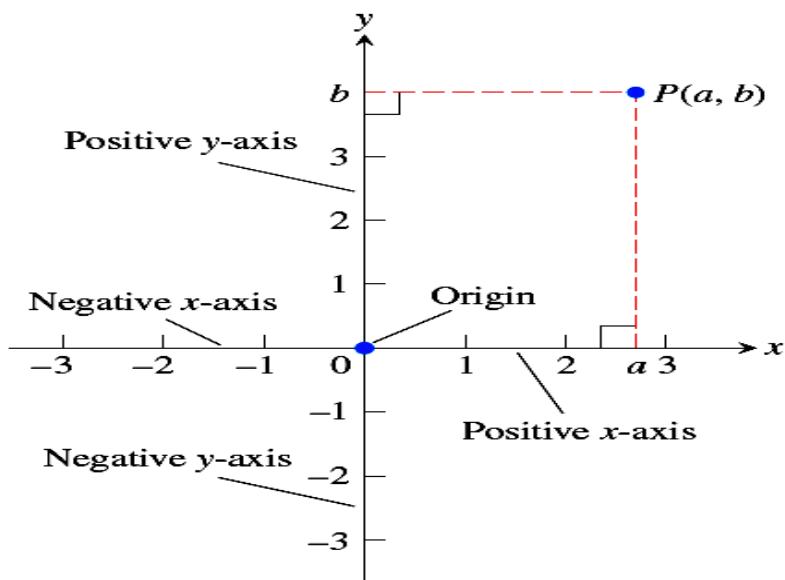
	Notation	Set description	Type	Picture
<b>Finite:</b>	$(a, b)$	$\{x   a < x < b\}$	Open	
	$[a, b]$	$\{x   a \leq x \leq b\}$	Closed	
	$[a, b)$	$\{x   a \leq x < b\}$	Half-open	
	$(a, b]$	$\{x   a < x \leq b\}$	Half-open	
<b>Infinite:</b>	$(a, \infty)$	$\{x   x > a\}$	Open	
	$[a, \infty)$	$\{x   x \geq a\}$	Closed	
	$(-\infty, b)$	$\{x   x < b\}$	Open	
	$(-\infty, b]$	$\{x   x \leq b\}$	Closed	
	$(-\infty, \infty)$	$\mathbb{R}$ (set of all real numbers)	Both open and closed	

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### (الاحداثيات في الفراغ او المستوى)

Each point in the plane can be represented with a pair of real numbers  $(a,b)$ , the number  $a$  is the horizontal distance from the origin to point  $P$ , while  $b$  is the vertical distance from the origin to point  $P$ . The origin divides the  $x$ -axis into positive  $x$  axis to the right and the negative  $x$ -axis to the left, also, the origin divides the  $y$ -axis into positive  $y$ -axis upward and the negative  $y$ -axis downward. The axes divide the plane into four regions called quadrants.



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### Distance between Points and (Mid-Point Formula):

Distance between points in the plane is calculated with a formula that comes from Pythagorean Theorem:

#### ❖ Distance Formula for Points in the Plane

The distance between  $P(x_1, y_1)$  and  $Q(x_2, y_2)$

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

and the mid-point formula:

$$x_0 = \frac{x_1 + x_2}{2}, y_0 = \frac{y_1 + y_2}{2}$$

**Example 2:** find the distance between  $P(-1,2)$  and  $Q(3,4)$  and find the mid-point:

Sol.:

$$\begin{aligned} d &= \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3 - (-1))^2 + (4 - 2)^2} = \sqrt{20} = 2\sqrt{5} \end{aligned}$$

$$x_0 = \frac{x_1 + x_2}{2}, x_0 = \frac{-1+3}{2} = 1 \text{ and } y_0 = \frac{y_1 + y_2}{2}, y_0 = \frac{2+4}{2} = 3.$$

**Example 3:** find the distance between  $R(2,-3)$  and  $S(6,1)$  and find the mid-point:

Sol.:

$$\begin{aligned} d &= \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(6 - 2)^2 + (1 - (-3))^2} = \sqrt{16 + 16} = \sqrt{32} = 2\sqrt{8} \\ x_0 &= \frac{x_1 + x_2}{2}, x_0 = \frac{2+6}{2} = 4 \text{ and } y_0 = \frac{y_1 + y_2}{2}, y_0 = \frac{-3+1}{2} = -1. \end{aligned}$$

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### Slope and Equation of Line

❖ Slope (الميل): The constant

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

is the slope of non-vertical line  $P_1 P_2$ .

**Note1:** Horizontal line have ( $m=0$ ) ( $\Delta y=0$ ), and the vertical line has no slope or the slope of vertical line is undefined ( $\Delta x=0$ ).

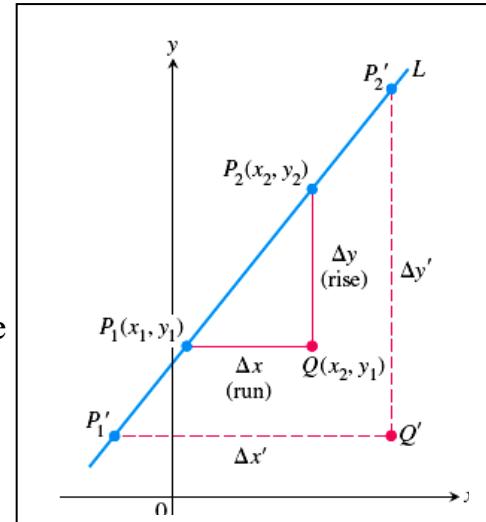
**Note2:** Parallel lines have the same slope  
In the the lines are parallel then ( $m_1 = m_2$ ).

**Note3:** If two non-vertical lines  $L_1$  and  $L_2$  are perpendicular, their slopes  $m_1$  and  $m_2$  satisfy

$$m_1 * m_2 = -1,$$

so each slope is the negative reciprocal of the other.

$$m_1 = \frac{1}{m_2} \text{ and } m_2 = \frac{1}{m_1}$$



**Example 4:** Find the slope of the straight line through the two points  $P(3,2)$  and  $Q(4,4)$ :

**Sol.:**

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{4 - 3} = 2.$$

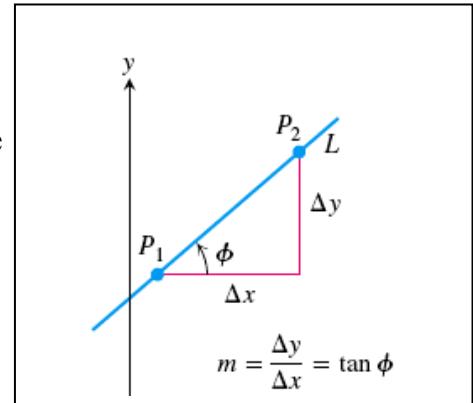
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### ❖ Point-Slope Equation:

We can write an equation for a non-vertical straight line  $L$  if we know its slope  $m$  and the coordinate of one point  $P_1(x_1, y_1)$  on it. If  $P(x, y)$  is any other point on  $L$ , then we can use two points  $P_1$  and  $P$  to compute the slope,

$$m = \frac{y - y_1}{x - x_1}$$



so that  $y - y_1 = m(x - x_1)$

or  $y = y_1 + m(x - x_1)$

**The equation  $y = y_1 + m(x - x_1)$**

**is the point-slope equation of the line that passes through the point  $P_1(x_1, y_1)$  and has slope  $m$ .**

**Example 5:** write an equation for the line pass through the point (2,3) with slope (-3/2).

**Sol.:** we substitute  $x_1 = 2$ ,  $y_1 = 3$ , and  $m = -3/2$  into the point-slope equation and obtain

$$y = y_1 + m(x - x_1)$$

$$y = 3 + \frac{-3}{2}(x - 2)$$

$$y = -\frac{3}{2}x + 6.$$

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**Example 6:** A line pass through two points: write an equation for the line through

(-2,-1) and (3,4)

**Sol.:** The line's slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 4}{2 - 3} = \frac{5}{5} = 1.$$

We can use this slope with either of the two given points in the point-slope equation;

**With (x1,y1) = (-2, -1)**

$$y = -1 + 1 \cdot (x - (-2))$$

$$y = -1 + x + 2$$

$$y = x + 1$$

**With (x2,y2) = (3, 4)**

$$y = 4 + 1 \cdot (x - 3)$$

$$y = 4 + x - 3$$

$$y = x + 1$$

**Note:** The equation:

$$y = mx + b$$

is called the **slope-intercept equation** of the line with slope m and y-intercept b

**Note:** The general form of straight line equation is

$$Ax + By + C = 0$$

**Example 7:** finding the slope and y-Intercept for the line  $8x + 4y = 20$ .

**Sol.:** solve the equation for y to put it in slope-intercept form :

$$8x + 4y = 20$$

$$4y = -8x + 20$$

$$y = -2x + 5$$

$$y = -2x + 5$$

The slope  $m = -2$  the y-intercept is  $b = 5$ .

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### **H.W:**

1. finding the slope and y-Intercept for the line  $4x + 2y = 4$ .
2. write an equation for the line pass through  $(-1,-1)$  and  $(1,2)$ .
3. write an equation for the line pass through the point  $(1,-1)$  with slope  $(4)$ .
4. Find the **slope** of the straight line through the two points **P(3,-2)** and **Q(3,6)**.
5. write an equation for the horizontal line pass through the point **(2,-2)**