Electromagnetic waves

Lecture 2

System of Coordinates and Application

Dr. Mokhalad Ali

Second stage

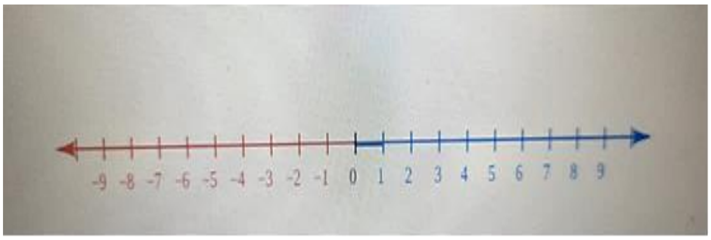
Department of Medical Physics
Al-Mustaqbal University-College

❖ Coordinate System

- Used to describe the position of a point in space
- Coordinate System consists of
  - A fixed reference point called the origin
  - Specific axes with scales and labels
  - Instruction on how to label a point relative to the origin and the axes

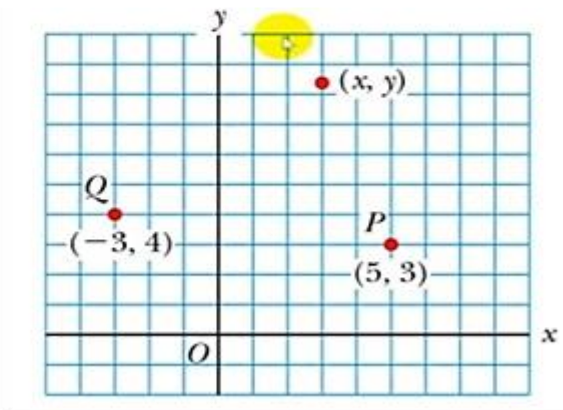
❖ Types of Coordinate Systems

- Number line. . A number line (also called a number axis) is an infinite line on which real numbers are designated: every point on the number line fits a real number, which may be a positive integer, a negative integer, zero, a fraction, or an irrational decimal number.
- Cartesian coordinate system.
- Polar coordinate system.
- Cylindrical and spherical coordinate systems.
- Homogeneous coordinate system.
- Other commonly used systems.
- Relativistic coordinate systems.



❖ Cartesian Coordinate System

- Also called rectangular Coordinate System
- x- and y- axes intersect at the origin
- Points are labeled (x , y)
- The plural of axis is axes
- Ordered pair (x , y) with x-value first



Example /Determine the following points

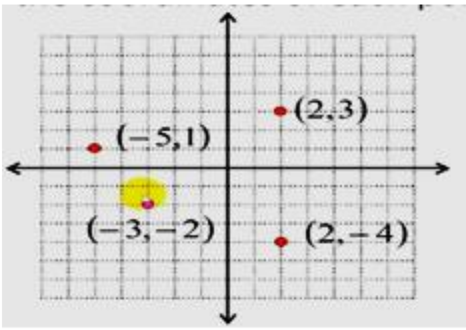
a.(2 ,3)

b.(-5 ,1)

c.(-3,-2)

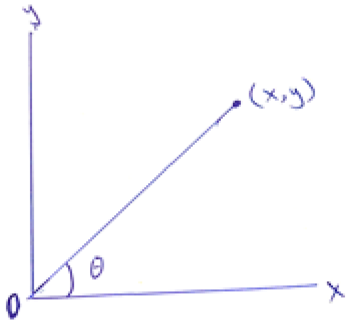
d.(2,-4)

solution :



❖ Polar coordinate system

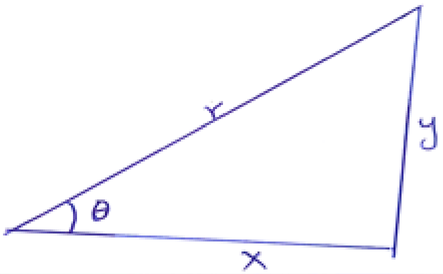
- Origin and reference line are noted.
- Point is distance r from the origin in the direction of angle θ , from reference line .
- Points are labeled (r , θ) .



❖ Polar to Cartesian coordinates

$sin\theta = \frac{y}{r}$, $cos\theta = \frac{x}{r}$, $tan\theta = \frac{y}{x}$

- Based on forming a right triangle from r and θ .
- $x = r\cos\theta$
- $y = r\sin\theta$



❖ Cartesian to Polar coordinates

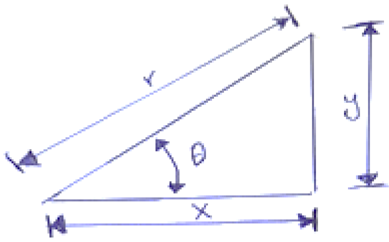
- If the Cartesian coordinates are known :
  - r is the hypotenuse and θ an angle

    $$tan\theta = \frac{y}{x}$$

    $$r = \sqrt{x^2 + y^2}$$

  - θ must be ccw from positive x axis for these equations to be valid

Example : The Cartesian coordinates of a point in the xy Plane are (x,y)=(12 , 5) . Fined the polar coordinates of this point .



Solution : from equation

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{12^2 + 5^2}$$

$$r = 13$$

And from equation ,

$$tan\theta = \frac{y}{x}$$

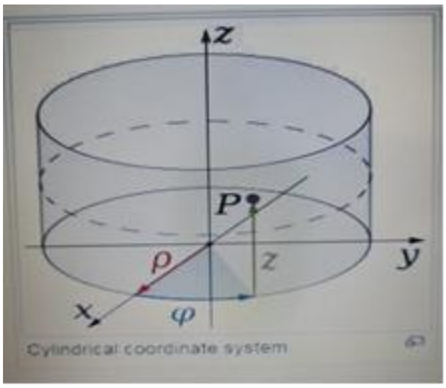
$$tan\theta = \frac{5}{12}$$

$$\theta = tan^{-1}\left(\frac{5}{12}\right)$$

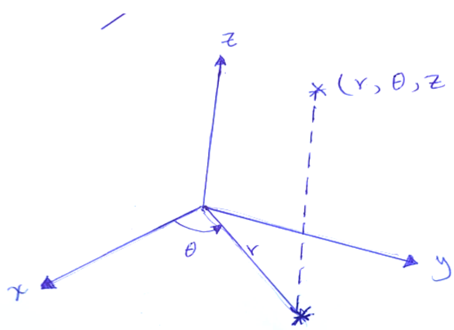
θ=22,6°

❖ Cylindrical coordinate systems

- There are two common methods for extending the polar coordinate system to three dimensions.
- In the cylindrical coordinate system, a z coordinate with the same meaning as in Cartesian coordinates is added to the r and θ polar coordinates giving a triple (r , θ , z).



❖ Convert from Cartesian coordinates to cylindrical coordinates



(x, y,z) → (r,θ,z)

$$\theta = tan^{-1}\left(\frac{y}{x}\right)$$

$$z = z$$

❖ Convert from cylindrical coordinates to Cartesian coordinates (r,θ,z) → (x,y,z)

$x = r\cos\theta$

$y = r\sin\theta$

z=z

Example: Convert from cylindrical coordinates to Cartesian coordinates $(4, \frac{2\pi}{3}, -2)$

Solution : $x = r\cos\theta$

$x = 4\cos\frac{2\pi}{3}$

$x = -2$

$y = r\sin\theta$

$y = 4\sin\frac{2\pi}{3}$

$y = 2\sqrt{3}$

z=z

z=-2

p=(-2,$2\sqrt{3}$,-2)

Example: Convert from Cartesian coordinates to cylindrical coordinates $(1, -3, 5)$

Solution :

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{1^2 + -3^2}$

$r = \sqrt{10}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$\theta = \tan^{-1}\left(\frac{-3}{1}\right)$

$\theta = -71.56$

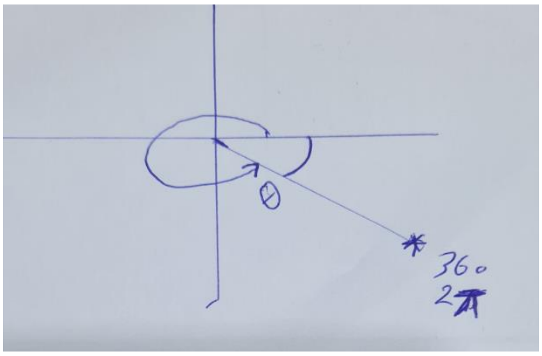
$\theta = 2\pi + 71.56$

$\theta = 431.56$

z = z

z = 5

(r,θ,z) = ($\sqrt{10}$, 5.03 , 5)



Homework :

Convert from Cartesian coordinates to polar coordinates (3,-3,-7) .

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{18}$

$\theta = tan^{-1} \left(\frac{y}{x}\right)$

$\theta = tan^{-1} \left(\frac{-3}{3}\right)$

$\theta = tan^{-1} (-1)$

$\theta = -45$

$\theta = -45 + 2\pi = \cdots.$

(r, θ) = ($\sqrt{18}$, ...)

Spherical Coordinate System

In mathematics, a spherical coordinate system is a coordinate system for three-dimensional space where the position of a point is specified by three numbers: the radial distance of that point from a fixed origin, its polar angle measured from a fixed zenith direction, and the azimuthal angle of its orthogonal projection on a reference plane that passes through the origin and is orthogonal to the zenith, measured from a fixed reference direction on that plane. It can be seen as the three-dimensional version of the polar coordinate system.

- The radial distance is also called the radius or radial coordinate. The polar angle may be called colatitude, zenith angle, normal angle, or inclination angle.

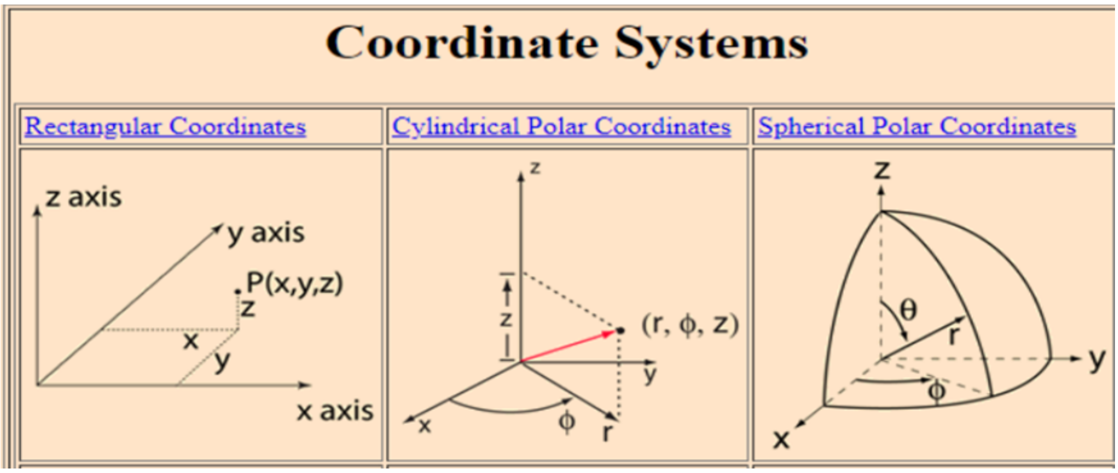
Definition

To define a spherical coordinate system, one must choose two orthogonal directions, the zenith and the azimuth reference, and an origin point in space. These choices determine a reference plane that contains the origin and is perpendicular to the zenith. The spherical coordinates of a point P are then defined as follows:

1-The radius or radial distance is the Euclidean distance from the origin O to P.

2-The inclination (or polar angle) is the angle between the zenith direction and the line segment OP.

3-The azimuth (or azimuthal angle) is the signed angle measured from the azimuth reference direction to the orthogonal projection of the line segment OP on the reference plane.

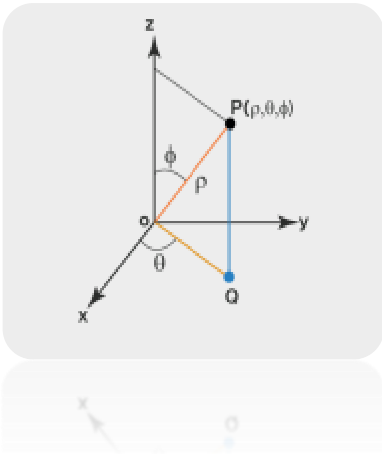


=r sin ɵ cos Φ

Y= r sin ɵ sin Φ

Z= r cos ɵ

Example: Convert from Spherical coordinates to Cartesian Coordinate System (4,30,60)



Solution:-

X=r sin ɵ cos Φ

X= 4sin(30) cos (60)

x=1

Y= 4sin(30).sin (60)

Y=$\sqrt{3}$

Z= 4cos (30)

Z= 2$\sqrt{3}$

r=$\sqrt{x^2 + y^2 + z^2}$

Φ = $\tan^{-1}$ (y/x)

ɵ = $\tan^{-1}$ ($\frac{\sqrt{x2+y2}}{z}$)

➢ Homogeneous coordinate system

A point in the plane may be represented in homogeneous coordinates by a triple (x,y,z) where x/z and y/z are the Cartesian coordinates of the point. this introduces an extra coordinate since only two are needed to specify a point on the plane, but this system is useful in that it represents any point on the projective plane without the use of infinity . in general, a homogeneous coordinate system is one where only the ratios of the coordinates are significant and not the actual values.

➢ Other commonly used systems

Some other common coordinate systems are the following:

1- Curvilinear coordinates are a generalization of coordinate systems generally; the system is based on the intersection of curves.

- ❖ Orthogonal coordinates: coordinate surfaces meet at right angles
- ❖ Skew coordinates: coordinate surfaces are not orthogonal

2- The log-polar coordinate system represents a point in the plane by the logarithm of the distance from the origin and an angle measured from a reference line intersecting the origin.

3- Plücker coordinates are a way of representing lines in 3D Euclidean space using a six-tuple of numbers as homogeneous coordinates.

4- Generalized coordinates are used in the Lagrangian treatment of mechanics.

5- Trilinear coordinates are used in the context of triangles.

➢ Applications

- ❖ Just as the two-dimensional Cartesian coordinate system is useful on the plane, a two-dimensional spherical coordinate system is useful on the surface of a sphere. In this system, the sphere is taken as a unit sphere, so the radius is unity and can generally be ignored. This simplification can also be very useful when dealing with objects such as rotational matrices.

- Three dimensional modeling of loudspeaker output patterns can be used to predict their performance. A number of polar plots are required, taken at a wide selection of frequencies, as the pattern changes greatly with frequency. Polar plots help to show that many loudspeakers tend toward Omni directionality at lower frequencies.

- Spherical coordinates are useful in analyzing systems that have some degree of symmetry about a point, such as volume integrals inside a sphere, the potential energy field surrounding a concentrated mass or charge, or global weather simulation in a planet's atmosphere. A sphere that has the Cartesian equation x2 + y2 + z2 = c2 has the simple equation r = c in spherical coordinates.

Homework:-

Convert from Spherical coordinates to Cartesian Coordinate System (8,30,60) .