Quantum Mechanics in Medicine

Lecture 1

Presented by

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Third-year students

Chapter One: The Origen of Quantum theory

Introduction

The Physical Foundations of Quantum Mechanics

Classical mechanics: can explain MACROSCOPIC phenomena such as motion of billiard balls or rockets.

Newton's mechanics (particle is particle)

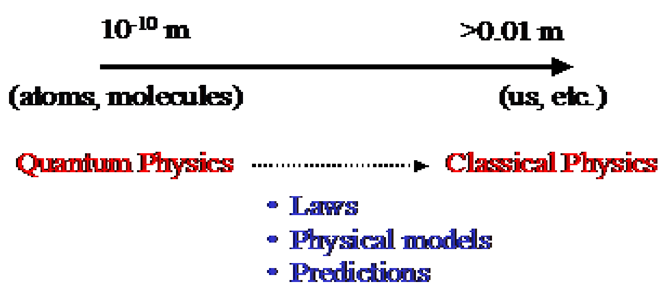
Properties (mass m, position $\mathbf{r}$, velocity $\boldsymbol{v}$) ⇒ Behaviour (collisions, momentum)

Maxwell's equations (electromagnetics theory) (wave is wave)

Properties (wavelength, frequency) ⇒ Behaviour (diffraction, interference)

Classical mechanics fail when we go to the atomic regime then we need to consider Quantum Mechanics.

Quantum mechanics is used to explain microscopic phenomena such as photon-atom scattering and flow of the electrons in a semiconductor.



Experimental Evidence for the Breakdown of Classical Mechanics

By the 20th Century, there were number of experimental results and phenomena that could not be explained by classical mechanics.

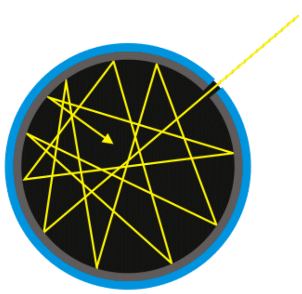
- **Black-body radiation (**1860-**1901)**
- **Atomic Spectroscopy (**1888-**)**
- **Photoelectric Effect (**1887-**1905)**
- **Compton Effect (1923).**
- **Electron Diffraction Davisson and Germer (1925).**

1.1 Black Body Radiation

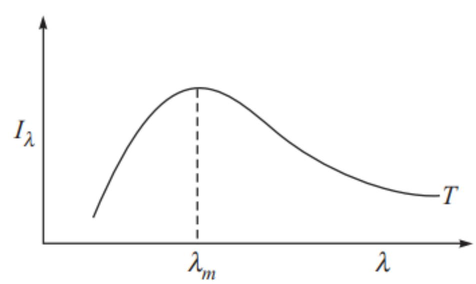
The quantum theory had its origin in the search for an explanation of the spectral distribution of radiant energy emitted by a blackbody.

An ideal blackbody is defined as one that absorbs all electromagnetic radiation incident upon it. that such a body is also a better radiator of energy, of all frequencies, than any other body at the same temperature. *An ideal blackbody does not exist.*

The nearest approximation is a hollow enclosure having blackened inner walls and a small hole. Any radiation entering the enclosure through the hole will suffer reflections repeatedly and get absorbed inside. There is very little chance of its coming out. If the enclosure is heated to a certain temperature *T*, it emits radiation. A very small fraction of the radiation will pass out



Conceptual Black Body



Wien's law: Wien obtained the following semiempirical formula to explain the shape of blackbody radiation curve, known as **Wien's law**:

$$I(\lambda, T) = \frac{ae^{-b/\lambda T}}{\lambda^5} \quad \ldots(1.1)$$

where a and b are adjustable parameters. This law fitted the experimental curve fairly well except at long wavelengths.

The Rayleigh-Jeans Law was derived by applying the principles of classical physics based on the classical electrodynamics and thermodynamics.

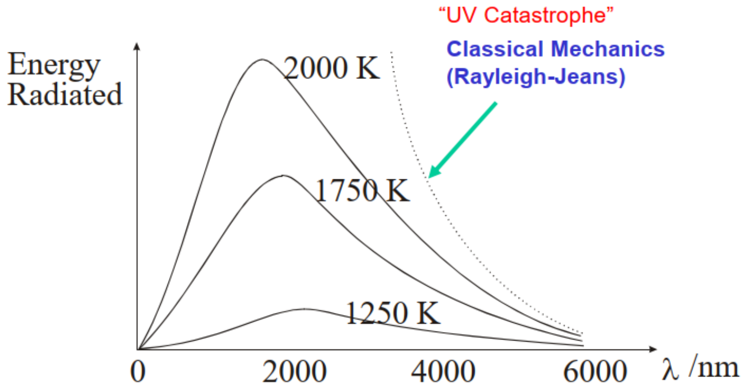
The energy density $U(\lambda, T)$ of the radiation of wave length λ in the cavity, at temperature T,

$$U(\lambda, T) = \frac{8\pi}{4\lambda^4} kT$$

The intensity $I(\lambda, T)$ of the radiation emitted by the cavity hole is;

$$\boxed{I(\lambda, T) = \frac{8\pi c}{4\lambda^4} kT} \qquad \ldots(1.2)$$

The law was derived by applying the principles of classical physics based on the classical electrodynamics and thermodynamics.



It is found that the Rayleigh-Jeans law agrees with the experimental results in the long wavelength region. However, it diverges as the wavelength tends to zero.

Planck's Quantum Theory

Planck (1900) proposed that the light energy emitted by the black body is quantized in units of $h\nu$ (ν = frequency of light)

Planck's quantum hypothesis: *(The material oscillators (in the walls of the cavity) can have only discrete energy levels rather than a continuous range of energies as assumed in classical physics. If a particle is oscillating with frequency ν, its energy can take only the values:*

$$\Delta E = nh\nu \quad (n = 1, 2, 3, \ldots) \qquad \ldots(1.3)$$

- High frequency light only emitted if thermal energy $KT \geq h\nu$.
- $h\nu$ a quantum of energy.
- Planck's constant $h = 6.626 \times 10^{-34}$Js
- If $h \to 0$ we regain classical mechanics.

The Planck's radiation law. In terms of the wavelength λ of the radiation,

$$\boxed{U(\lambda, T) = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1}} \qquad \ldots(1.4)$$

Planck's law agrees very closely with the observed spectral distribution curves for all values of λ and T.

PROBLEM: Show that Planck's law reduces to Wien's law in the short wavelength limit and Rayleigh-Jeans' law in the long wavelength limit.

Solution: When λ is small, $e^{hc/\lambda kT} >> 1$. Therefore,

$$U(\lambda,\ T) \sim \frac{8\pi hc}{\lambda^5}e^{-hc/\lambda kT}$$

which is *Wien's law* (see Equation 2.1).

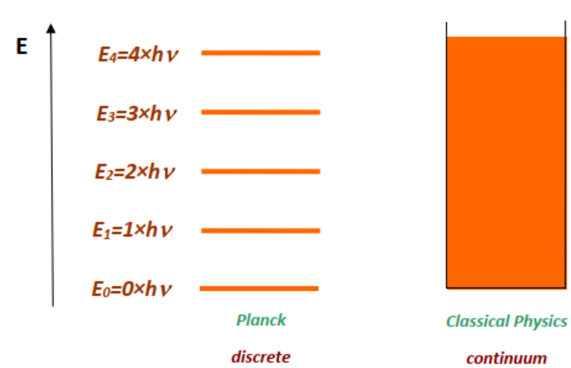
When λ is large,

$$e^{hc/\lambda kT} \sim 1 + \frac{hc}{\lambda kT}$$

Therefore,

$$U(\lambda,\ T) \sim \frac{8\pi hc}{\lambda^5}\frac{\lambda kT}{hc} = \frac{8\pi kT}{\lambda^4}$$

which is Rayleigh-Jeans' law



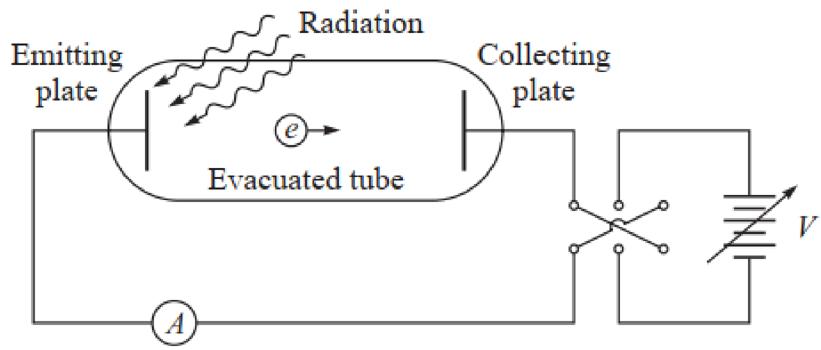
Conclusions:

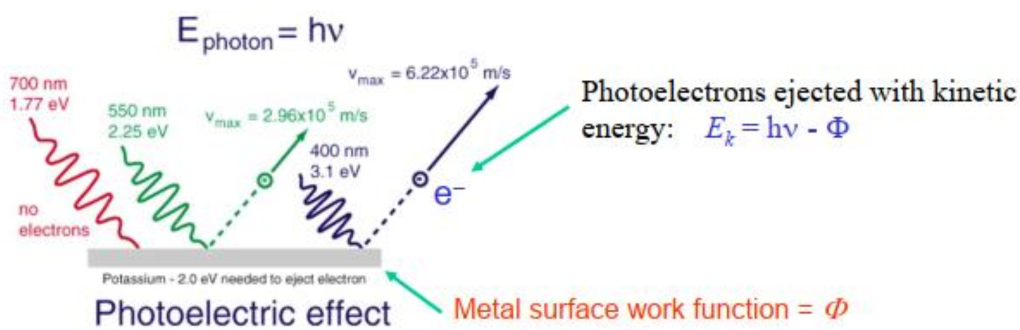
Energy is quantized (not continuous).

Energy can only change by well-defined amounts.

1.2 Photoelectric Effect (Einstein 1905)

When electromagnetic radiation of high enough frequency is incident on a metal surface, electrons are emitted from the surface. This phenomenon is called photoelectric effect. The emitted electrons are generally called photoelectrons. This effect was discovered by Heinrich Hertz in 1887.





The following interesting results were obtained in the study:

(1) No electrons are emitted if the incident radiation has a frequency less than a threshold value ν_o. The value of ν_o varies from metal to metal.

(2) The kinetic energy of the emitted electrons varies from zero to a maximum value. The maximum value of energy depends on the frequency and not on the intensity of radiation. It varies linearly with the frequency.

(3) The number of photoelectrons emitted per second, or the photoelectric current, is proportional to the intensity of radiation but is independent of the frequency.

(4) The photoelectric emission is an instantaneous process, i.e., there is negligible time lag between the incidence of radiation and the emission of electrons, regardless of how low the intensity of radiation is.

Light consists of discrete packets (quanta) of energy = photons.

…(1.5)

$$E = h\nu = E_k + \Phi$$

$$\nu = \frac{\lambda}{c} \qquad \Phi = h\nu_o \qquad E_k = \frac{1}{2}mv^2$$

Photoelectrons ejected with kinetic energy:

$$E_k = h\nu - \Phi = h\nu - h\nu_o \qquad \ldots(1.6)$$

Each photon has: Energy = Planks constant × Frequency

Energy in Joules: $E = h\nu = 6.626 \times 10^{-34}(J.s) \times \nu(s^{-1})$

Or, $E = \frac{hc}{\lambda} = \frac{1.99\times10^{-25}(J.m)}{\lambda(m)}$

Energy in (eV): $E = h\nu = 4.14 \times 10^{-15}(eV.s) \times \nu(s^{-1})$

Or, $E = \frac{hc}{\lambda} = \frac{1.99\times10^{-25}(J.m)}{\lambda(m)} = \frac{1240(eV.nm)}{\lambda(nm)}$

Example: The threshold (cutoff) frequency is 1.2 x 10^{15}Hz. What is the threshold wavelength? What is the work function of tin?

$$\nu_o = 1.2 \times 10^{15} Hz$$

$$\lambda_o = \frac{c}{\nu_o} = \frac{3 \times 10^8}{1.2 \times 10^{15}} = 2.5 \times 10^{-7} m = 250\ nm$$

$$\Phi = h\nu_o = 6.625 \times 10^{-34} \times 1.2 \times 10^{15} = 7.95 \times 10^{-19} J$$

$$\Phi = \frac{7.95 \times 10^{-19}}{1.6 \times 10^{-19}} = 4.97\ eV$$

Einstein's Theory—Photons

- **Einstein** explained the photoelectric effect using Planck's quantum hypothesis. In order to explain the spectral distribution of **blackbody radiation**,

- **Planck** had assumed that the exchange of energy between the walls of a cavity and the radiation of frequency ν takes place in quanta of magnitude $h\nu$, where h is called Planck's constant. ***Einstein*** *suggested that the incident radiation itself acts like quanta of energy* $(h\nu)$. These quanta later came to be known as photons.
- When a photon collides with an electron in the metal surface, it can be absorbed, imparting all its energy to the electron immediately.
- If the work function of the metal is Φ, then this much energy is expanded to remove the electron from the surface.

Therefore, the maximum kinetic energy E_{max}, and the corresponding velocity v_{max}, of the emitted electron are given by;

$$E_{max} = \frac{1}{2}mv_{max}^2 = h\nu - \Phi \qquad \ldots(1.7)$$

This is called Einstein's photoelectric equation. It shows that E_{max} varies linearly with the frequency ν of the incident radiation.

ν_o is the threshold (cutoff) frequency Thus; $\Phi = h\nu_o$ then

$$E_{max} = h\nu - \Phi = h(\nu - \nu_o) \qquad \ldots(1.8)$$

Clearly, no emission is possible if $\nu \leq \nu_o$.

Stopping or cut-off Potential; For a certain value V_o of this negative potential, the most energetic electrons are just turned back and therefore the photoelectric current becomes zero.

$$eV_0 = E_{\max}$$

$$eV_0 = h(\nu - \nu_0)$$

$$\boxed{V_0 = \frac{h}{e}(\nu - \nu_0)} \qquad \ldots(1.9)$$

Conclusion:

An increase in the intensity of radiation results in an increase in the number of photons striking the metal per second but not in the energy of single photons.

Therefore, the number of photoelectrons emitted per second, and hence the photoelectric current, increases, but not the energy of photoelectrons.

Since the electron emission is the result of a direct collision between an electron and a photon, there is no time delay before emission starts.

PROBLEM: Find the number of photons emitted per second by a 40 W source of monochromatic light of wavelength 6000 Å.

Solution: Let the number of photons be *n*. Then

$$nh\nu = E$$

$$n = \frac{E}{h\nu} = \frac{E\lambda}{hc}$$

$$= \frac{40 \times 6000 \times 10^{-10}}{6.63 \times 10^{-34} \times 3 \times 10^{8}}$$

$$= \boxed{12.06 \times 10^{19}}$$

H.W

1- The work function of a metal is 3.45 eV. What is the maximum wavelength of a photon that can eject an electron from the metal?

2- A metal of work function 3.0 eV is illuminated by light of wavelength 3000 Å. Calculate (a) the threshold frequency, (b) the maximum energy of photoelectrons, and (c) the stopping potential.

3- (a) A stopping potential of 0.82 V is required to stop the emission of photoelectrons from the surface of a metal by light of wavelength 4000 Å. For light of wavelength 3000 Å, the stopping potential is 1.85 V. Find the value of Planck's constant.

(b) At stopping potential, if the wavelength of the incident light is kept fixed at 4000 Å but the intensity of light is increased two times, will photoelectric current be obtained? Give reasons for your answer.

4- Light of wavelength 4560 Å and power $1mW$ is incident on a Caesium surface. Calculate the photoelectric current, assuming a quantum efficiency of 0.5%. Work function of Caesium =1.93 eV.

Multiple Choice Questions (MCQs) - Lecture 1: Origin of Quantum Theory

Q1. Classical mechanics fails to explain phenomena occurring at the:
A) Macroscopic level
B) Atomic regime
C) Planetary scale
D) Gravitational interactions

Q2. The breakdown of classical mechanics was first noticed in which area?
A) Thermodynamics
B) Quantum tunneling
C) Black-body radiation
D) Relativity

Q3. Planck’s hypothesis states that the energy of an oscillator is:
A) Continuous
B) Discrete in multiples of hν
C) Dependent on intensity
D) Randomly distributed

Q4. A photon has a frequency of 6×10^{14} Hz. What is its energy?

A) 3.98×10^{-19} J
B) 6.63×10^{-34} J
C) 1.20×10^{-21} J
D) 2.5×10^{-15} J

Q5. In the photoelectric effect, no electrons are emitted if:
A) Intensity is too low
B) Frequency < threshold frequency
C) Wavelength is very short
D) Metal has no work function

Q6. The photoelectric current is directly proportional to:
A) Frequency of incident light
B) Intensity of incident light
C) Wavelength of incident light

D) Work function of metal

Q7.The work function of sodium is 2.3 eV. What is the threshold wavelength?
A) 540 nm
B) 430 nm
C) 620 nm
D) 300 nm

Q8. According to Einstein's photoelectric equation:
A) $E_k = h\nu - \phi$
B) $E_k = mc^2$
C) $E_k = h\nu + \phi$
D) $E_k = \phi - h\nu$

Q9. The stopping potential is used to measure:
A) Work function
B) Maximum kinetic energy of photoelectrons
C) Intensity of light
D) Frequency of radiation

Q10. Light of wavelength 400 nm is incident on a metal with work function 2.5 eV. What is the maximum kinetic energy of emitted electrons?
A) 0.6 eV
B) 1.1 eV
C) 2.5 eV
D) 3.1 eV

Q11. The Compton effect proved that light behaves as:
A) Pure wave
B) Pure particle
C) Photon with momentum
D) Stationary wave

Q12. Wien's law is accurate in which region of the blackbody spectrum?
A) Long wavelength
B) Short wavelength

C) Entire spectrum
D) UV catastrophe

Q13. If a 40 W monochromatic light source of λ = 600 nm is used, how many photons are emitted per second?
A) 1.2×10^{18}
B) 2.0×10^{19}
C) 5.0×10^{17}
D) 3.3×10^{20}

Q14. Planck's law reduces to Rayleigh-Jeans law in the:
A) Short wavelength limit
B) Long wavelength limit
C) Medium wavelength region
D) Infrared only

Q15. The photoelectric emission is instantaneous because:
A) Light speed is high
B) No time lag between photon absorption and electron emission
C) Intensity is proportional to frequency
D) Electrons are already free

Q16. (Numerical) The threshold frequency for a metal is 1.2×10^{15} Hz. What is the work function in eV?
A) 3.8 eV
B) 4.9 eV
C) 6.2 eV
D) 7.5 eV

Q17. Increasing intensity of incident light increases:
A) Energy of photons
B) Work function
C) Number of photoelectrons
D) Threshold frequency

Q18. Which scientist explained the photoelectric effect using photon theory?
A) Planck
B) Hertz

C) Einstein
D) Compton

Q19. (Numerical) A stopping potential of 0.82 V is observed for light of λ = 4000 Å. What is the maximum kinetic energy of photoelectrons?
A) 0.41 eV
B) 0.82 eV
C) 1.6 eV
D) 2.0 eV

Q20. In Planck’s theory, if h $\rightarrow$ 0:
A) Relativity is restored
B) Classical mechanics is regained
C) Quantum effects increase
D) Light stops behaving as a wave