Mechanics

Lecture 2: Particle Kinematics
Rectangular and Plane Polar Coordinates

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First Year

Academic Year 2025–2026

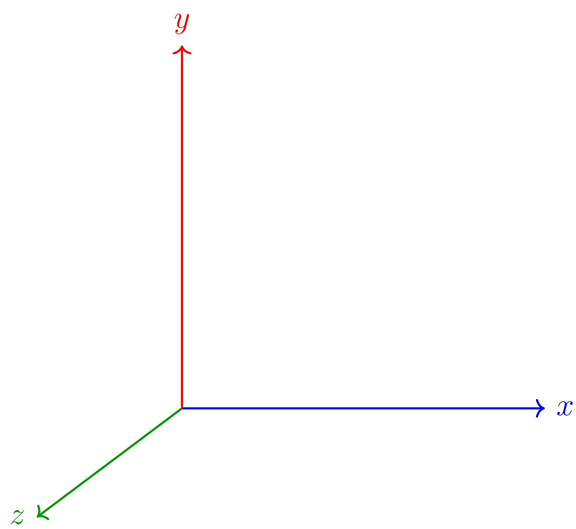
1 Particle Kinematics

Kinematics is the branch of mechanics that deals with the description of motion without reference to the forces that produce it. The central aim of kinematics is to describe how the position of a particle changes with time. To achieve this, we introduce the fundamental kinematic quantities: position, displacement, velocity, and acceleration.

In medical physics, kinematics plays a vital role in understanding the motion of charged particles in radiation therapy, the transport of radioactive tracers, and the mechanical motion of imaging systems such as CT and PET scanners. In many situations, the size of the object under consideration is very small compared to the distance it travels, allowing us to treat it as a particle.

2 Rectangular (Cartesian) Coordinate System

To describe motion quantitatively, a coordinate system must first be established. In three-dimensional space, the rectangular coordinate system consists of three mutually perpendicular axes labeled x, y, and z.



Associated with these axes are the unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$, which indicate direction and have unit magnitude. Once the coordinate system is fixed, all positions and motions are described relative to it.

2.1 Position Vector

The position of a particle relative to the origin is described by the position vector

$$\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath} + z(t)\hat{k}.$$

This vector specifies both the distance and the direction of the particle from the origin.

Solved Example 1 (Position as a Vector)

A particle is located at the point $(3, -2, 4)$ m.

Solution

The position vector is obtained directly from the coordinates:

$$\vec{r} = 3\hat{\imath} - 2\hat{\jmath} + 4\hat{k} \text{ m}.$$

Solved Example 2 (Time-Dependent Position)

The motion of a particle is described by

$$x(t) = 2t, \quad y(t) = t^2, \quad z(t) = 5,$$

where t is in seconds. Find the position vector at $t = 3$ s.

Solution

At $t = 3$ s,

$$x = 6, \quad y = 9, \quad z = 5.$$

Thus,

$$\vec{r}(3) = 6\hat{\imath} + 9\hat{\jmath} + 5\hat{k} \text{ m}.$$

This example illustrates how the position vector evolves with time.

3 Displacement

Displacement is defined as the change in position of a particle during a given time interval. If a particle moves from an initial position $\vec{r}_1$ to a final position $\vec{r}_2$, the displacement vector is

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1.$$

Displacement depends only on the initial and final positions and not on the path taken.

Solved Example 3 (Vector Displacement)

A particle moves from $(1,2,0)$ m to $(5,-1,3)$ m.

Solution

$$\Delta\vec{r} = (5-1)\hat{i} + (-1-2)\hat{j} + (3-0)\hat{k} = 4\hat{i} - 3\hat{j} + 3\hat{k}.$$

The magnitude of the displacement is

$$|\Delta\vec{r}| = \sqrt{4^2 + (-3)^2 + 3^2} = \sqrt{34}\ \text{m}.$$

This result emphasizes that displacement is a vector quantity with both magnitude and direction.

4 Velocity in Rectangular Coordinates

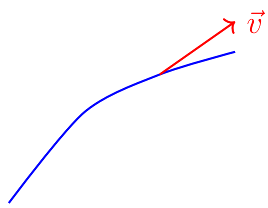
Velocity describes how fast and in what direction a particle's position changes. The instantaneous velocity is defined as the time derivative of the position vector:

$$\vec{v} = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t}.$$

In rectangular coordinates,

$$\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}.$$

Velocity is always tangent to the particle's trajectory.



Solved Example 4 (Velocity Components)

The position of a particle is given by

$$x(t) = t^2, \quad y(t) = 4t, \quad z(t) = 0.$$

Solution

The velocity components are

$$v_x = \frac{\mathrm{d}x}{\mathrm{d}t} = 2t, \quad v_y = \frac{\mathrm{d}y}{\mathrm{d}t} = 4, \quad v_z = 0.$$

Hence,

$$\vec{v}(t) = 2t\hat{i} + 4\hat{j}.$$

Solved Example 5 (Speed and Direction)

At $t = 3$ s, find the speed and direction of motion.

Solution

At $t = 3$ s,

$$\vec{v} = 6\hat{\imath} + 4\hat{\jmath}.$$

The speed is

$$v = \sqrt{6^2 + 4^2} = \sqrt{52} \text{ m/s}.$$

The direction relative to the x-axis is

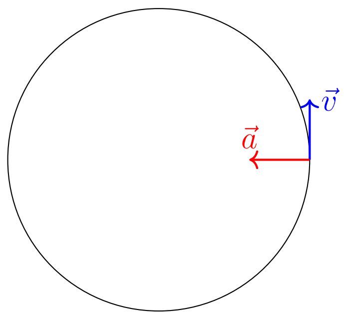
$$\tan\theta = \frac{4}{6} \Rightarrow \theta = 33.7^\circ.$$

5 Acceleration in Rectangular Coordinates

Acceleration describes the rate at which velocity changes with time and is defined as

$$\vec{a} = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t}.$$

Acceleration may result from a change in speed, a change in direction, or both.



Solved Example 6 (Acceleration from Position)

The position of a particle is

$$x(t) = t^3, \quad y(t) = 2t^2.$$

Solution

Velocity:

$$\vec{v} = 3t^2\hat{\imath} + 4t\hat{\jmath}.$$

Acceleration:

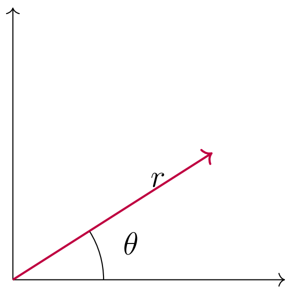
$$\vec{a} = 6t\hat{\imath} + 4\hat{\jmath}.$$

This example shows how acceleration can be obtained by successive differentiation.

6 Plane Polar Coordinate System

When motion involves curved paths or rotation, polar coordinates provide a more natural description. In plane polar coordinates, the position of a particle is specified by the radial distance r and the angular coordinate θ.

$$\vec{r} = r\hat{e}_r.$$



Solved Example 7 (Polar Position)

A particle is located 6 m from the origin.

Solution

$$\vec{r} = 6\hat{e}_r.$$

7 Velocity and Acceleration in Polar Coordinates

In polar coordinates, the velocity of a particle is given by

$$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta.$$

The acceleration is

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta.$$

Solved Example 8 (Polar Velocity and Acceleration)

The motion of a particle is described by

$$r(t) = 2 + t, \quad \theta(t) = t^2.$$

Solution

$$\dot{r} = 1, \quad \ddot{r} = 0, \quad \dot{\theta} = 2t, \quad \ddot{\theta} = 2.$$

At $t = 1$ s,

$$r = 3.$$

Velocity:

$$\vec{v} = \hat{e}_r + 6\hat{e}_\theta.$$

Acceleration:

$$\vec{a} = -12\hat{e}_r + 10\hat{e}_\theta.$$

8 Uniform Circular Motion

In uniform circular motion, a particle moves along a circular path with constant speed. Although the speed is constant, the velocity changes direction continuously, resulting in a centripetal acceleration directed toward the center of the circle.

$$\vec{v} = r\omega\hat{e}_\theta, \quad \vec{a} = -r\omega^2\hat{e}_r.$$

Solved Example 9 (Circular Motion)

A particle moves in a circle of radius 0.50 m with constant speed 2.0 m/s.
Solution

$$\omega = \frac{v}{r} = 4.0 \text{ rad/s}, \quad a = \frac{v^2}{r} = 8.0 \text{ m/s}^2.$$

9 Multiple Choice Questions (MCQs)

1. **Kinematics is the study of motion without considering**
   A- time
   B- displacement
   C- forces
   D- velocity
   E- None of them

2. **A particle is defined as**
   A- an object with volume
   B- an object with mass only
   C- an object of negligible size
   D- an object at rest
   E- None of them

3. **Which of the following quantities is a vector?**
   A- speed
   B- distance
   C- displacement
   D- time
   E- mass

4. **The SI unit of displacement is**
   A- meter
   B- meter per second
   C- meter per second squared
   D- kilogram
   E- newton

5. **The position vector of a particle specifies**
   A- only its distance from the origin
   B- only its direction of motion
   C- its location relative to the origin
   D- the path length traveled
   E- None of them

6. **Displacement depends on**
   A- the path followed
   B- the time taken
   C- initial and final positions only
   D- the speed of motion
   E- None of them

7. **Distance traveled is always**
   A- a vector quantity
   B- equal to displacement
   C- zero for closed paths
   D- a scalar quantity
   E- negative

8. **Velocity is defined as the rate of change of**
   A- distance
   B- speed
   C- displacement
   D- acceleration
   E- force

9. **Instantaneous velocity is obtained from**
   A- $\Delta x/\Delta t$ for large Δt
   B- the area under a position–time graph
   C- the slope of the position–time graph
   D- the slope of the velocity–time graph
   E- None of them

10. **Velocity is always**
    A- parallel to acceleration
    B- perpendicular to the path
    C- tangent to the trajectory
    D- directed toward the origin
    E- constant in magnitude

11. **Speed is defined as**
    A- the rate of change of displacement
    B- the magnitude of velocity
    C- the direction of motion
    D- the rate of change of acceleration
    E- None of them

12. **Which of the following quantities can never be negative?**
    A- velocity
    B- displacement
    C- acceleration
    D- speed
    E- position

13. **Acceleration is defined as the rate of change of**
    A- position
    B- displacement
    C- speed only
    D- velocity
    E- distance

14. **Acceleration can exist even when**
    A- velocity is increasing

B- velocity is decreasing
C- velocity is zero
D- speed is changing
E- None of them

15. **If velocity is constant, acceleration is**
A- positive
B- negative
C- increasing
D- zero
E- maximum

16. **In one-dimensional motion, acceleration is**
A- always positive
B- always zero
C- along the line of motion
D- perpendicular to velocity
E- None of them

17. **The slope of a velocity–time graph represents**
A- displacement
B- speed
C- acceleration
D- distance
E- position

18. **Which coordinate system is most suitable for circular motion?**
A- Cartesian
B- rectangular
C- polar
D- linear
E- None of them

19. **In polar coordinates, position is described by**
A- x and y
B- x, y, and z
C- r and θ
D- distance only
E- velocity only

20. **The radial unit vector $\hat{e}_r$ in polar coordinates**
A- has fixed direction
B- is always horizontal
C- rotates with the particle
D- is perpendicular to radius
E- None of them

21. **The tangential component of velocity in polar coordinates is**
A- $\dot{r}$
B- $\ddot{r}$

C- $r\dot{\theta}$
D- $r\ddot{\theta}$
E- $\dot{\theta}$

22. **Centripetal acceleration is directed**
A- tangentially
B- outward from the center
C- along the velocity
D- toward the center of the circle
E- vertically downward

23. **In uniform circular motion, speed is**
A- zero
B- increasing
C- decreasing
D- constant
E- changing direction only

24. **In uniform circular motion, acceleration is**
A- zero
B- constant in direction
C- tangent to the path
D- directed toward the center
E- parallel to velocity

25. **The magnitude of centripetal acceleration depends on**
A- mass only
B- radius only
C- speed and radius
D- direction only
E- time only

26. **If the radius of circular motion increases while speed is constant, the centripetal acceleration**
A- increases
B- decreases
C- remains constant
D- becomes zero
E- reverses direction

27. **The angular velocity ω is related to linear speed by**
A- $\omega = vr$
B- $\omega = v^2/r$
C- $\omega = v/r$
D- $\omega = r/v$
E- None of them

28. **Which of the following quantities changes continuously in uniform circular motion?**
A- speed

B- mass
C- magnitude of velocity
D- direction of velocity
E- radius

29. **Which of the following best describes acceleration in circular motion?**
A- Zero acceleration
B- Tangential acceleration only
C- Radial acceleration only
D- Both radial and tangential acceleration
E- None of them

30. **In medical imaging devices involving rotation, the most important kinematic concept is**
A- linear displacement
B- rectilinear motion
C- circular motion
D- free fall
E- None of them