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College of Science
Department of Medical
Physics



Electrical Material

First Stage

Lecture name : Gauss's Law

Lecture number : 3

Name of lecturer

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Introduction and Historical Background

Gauss's Law is one of the four fundamental equations of classical electromagnetism, collectively known as Maxwell's Equations. It provides a deep connection between electric fields and electric charges and plays a central role in understanding electrostatic phenomena.

The law was formulated by the German mathematician and physicist Carl Friedrich Gauss. Although Gauss's Law is always valid, it becomes especially powerful when applied to systems with a high degree of symmetry.

This lecture focuses on:

- ✚ The physical meaning of Gauss's Law
 - ✚ Mathematical formulation
 - ✚ Concept of electric flux
 - ✚ Applications to symmetric charge distributions
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Review of Electric Charge and Electric Field

Electric Charge

Electric charge is a fundamental property of matter. It exists in two forms:

- Positive charge
- Negative charge

Charges interact through the electric force, and the source of this interaction is the **electric field**.

Electric Field

The electric field \mathbf{E} at a point in space is defined as the force experienced by a unit positive test charge placed at that point:

$$\mathbf{E} = \frac{\mathbf{F}}{q}$$

The electric field is a vector quantity:

- Magnitude: strength of the field
 - Direction: direction of force on a positive test charge
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Electric Field Lines and Physical Interpretation

Electric field lines are a visual tool used to represent electric fields.

Properties of electric field lines:

- They originate from positive charges
- They terminate on negative charges
- The density of lines indicates field strength
- Field lines never intersect

Although field lines are imaginary, they help us understand how electric fields propagate in space and how they interact with surfaces.

Electric Flux – Concept and Definition

Electric flux is a measure of how much electric field passes through a given surface.

For a uniform electric field passing through a flat surface:

$$\Phi_E = \vec{E} \cdot \vec{A} = EA \cos \theta$$

Where:

- \vec{A} is the area vector (normal to the surface)
- θ is the angle between the electric field and the area vector

Physical interpretation:

- Maximum flux when the field is perpendicular to the surface
- Zero flux when the field is parallel to the surface

Closed Surfaces and Gaussian Surfaces

A **closed surface** completely encloses a volume. Examples include:

- Spheres
- Cylinders
- Cubes

A **Gaussian surface** is an imaginary closed surface chosen to apply Gauss's Law.

Important characteristics:

- It is a mathematical tool
- It does not need to correspond to a physical object
- Its shape is chosen to exploit symmetry in the electric field

Statement and Mathematical Form of Gauss's Law

Integral Form of Gauss's Law

$$\oint \vec{E} \cdot d\vec{A} = Q_{\text{enclosed}} / \epsilon_0$$

Where:

- The integral is taken over a closed surface
- Q_{enclosed} is the total charge inside the surface
- ϵ_0 is the permittivity of free space

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / (\text{N} \cdot \text{m}^2)$$

Physical Meaning of Gauss's Law

Gauss's Law states that:

- The net electric flux through a closed surface depends only on the total charge enclosed
- Charges outside the surface do not contribute to the net flux

Reason:

- Field lines from external charges enter and leave the surface in equal numbers
- Their contributions cancel out

This highlights a fundamental principle: **electric fields are sourced by charges.**

Application 1 – Point Charge with Spherical Symmetry

Consider a point charge q located at the center of a spherical Gaussian surface.

Due to spherical symmetry:

- The electric field has the same magnitude everywhere on the surface
- The field is radial and perpendicular to the surface

Applying Gauss's Law:

$$E(4\pi r^2) = q/\epsilon_0$$

Solving for E :

$$E = q/4\pi\epsilon_0 r^2$$

This result is identical to Coulomb's Law, confirming the consistency of Gauss's Law.

Application 2 – Infinite Line and Plane of Charge

Infinite Line of Charge

For a long uniformly charged wire with linear charge density λ :

$$E = \lambda/2\pi\epsilon_0 r$$

A cylindrical Gaussian surface is used because of cylindrical symmetry.

Infinite Plane of Charge

For σ uniformly charged infinite plane with surface charge density:

$$E = \sigma / 2\epsilon_0$$

The electric field is constant and independent of distance from the plane.

Importance, Limitations, and Summary

Importance of Gauss's Law

- Fundamental law of electromagnetism
- Simplifies electric field calculations
- Essential for understanding conductors and capacitors
- Forms part of Maxwell's Equations

Limitations

- Useful mainly for highly symmetric charge distributions
- Not practical for arbitrary charge configurations

Summary

- Gauss's Law relates electric flux to enclosed charge
- Electric flux measures field through a surface
- Gaussian surfaces are chosen based on symmetry
- The law is exact and universally valid

Final Equation

$$\oint \vec{E} \cdot d\vec{A} = Q / \epsilon_0$$