Atomic and Molecular Physics

Presented by

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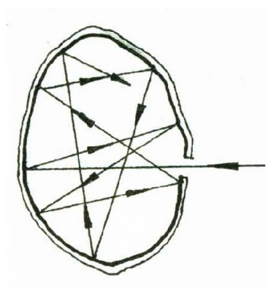
Second-year students

Chapter Two: Quantum Theory

2.1 Black body

A blackbody is defined as the body which can absorb all energies that fall on it. It is something like a black hole. No lights or material can get away from it as long as it is trapped. A large cavity with a small hole on its wall can be taken as a blackbody.



2.2 Black body radiation

Any radiation that enters the hole is absorbed in the interior of the cavity, and the radiation emitted from the hole is called blackbody radiation.

2.2.1 Stefan-Boltzmann Law :

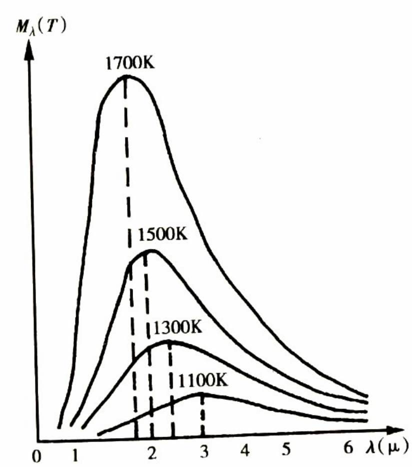
It is found that the radiation energy is proportional to the fourth power of the associated temperature.

$$M_\lambda(T) = \sigma T^4$$

Where $M_\lambda(T)$ **is the area under the curve**

σ **is called stefan's constant**

T is the absolute temprature in Kelvin



2.2.2 Wien's displacement law:

The peak of the curve shifts towards longer wavelength as the temperature falls and it satisfies

$$\lambda_{\max} T = b$$

Where *b* is called the Wien's constant

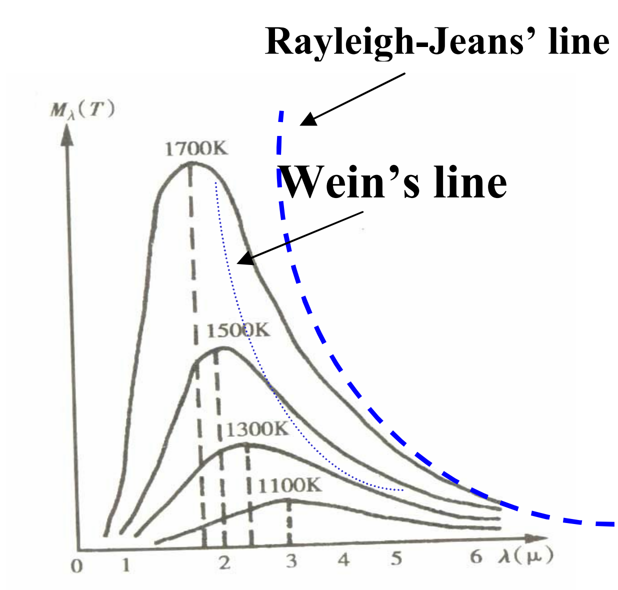
This law is quite useful for measuring the temperature of a blackbody with a very high temperature.

2.3 Rayleigh-Jeans Law

In 1890, Rayleigh and Jeans obtained a formula using the classical electromagnetic (Maxwell) theory and the classical equipartition theorem of energy in thermodynamics. The formula is given by;

$$M_{\lambda}(T) = C_1 \lambda^{-4} T$$ **where C_1 is a constant**

Rayleigh-Jeans formula was correct for very long wavelength in the far infrared but hopelessly wrong in the visible light and ultraviolet region. Maxwell's electromagnetic theory and thermodynamics are known as correct theory. The failure in explaining blackbody radiation puzzled physicists! It was regarded as ultraviolet Catastrophe (disaster).

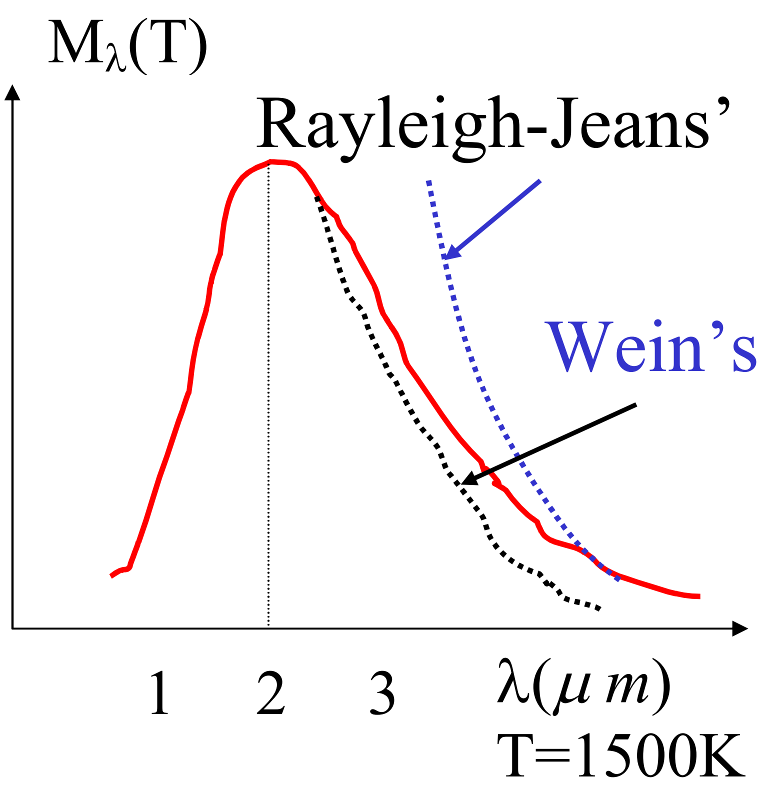


2.4 Wein's formula:

Later on in 1896, Wein derived another important formula using thermodynamics.

$$M_\lambda(T) = C_2\,\lambda^{-5}\,e^{-\frac{C_3}{\lambda T}}$$

Unfortunately, this formula is only valid in the region of short wavelengths.



2.5 Planck's Magic formula

In 1900, after studying the above two formulas carefully, Planck proposed an empirical formula

$$M_\lambda(T) = 2\pi \hbar c^2 \lambda^{-5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$

Where c is the speed of light, *k* is Boltzmann's constant, h is Planck's constant and e is the base of natural logarithm.

It is surprising that the experience formula can describe the curve of blackbody radiation exactly for all wavelengths.

- **Other unbelievable deductions:**

(1) For very large wavelength, the Rayleigh-Jeans formula can be obtained from Planck's formula;

$$\frac{hc}{k\lambda T} << 1$$

$$e^{\frac{hc}{k\lambda T}} = 1 + \frac{hc}{k\lambda T} + \frac{1}{2}\left(\frac{hc}{k\lambda T}\right)^2 + \cdots$$

Drop the second order and higher order terms, and RJ formula could be obtained.

(2) For smaller wavelength of blackbody radiation, the Wein's formula can be achieved also from Planck's experience formula;

$$\frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \approx e^{-\frac{hc}{\lambda kT}}$$

Then Wein's formula could be obtained

(3) Integrating Planck's formula with respect to wavelength, the Stefan and Boltzmann's law can be obtained as well.

$$M(T) = \int_0^{\infty} M_\lambda(T) d\lambda = \sigma T^4$$

, where σ is called Stefen constant.

(4) Finally, according to the basic mathematical theory and differentiating the Planck's formula with respect to wavelength, Wien's displacement law can also be derived!

$$\frac{dM_\lambda(T)}{d\lambda} = 0$$

$$\Rightarrow \lambda_{\max} T = b$$

Planck's empirical formula matched all the different classical physics results obtained by the Maxwell electromagnetic theory, thermodynamics and statistics! However, no one knew why at that time. This phenomenon seemed unbelievable, incredible and even impossible, but is true!

In order to derive this formula theoretically, Planck proposed a brave hypothesis which is also incredible.

Planck's Hypotheses:

- **The molecules and atoms composing the blackbody concave can be regarded as the linear harmonic oscillator with electrical charge;**
- **The oscillators can only be in a special energy state. All these energies must be the integer multiples of a smallest energy ($\varepsilon_0 = h\nu$). Therefore the energies of the oscillators are $E = n\, h\nu$ with n = 1, 2, 3, …**

Using the hypothesis and classical physics, Planck arrived at his experience formula in two months later. The correct result shows that Planck's hypothesis is correct!

Quantum theory and modern physics was founded by these hypotheses!

2.6 Planck-Einstein Energy Quantization Law:

$$E = h\nu = \frac{hc}{\lambda}$$

E → Quantum energy; h → Planck constant; ν → Frequency

$$h = 6.626 \times 10^{-34} J \cdot s$$
$$= 4.136 \times 10^{-15} eV \cdot s$$

$$1eV = 1.602 \times 10^{-19} J$$
$$1J = 6.242 \times 10^{18} eV$$

Example: **Calculate the photon energies for the following types of electromagnetic radiation: (a) a 600kHz radio wave; (b) the 500nm (wavelength of) green light; (c) a 0.1 nm (wavelength of) X-rays.**

Solution: (a) for the radio wave, we can use the Planck-Einstein law directly

$$E = h\nu = 4.136 \times 10^{-15} eV \cdot s \times 600 \times 10^{3} Hz$$
$$= 2.48 \times 10^{-9} eV$$

(b) The light wave is specified by wavelength, we can use the law explained in wavelength:

$$E = \frac{hc}{\lambda} = \frac{1.241 \times 10^{-6} eV \cdot m}{550 \times 10^{-9} m} = 2.26eV$$

(c) For X-rays, we have

$$E = \frac{hc}{\lambda} = \frac{1.241 \times 10^{-6} eV \cdot m}{0.1 \times 10^{-9} m} = 1.24 \times 10^{4} eV = 12.4keV$$

Therefore you can see that the higher frequency corresponds to the higher energy. The X-rays have quite high energy, so they have high power of penetration.

Here we emphasize that the particle properties of light and the photon will be defined. As we know the light is electromagnetic waves and it has the properties of waves.

PlPanck associated the energy quanta only with the light emission in the cavity walls and Einstein extended them to the absorption of radiation in his explanation of the *photoelectric effect*.

2.7 Photoelectric effect

The quantum nature of light had its origin in the theory of thermal radiation and was strongly reinforced by the discovery of the photoelectric effect.

In figure 2.1, a glass tube contains two electrodes of the same material, one of which is irradiated by light. The electrodes are connected to a battery and a sensitive current detector measures the current flow between them.

The current flow is a direct measure of the rate of emission of electrons from the irradiated electrode.

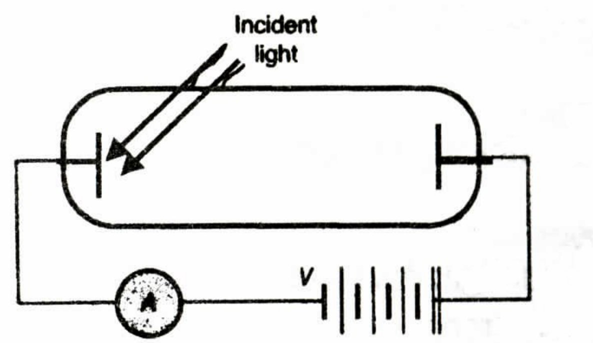


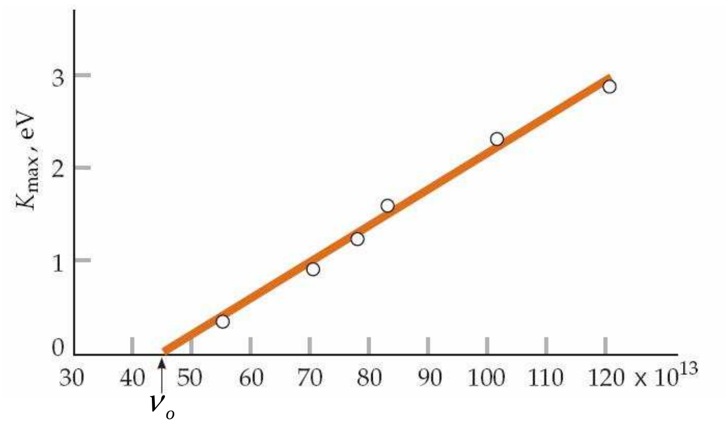
Fig. 2.1 Apparatus to investigate the photoelectric effect that was first found in 1887 by Hertz.

The electrons in the electrodes can be ejected by light and have a certain amount of kinetic energy.Now we change:

(1) the frequency and intensity of light,
(2) the electromotive force (e.m.f. or voltage),
(3) the nature of electrode surface.

It is found that:

1. **For a given electrode material, no photoemission exists at all below a certain frequency of the incident light. When the frequency increases, the emission begins at a certain frequency. The frequency is called threshold frequency (ν_o) of the material. The threshold frequency has to be measured in the existence of e.m.f. (electromotive force) as at such a case the photoelectrons have no kinetic energy to move from the cathode to anode. Different electrode material has different threshold frequency.**



2. **The rate of electron emission is directly proportional to the intensity of the incident light.**
3. **Increasing the intensity of the incident light does not increase the kinetic energy of the photoelectrons.**
4. **There is no measurable time delay between irradiating the electrode and the emission of photoelectrons, even when the light is of very low intensity. As soon as the electrode is irradiated, photoelectrons are ejected.**
5. **The photoelectric current is deeply affected by the nature of the electrodes and chemical contamination of their surface.**

$$K_{\max} = eV$$

(1) In 1905, Einstein solved the photoelectric effect problem by applying the Planck's hypothesis. He pointed out that Planck's quantization hypothesis applied not only to the emission of radiation by a material object but also to its transmission and its absorption by another material object. The light is not only electromagnetic waves but also a quantum. All the effects of photoelectric emission can be readily explained from the following assumptions: The photoemission of an electron from a cathode occurs when an electron absorbs a photon of the incident light;

(2) The photon energy is calculated by the Planck's quantum relationship: E = hν.

(3) The minimum energy is required to release an electron from the surface of the cathode. The minimum energy is the characteristic of the cathode material and the nature of its surface. It is called work function (Φ).

The equation for the photoelectric emission can be written out by supposing the photon energy is completely absorbed by the electron. After this absorption, the kinetic energy of the electron should have the energy of the photon. If this energy is greater than the work function of the material, the electron should become a photoelectron and jumps out of the material and probably have some kinetic energy.

Therefore we have the equation of photoelectric effect:

$$K_{\max} = h\nu - \Phi$$

Where ($K_{\max}$) is the photoelectron kinetic energy, (*h*) is Planck's constant, (ν) is the frequency of the incident light and Φ is the work function.

$$K_{\max} = \frac{1}{2}\mathrm{mv}^2 \ \textbf{and}\ \Phi = h\nu_o$$

$$hc = (4.136\times10^{-15}\ \mathrm{eV.s})(3\times10^{8}\,\mathrm{m/s}) = 1.240\times10^{-6}\ \mathrm{eV.m}$$

Or

$$hc = 1240 \quad \mathrm{eV.nm}$$

Example: **Ultraviolet light of wavelength 150nm falls on a chromium electrode. Calculate the maximum kinetic energy and the corresponding velocity of the photoelectrons (the work function of chromium is 4.37eV).**

Solution: **using the equation of the photoelectric effect, it is convenient to express the energy in electron volts. The photon energy is**

$$E = h\nu = \frac{hc}{\lambda} = \frac{1.241\times10^{-6}\,eV\cdot m}{150\times10^{-9}\,m} = 8.27eV$$

$$K_{\max} = h\nu - \Phi$$

$$\Rightarrow \frac{1}{2}\mathrm{mv}^2 = (8.27-4.37)eV = 3.90eV$$

$$1eV = 1.602\times10^{-19}\,J = 1.602\times10^{-19}\,N\cdot m = 1.602\times10^{-19}\,kg\cdot m^2\cdot s^{-2}$$

$$\frac{1}{2}\mathrm{mv}^2 = 3.90eV = 3.90\times1.602\times10^{-19}\,kg\cdot m^2\cdot s^{-2}$$

$$\mathrm{v} = \sqrt{\frac{2\times3.90eV}{m}} = \sqrt{\frac{12.496\times10^{-19}}{9.11\times10^{-31}}} = 1.17\times10^{6}\,m/s$$

2.8 Compton effect

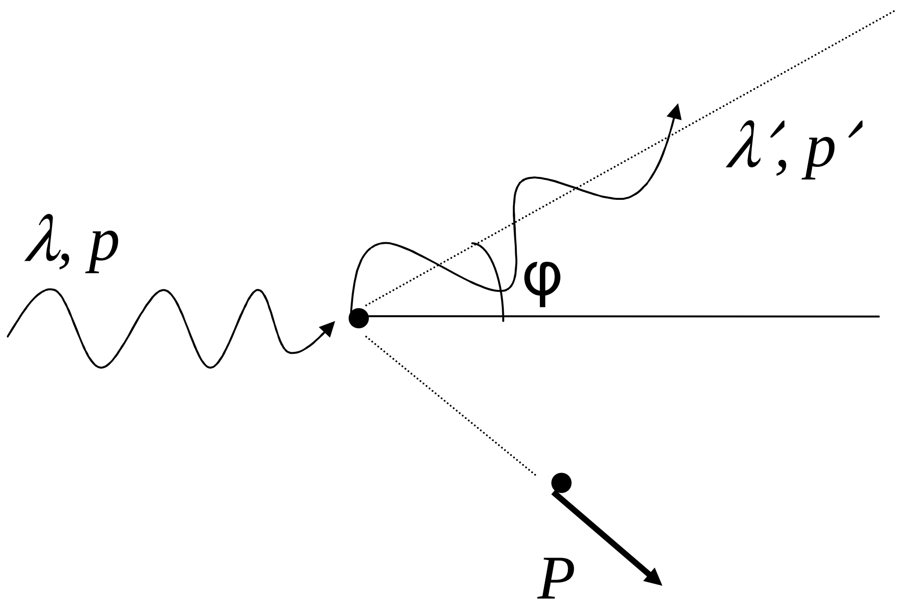
A phenomenon called Compton scattering, first observed in 1924 by Compton, provides additional direct confirmation of the quantum nature of electromagnetic radiation. When X-rays impinges on matter, some of the radiation is scattered, just as the visible light falling on a rough surface undergoes diffuse reflection. Observation shows that some of the scattered radiation has smaller frequency and longer wavelength than the incident radiation, and that the change in wavelength depends on the angle through which the radiation is scattered. Specifically, if the scattered radiation emerges at an angle φ with the respect to the incident direction, and if λ and λ' are the wavelength of the incident and scattered radiation, respectively, it is found that

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\varphi) = 2\frac{h}{mc}\sin^2\left(\frac{\varphi}{2}\right)$$

where *m is the electron mass.* $\Delta\lambda = \lambda' - \lambda$

$\lambda_c = \frac{h}{mc} = 0.00243 \; nm$ **called Compton wavelength**

In figure 2.1, the electron is initially at rest with incident photon of wavelength λ and momentum *p*; scattered photon with longer wavelength λ' and momentum *p′* and recoiling electron with momentum P. The direction of the scattered photon makes an angle φ with that of the incident photon, and the angle between p and p′ is also φ.



Q1. A blackbody is defined as a body that:
A) Reflects all incident radiation
B) Absorbs all incident radiation of any wavelength
C) Emits radiation only at short wavelengths
D) Allows only visible light to pass through

Q2. In a practical sense, a large cavity with a small hole is considered a blackbody because:
A) Radiation entering never escapes and is absorbed
B) It reflects radiation uniformly
C) It transmits light only at resonance frequencies
D) It blocks all electromagnetic waves

Q3. The Stefan–Boltzmann law states that the total energy radiated per unit area is proportional to:
A) T
B) T^2
C) T^3
D) T^4

Q4. Wien's displacement law states that the wavelength corresponding to maximum emission varies as:
A) $\lambda_{\max} T = b$
B) $\lambda_{\max}/T = b$
C) $\lambda_{\max} \propto T^2$
D) $\lambda_{\max} = \text{constant}$

Q5. Rayleigh–Jeans law is valid for:
A) Long wavelengths (infrared region)
B) Short wavelengths (ultraviolet region)
C) Only visible spectrum
D) X-ray region

Q6. The failure of the Rayleigh–Jeans law at short wavelengths was termed the:
A) Gamma catastrophe
B) Ultraviolet catastrophe
C) Infrared anomaly
D) Photoelectric paradox

Q7. Wien's formula derived in 1896 successfully explained:
A) Only short wavelength region of blackbody radiation
B) Only long wavelength region of blackbody radiation
C) All wavelength regions
D) No region correctly

Q8. Planck's empirical formula for blackbody radiation was unique because:
A) It agreed with experiments for all wavelengths
B) It only applied to X-rays
C) It ignored temperature dependence
D) It predicted no radiation beyond visible light

Q9. According to Planck's hypothesis, the energy of oscillators in a blackbody cavity is:
A) Continuous
B) Quantized in integer multiples of hνh\nu
C) Dependent only on temperature
D) Randomly distributed

Q10. Which classical laws can be derived as limiting cases of Planck's formula?
A) Only Rayleigh–Jeans law
B) Only Wien's law
C) Stefan–Boltzmann and Wien's law
D) Rayleigh–Jeans, Wien, and Stefan–Boltzmann laws

Q11. The Planck–Einstein relation connects photon energy with:
A) Mass of the nucleus
B) Frequency of light (E=hνE = h\nu)
C) Velocity of electrons
D) Wavelength squared

Q12. Among the following, which electromagnetic radiation has the highest photon energy?
A) 600 kHz radio waves
B) Green light (λ=500 nm\lambda = 500 \, nm)
C) X-rays (λ=0.1 nm\lambda = 0.1 \, nm)
D) Infrared radiation

Q13. The photoelectric effect demonstrates the particle nature of light because:
A) Emission occurs instantly when photons exceed threshold frequency
B) Emission is delayed at low intensity
C) Emission depends only on intensity, not frequency
D) Emission is explained by continuous wave theory

Q14. In the photoelectric effect, no photoemission occurs if:
A) Intensity of light is low
B) Frequency of incident light is below threshold
C) Voltage is applied across electrodes
D) Surface is polished

Q15. The maximum kinetic energy of photoelectrons is given by:

A) $K_{max} = h\nu$

B) $K_{max} = h\nu - \phi$

C) $K_{max} = \phi - h\nu$

D) $K_{max} = eV$

Q16. Increasing the intensity of light in a photoelectric experiment increases:

A) Maximum kinetic energy of electrons
B) Threshold frequency
C) Number of photoelectrons emitted
D) Work function of the surface

Q17. Einstein's explanation of the photoelectric effect applied Planck's quantum concept to:

A) Emission only
B) Absorption and emission of light
C) Only reflection phenomena
D) Nuclear binding energies

Q18. The work function ϕ\phi of a metal is defined as:

A) Minimum energy required to release an electron from the metal surface
B) Maximum kinetic energy of photoelectrons
C) Energy gap between atomic levels
D) Binding energy of nucleus

Q19. Compton scattering provided direct evidence of photon momentum because:

A) Scattered X-rays had higher frequency than incident
B) Scattered X-rays had longer wavelength depending on angle
C) X-rays passed unaffected through matter
D) Photon energy was independent of wavelength

Q20. The change in wavelength in Compton effect is given by:

A) $\Delta\lambda = \frac{h}{mc}$

B) $\Delta\lambda = \frac{h}{mc}(1 - \cos\phi)$

C) $\Delta\lambda = \frac{h}{mc}(1 + \cos\phi)$

D) $\Delta\lambda = h\nu - \phi$