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Medical Nuclear Physics

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Introduction

We covered radioactive decay from a phenomenological perspective in the last lecture. In this lecture, we consider the topic from a more general analytical perspective.

The reason for doing this is so that we can develop a form of thinking which will help us to understand what is going on in a quantitative, mathematical sense. We will be introduced to concepts such as the Decay Constant and the Half Life as well as units used for the measurement of radioactivity. You will also have a chance to develop your understanding by being brought through three questions on this subject.

Assumptions

The usual starting point in most forms of analysis in physics is to make some assumptions which simplify the situation.

- For our radioactive decay situation, we first of all consider that we have a sample containing a large number of radioactive nuclei all of the same kind.
- Secondly, we assume that all of the radioactive nuclei decay by the same process be it alpha, beta or gamma decay.
- Thirdly, we take a few moments to ponder the fact that we can only really consider what is going on from a statistical perspective.

Introduction

- Radioactive decay will be covered from a mathematical perspective.
- Analogy: Popcorn popping.
  - Individual kernels pop randomly.
  - Radioactive nuclei decay randomly.
- Key Idea: We can control popcorn popping with heat, but we cannot control radioactive decay by changing the environment.
- Conclusion: Decay depends on the nucleus itself.

The Radioactive Decay Law

- Notation:
  - N: Number of radioactive nuclei at time t.
  - dN: Change in the number of radioactive nuclei.
  - dt: Small time interval.
- Proportionality: $-dN \propto Ndt$
  - More nuclei $\rightarrow$ more decays.
  - Longer time interval $\rightarrow$ more decays.
- Decay Constant: $-dN = \lambda Ndt$
  - λ: Decay constant (characterizes the decay rate).
- Rearranging: $\frac{dN}{N} = -\lambda dt$
- Integration: $\int_{N_0}^{N_t} \frac{dN}{N} = \int_0^t -\lambda dt$
  - N_0: Initial number of nuclei.
  - N_t: Number of nuclei at time t.
  - $-\frac{N_t}{N_0} = \lambda t$
  - $N_t = N_0 \exp(-\lambda t)$

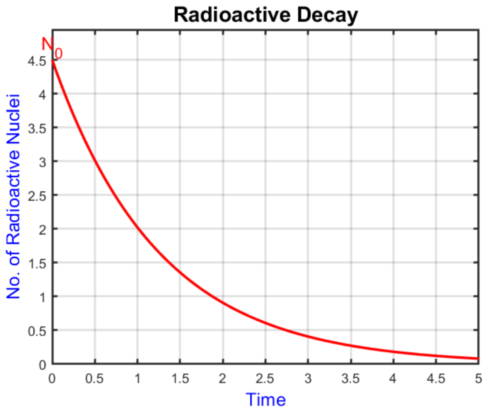
This final expression is known as the Radioactive Decay Law.

Mathematics Review

- Let us review the mathematics which we used above.
- First of all we used integral calculus to figure out what was happening over a period of time by integrating what we knew would occur in a brief interval of time.
- Secondly we used a calculus relationship that the $\int \frac{dx}{x} = \ln x$

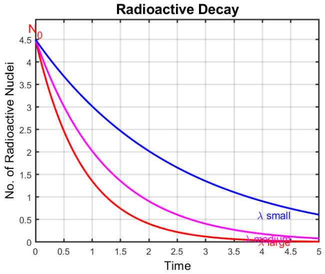
Decay Graphs

The graph plots the number of radioactive nuclei at any time, N_t, against time, t. We can see that the number of radioactive nuclei decreases from N_0 that is the number at $t = 0$ in a rapid fashion initially and then more slowly in the classic exponential manner.



Influence of Decay Constant

The influence of the Decay Constant can be seen in the following figure:



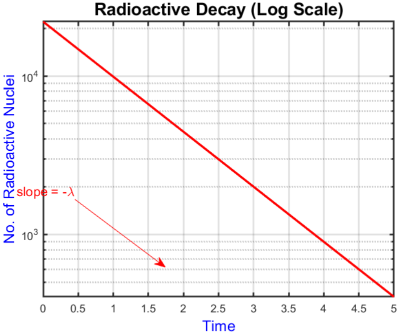
All three curves here are exponential in nature; only the Decay Constant is different. Notice that when the Decay Constant has a low value, the curve decreases relatively slowly, and when the Decay Constant is large, the curve decreases very quickly. The Decay Constant is characteristic of individual radionuclides.

Logarithmic Representation

It is also possible to consider the Radioactive Decay Law from another perspective by plotting the logarithm of N_t against time.

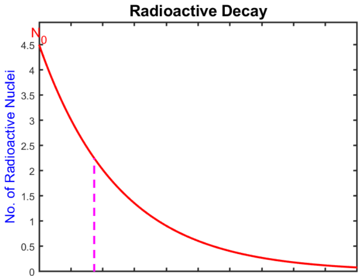
$\ln\left(\frac{N_t}{N_0}\right) = -\lambda t$

Notice that this expression is simply an equation of the form $y = mx + c$ where $m = -\lambda$ and $c = \ln N_0$.



Half-Life

Most of us have not been taught to think instinctively in terms of logarithmic or exponential terms, even though many natural phenomena display exponential behaviours.
This indicator is called the Half-Life, and it expresses the length of time it takes for the radioactivity of a radioisotope to decrease by a factor of two.
From a graphical point of view, we can say that when:
$N_t = \frac{N_0}{2}$
the time taken is the Half-Life:



Radioisotope Half-Life Examples

Note that the half-life does not express how long a material will remain radioactive, but simply the length of time for its radioactivity to halve.

Radioisotope	**Half-Life (approx.)**
^{81m}Kr	13 seconds
^{99m}Tc	6 hours
^{131}I	8 days
^{51}Cr	1 month
^{137}Cs	30 years
^{241}Am	462 years
^{226}Ra	1620 years
^{238}U	4.51×10^{9} years

Logistical Issues

But they do present a logistical problem when we wish to use them when there may not be a radioisotope production facility nearby. For example suppose we wish to use ^{99m}Tc for a patient study and the nearest nuclear facility for making this isotope is 5,000 km away. The production facility could...

Relationship Between Decay Constant and Half-Life

From the equation: $N_0 = N_0 \exp(-\lambda t_{1/2})$ we can derive that: $2^{-1} = \exp(-\lambda t_{1/2})$ Taking the natural logarithm on both sides: $\ln 2^{-1} = -\lambda t_{1/2} \quad \Rightarrow \quad \ln 2 = \lambda t_{1/2}$ Thus we get: $\lambda = \frac{0.693}{t_{1/2}}$ These last two equations express the relationship between the Decay Constant and the Half-Life. They are very useful when solving numerical questions relating to radioactivity and usually form the first step in solving a numerical problem.

Units of Radioactivity

The SI or metric unit of radioactivity is named after Henri Becquerel, in honour of his discovery of radioactivity, and is called the becquerel with the symbol Bq.
The becquerel is defined as the quantity of radioactive substance that gives rise to a decay rate of 1 decay per second.
In medical diagnostic work, 1 Bq is a rather small amount of radioactivity. For this reason, the kilobecquerel (kBq) and megabecquerel (MBq) are more frequently used.
The traditional unit of radioactivity is named after Marie Curie and is called the curie, with the symbol Ci.

Radioactivity Units

- 1 Ci is defined as the amount of radioactive substance which gives rise to a decay rate of 3.7×10^{10} decays per second.
- This makes 37 thousand million decays per second a substantial amount of radioactivity for practical uses.

Why two units?

- Like all units of measurement, it depends on the area you are in. For example, the kilometer is widely used in Europe and Australia as a unit of distance and the mile is used in the USA.
- Hence, one might need to know and understand both units, especially in the context of radioactivity.

Questions

Three questions are given below to help you develop your understanding:

(1) The first is relatively straightforward and will test your application of the Radioactive Decay Law and your understanding of the concept of Half-Life.

(2) The second question relates to the radioactive nuclei which are decaying in a sample.

(3) The third question will give you a similar scenario but from a different perspective.

Question 1

(a) The half-life of ^{99m}Tc is 6 hours. After how much time will $\frac{1}{16}$ of the radioisotope remain?
(b) Verify your answer by another means.

Answer to Question 1

(a) Starting with the relationship established earlier between the Decay Constant and the Half Life:

$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{6 \text{ hours}} = 0.1155 \text{ hr}^{-1}$

Now applying the Radioactive Decay Law: $N_t = N_0 \exp(-\lambda t)$

The question tells us that N_0 has reduced to $\frac{1}{16}$ of its value, that is: $\frac{N_0}{16}$

Question 2

Find the radioactivity of a 1 g sample of ^{226}Ra given that $t_{1/2} = 1620$ years and Avogadro's Number $= 6.023 \times 10^{23}$.

Answer to Question 2

1. Unit Conversion: Note that the length of a year used in converting from 'per year' to 'per second' is 365.25 days to account for leap years. The unit of radioactivity is expressed as the number of nuclei decaying per second, hence the conversion to 'per second'.

2. Number of Nuclei (N):

$$N = \frac{\text{(Avogadro's No.)(Mass)}}{\text{Molar Mass}} = \frac{(6.023 \times 10^{23}\ \text{nuclei/mol})(1\ \text{g})}{226\ \text{g/mol}} \approx 2.665$$

Important Correction: The image incorrectly states 2.7×10^{24} nuclei. The correct value is approximately 2.665×10^{21} nuclei.

3. Radioactive Decay Law and Activity: The Radioactive Decay Law is $N = N_0 e^{-\lambda t}$. Differentiating gives:

$$\frac{dN}{dt} = -\lambda N$$

The activity is the absolute value of the rate of change:

Continue Answer to Question 2

4. **Decay Constant (λ):** First, we calculate the decay constant from the half-life:

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{1620 \text{ years}} \approx 4.28 \times 10^{-4} \text{ year}^{-1}$$

Convert λ to s^{-1}:

$$\lambda \approx 4.28 \times 10^{-4} \text{ year}^{-1} \times \frac{1 \text{ year}}{3.156 \times 10^{7} \text{ s}} \approx 1.36 \times 10^{-11} \text{ s}^{-1}$$

5. **Calculating Activity:**

$$\text{Activity} = \lambda N \approx (1.36\times10^{-11} \text{ s}^{-1})(2.665\times10^{21} \text{ nuclei}) \approx 3.62\times10^{10} \text{ dec}$$

6. **Converting to Curies:** Since 1 Ci $= 3.7 \times 10^{10}$ decays/second:

$$\text{Activity} \approx \frac{3.62 \times 10^{10} \text{ decays/second}}{3.7 \times 10^{10} \text{ decays/second/Ci}} \approx 0.98 \text{ Ci} \approx 1 \text{ Ci}$$

Question 3

What is the minimum mass of ^{99m}Tc that can have a radioactivity of 1 MBq? Assume the half-life is 6 hours and that Avogadro's Number is 6.023×10^{23}.

Answer to Question 3

1. Decay Constant (λ): Starting with the relationship between the decay constant and the half-life:

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{6 \text{ hours}} = 0.1155 \text{ hr}^{-1}$$

Converting to s^{-1}:

$$\lambda = 0.1155 \text{ hr}^{-1} \times \frac{1 \text{ hr}}{3600 \text{ s}} \approx 3.21 \times 10^{-5} \text{ s}^{-1}$$

2. Activity and Number of Nuclei (N): The radioactivity is given as 1 MBq, which is 1×10^6 decays per second (dps). Since activity is defined as $|dN/dt| = \lambda N$, we can find N:

$$|dN/dt| = \lambda N = 1 \times 10^6 \text{ dps}$$

$$N = \frac{1 \times 10^6 \text{ dps}}{\lambda} \approx \frac{1 \times 10^6}{3.21 \times 10^{-5} \text{ s}^{-1}} \approx 3.116 \times 10^{10} \text{ nuclei}$$

Continue Answer to Question 3

3. Mass Calculation: The mass of ^{99m}Tc can be calculated as follows:

$$\text{Mass} = \frac{(\text{Number of Nuclei}) \times (\text{Mass Number})}{\text{Avogadro's Number}}$$

Substituting the values (mass number of ^{99m}Tc is 99):

$$\text{Mass} = \frac{(3.116 \times 10^{10})(99)}{6.023 \times 10^{23}} \approx 5.122 \times 10^{-12} \text{ g}$$

Conclusion: The minimum mass of ^{99m}Tc is approximately 5.122×10^{-12} g (about 5 picograms). This result emphasizes the incredibly high activity-to-mass ratio of radioactive materials. Handle radioactive materials with the same care as pathogenic bacteria!

MCQ: Lecture 3

Question 1

Which of the following best describes the Radioactive Decay Law?

1. The decay of radioactive nuclei increases linearly over time.
2. The number of radioactive nuclei decreases exponentially over time.
3. The decay rate depends on external conditions like temperature and pressure.
4. The decay of radioactive nuclei can be predicted exactly for each nucleus.

Question 2

The half-life of ^{131}I is 8 days. How much of a 200 g sample remains undecayed after 16 days?

1. 100 g
2. 50 g
3. 25 g
4. 200 g

Question 3

The decay constant (λ) for a radionuclide is 0.001 s^{-1}. What is its half-life?

1. 1000 s
2. 693 s
3. 25 s
4. 500 s

MCQ: Lecture 3

Question 4

What is the SI unit of radioactivity?

1. Curie (Ci)
2. Becquerel (Bq)
3. Gray (Gy)
4. Rad (rad)

Question 5

For a radionuclide with $\lambda = 0.05\,\text{min}^{-1}$, what fraction of the sample remains after 20 minutes?

1. 0.223
2. 0.367
3. 0.135
4. 0.50

Question 6

A radionuclide has a half-life of 6 hours. After how much time will only $\frac{1}{16}\ \frac{1}{161}\ \frac{1}{16}$ of the original sample remain?

1. 6 hours
2. 12 hours
3. 24 hours
4. 48 hours

Question 7

Which statement is true regarding the decay constant (λ) and the half-life ($t_{1/2}$)?

1. Larger λ results in a shorter $t_{1/2}$.
2. Larger λ results in a longer $t_{1/2}$.
3. λ independent of $t_{1/2}$.
4. λ and $t_{1/2}$ are directly proportional.

Question 8

For a 1 g sample of ^{226}Ra with a half-life of 1620 years, what is the approximate activity in curies?

1. 0.5 Ci
2. 1.0 Ci
3. 2.0 Ci
4. 0.1 Ci

Question 9

A plot of $\ln(N_t)$ versus time gives:

1. An exponential curve.
2. A straight line with a negative slope.
3. A straight line with a positive slope.
4. A parabolic curve.

Question 10

If a sample's radioactivity decreases to $\frac{1}{8}$ of its original activity in 9 hours, what is its half-life?

1. 3 hours
2. 6 hours
3. 4.5 hours
4. 2.25 hours