



جامعة المستقبل  
AL MUSTAQBAL UNIVERSITY

كلية العلوم  
قسم الانظمة الطبية الذكية

Lecture (1)

# FUNCTIONS AND THEIR GRAPHS

المادة : رياضيات

المرحلة : الاولى

اسم الاستاذ: م.م ريام ثائر احمد



- The General Aim
- The Behavioral objectives
- What is Function?
- The Domain and Range
- Graphs of Function



## **The General Aim**



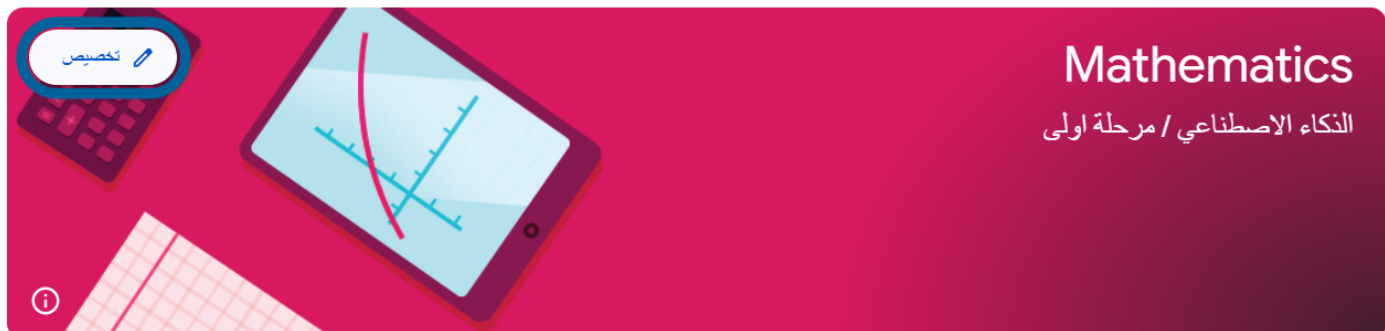
The general aim of a function is to describe the relationship between two quantities, where one depends on the other. It allows us to analyze, predict, and model real-life and mathematical situations accurately.

### **The Behavioral objectives**

By the end of the lecture, the student will be able to:

- ✓ Define a function and identify its domain and range correctly.
- ✓ Distinguish between relations and functions using tables, formulas, and graphs.
- ✓ Evaluate a function for given values of the independent variable.
- ✓ Plot the graph of a given function accurately on the Cartesian plane.
- ✓ Interpret the graph of a function to describe relationships between variables.
- ✓ Determine whether a graph represents a function using the vertical line test.





**juphblg7**

<https://classroom.google.com/c/ODM3Njc4MDI3NjYx?cjc=juphblg7>

## **I. What is a Function?**

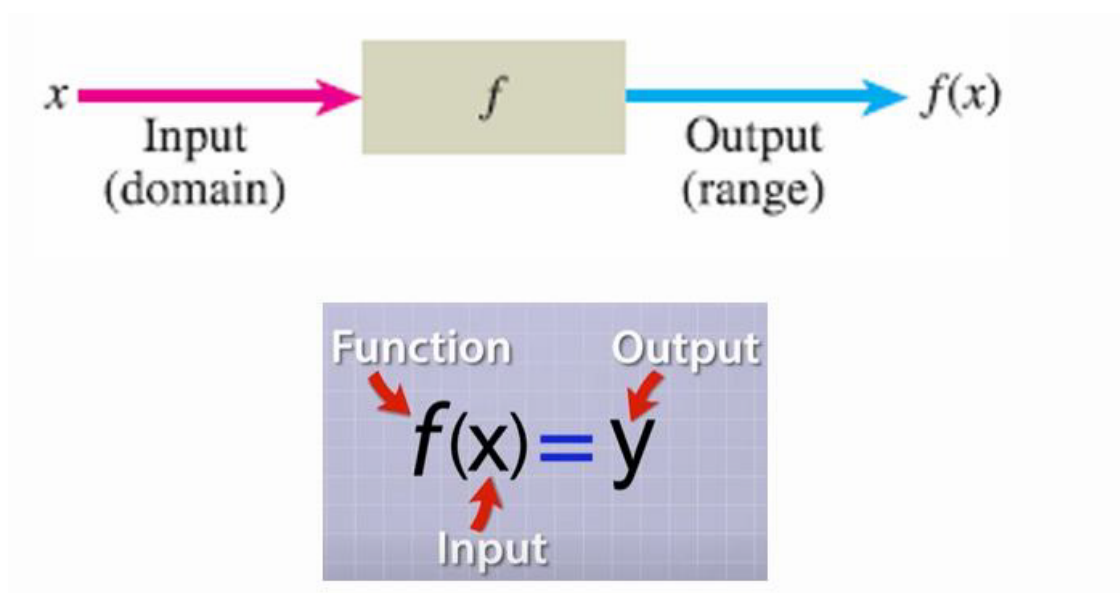




- Functions are a tool for describing the real world in mathematical terms.
- A function can be represented by an equation, a graph, a numerical table, or a verbal description.

A function takes an **input** (usually called  $x$ ) and produces an **output** (usually called  $f(x)$ ).

We write it as:



## II. The Domain and Range

A function  $f$  consists of a set of **inputs**, a set of **outputs**, and a rule for assigning each input to exactly one output. The set of inputs is called **Domain** of the function. The set of output is called the **Range** of the function.



## Example

For the function  $f(x) = 3x^2 + 2x - 1$ , evaluate

- a.  $f(-2)$
- b.  $f(\sqrt{2})$
- c.  $f(a + h)$

## Solution

Substitute the given value for  $x$  in the formula for  $f(x)$ .

- a.  $f(-2) = 3(-2)^2 + 2(-2) - 1 = 12 - 4 - 1 = 7$
- b.  $f(\sqrt{2}) = 3(\sqrt{2})^2 + 2\sqrt{2} - 1 = 6 + 2\sqrt{2} - 1 = 5 + 2\sqrt{2}$
- c.  $f(a + h) = 3(a + h)^2 + 2(a + h) - 1 = 3(a^2 + 2ah + h^2) + 2a + 2h - 1$   
 $= 3a^2 + 6ah + 3h^2 + 2a + 2h - 1$

**Example:** If  $f(x) = 2x^2 - 5x + 1$  and  $h \neq 0$ , evaluate  $\frac{f(a + h) - f(a)}{h}$

**Solution:** We first evaluate  $f(a + h)$  by replacing  $x$  by  $a + h$  in the expression for  $f(x)$ :

$$\begin{aligned} f(a + h) &= 2(a + h)^2 - 5(a + h) + 1 \\ &= 2(a^2 + 2ah + h^2) - 5(a + h) + 1 \\ &= 2a^2 + 4ah + 2h^2 - 5a - 5h + 1 \end{aligned}$$



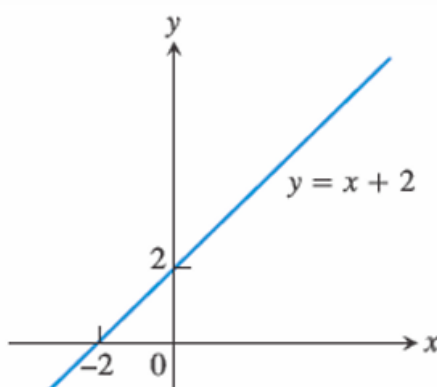
Then we substitute into the given expression and simplify:

$$\begin{aligned}\frac{f(a+h) - f(a)}{h} &= \frac{(2a^2 + 4ah + 2h^2 - 5a - 5h + 1) - (2a^2 - 5a + 1)}{h} \\ &= \frac{2a^2 + 4ah + 2h^2 - 5a - 5h + 1 - 2a^2 + 5a - 1}{h} \\ &= \frac{4ah + 2h^2 - 5h}{h} = 4a + 2h - 5\end{aligned}$$

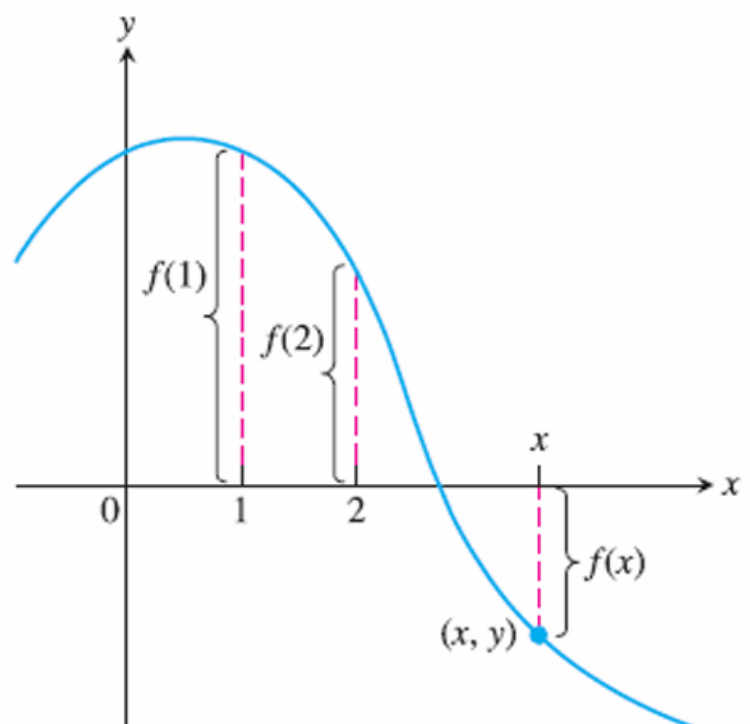
### III. Graphs of Function

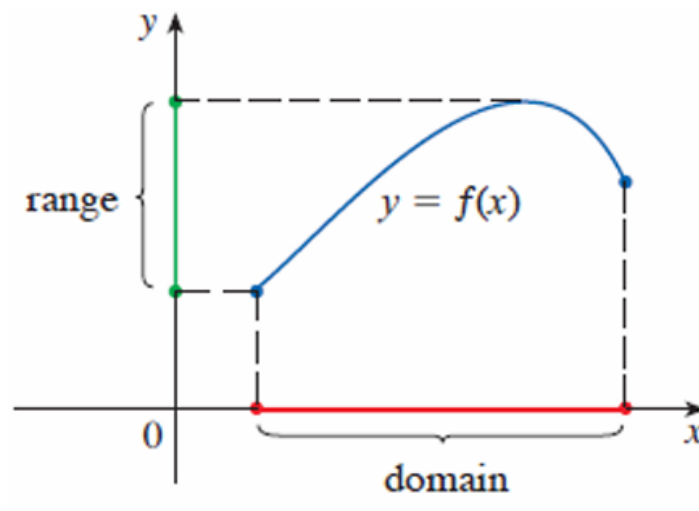
If  $f$  is a function with domain  $D$ , its graph consists of the points in the Cartesian plane whose coordinates are the input-output pairs for  $f$ . In set notation, the graph is:

$$\{(x, f(x)) \mid x \in D\}$$



The graph of  $f(x) = x + 2$  is the set of points  $(x, y)$  for which  $y$  has the value  $x + 2$ .

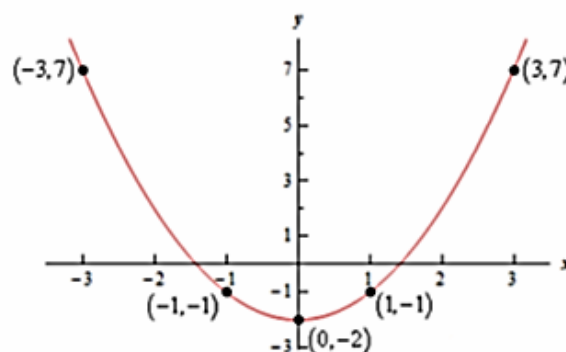




**Example 1:** Sketch the graph of the following function.

$$f(x) = x^2 - 2$$

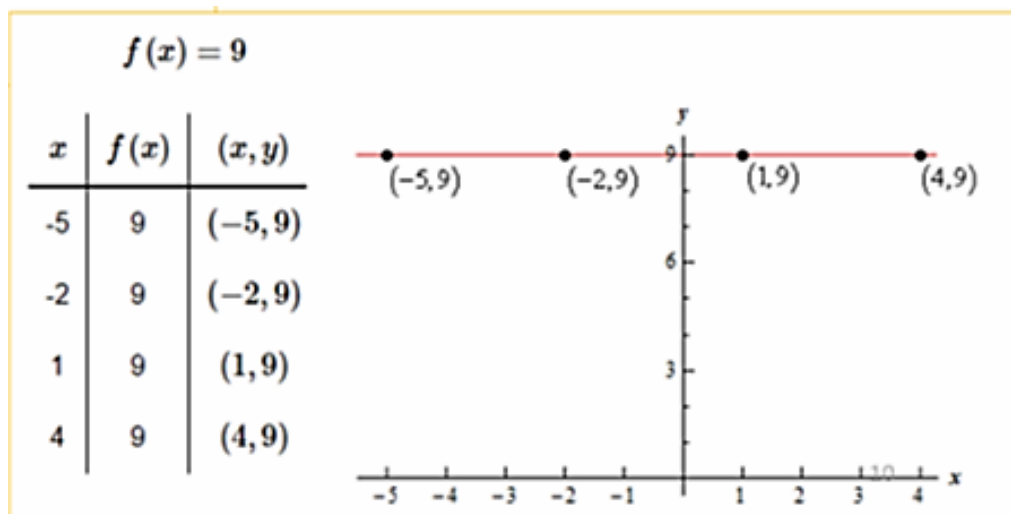
$x$	$f(x)$	$(x, y)$
-3	7	$(-3, 7)$
-1	-1	$(-1, -1)$
0	-2	$(0, -2)$
1	-1	$(1, -1)$
3	7	$(3, 7)$



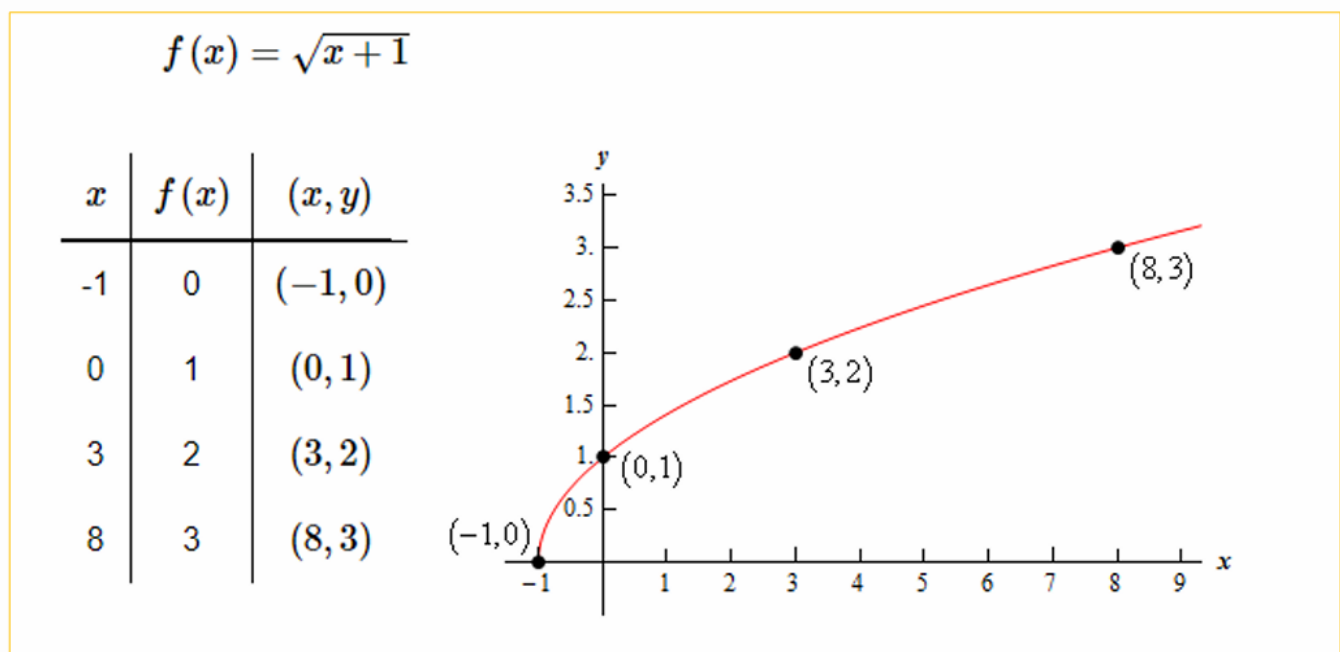
Domain is  $(-\infty, \infty)$  and  
Range is  $[-2, \infty)$



**Example 2:** Sketch the graph of the function:



**Example 3:** graph of the following function.

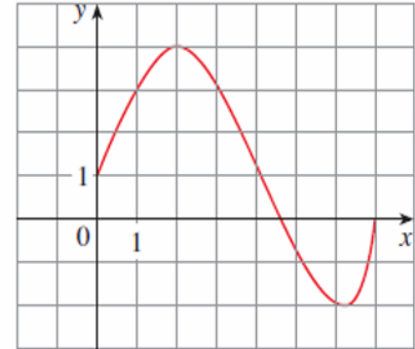


Domain is  $[-1, \infty)$  and  
Range is  $[0, \infty)$



**EXAMPLE** // The graph of a function  $f$  is shown in Figure shown.

- (a) Find the values of  $f(1)$  and  $f(5)$  .  
 (b) What are the domain and range of  $f$ ?



### SOLUTION

- (a) We see from Figure the value of  $f$  at 1 is  $f(1) = 3$ .  
 ( the point on the graph that lies above  $x = 1$  is 3 units above the  $x$ -axis.)

When  $x = 5$ , the graph lies about 0.7 units below the  $x$ -axis,  
 so we estimate that  $f(5) \approx -0.7$ .

- (b) We see that  $f(x)$  is defined when  $0 \leq x \leq 7$ , so the domain of  $f$  is the closed interval  $[0, 7]$ . Notice that  $f$  takes on all values from  $-2$  to  $4$ , so the range of  $f$  is

$$\{y \mid -2 \leq y \leq 4\} = [-2, 4]$$

**Example:** Find the Domains and Ranges of each of all of the following

(a)  $y = x^3 \quad -5 \leq x < 4$       (b)  $y = x^4$       (c)  $y = \frac{1}{(x-1)(x+2)} \quad 0 \leq x \leq 6$

### Solution

(a)  $y = x^3 \quad -5 \leq x < 4$   
 domain  $-5 \leq x < 4$ , range  $-125 \leq y < 64$

(b)  $y = x^4$   
 domain  $-\infty < x < \infty$ , range  $0 \leq y < \infty$

(c)  $y = \frac{1}{(x-1)(x+2)}, \quad 0 \leq x \leq 6$   
 domain  $0 \leq x < 1$  and  $1 < x \leq 6$ ,  
 range  $-\infty < y \leq -0.5, 0.25 \leq y < \infty$

$$\begin{aligned} (x-1)(x+2) &\geq 0 \\ x^2 + x - 2 &\geq 0 \end{aligned}$$

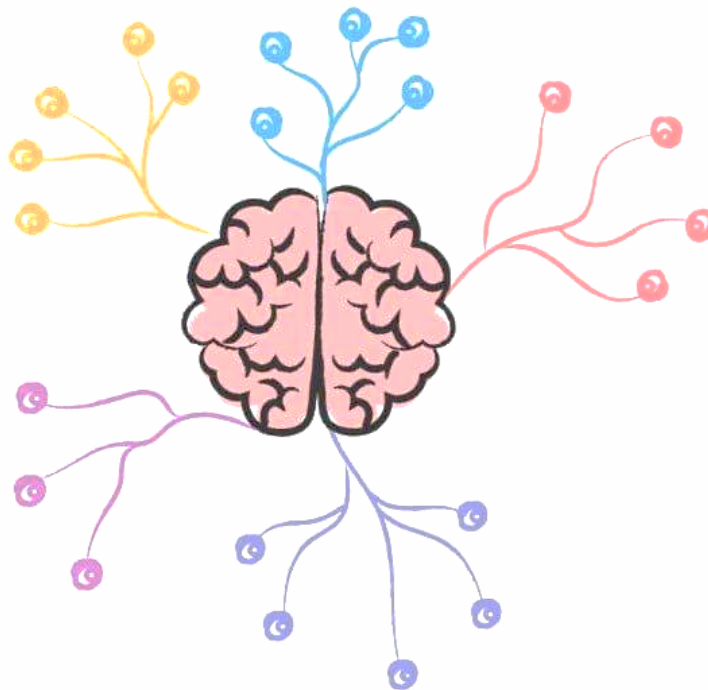


## **TASK:**

- Sketch the following functions:

1.

2.



**Note: The Answer must be sent to the Google Classroom**



[reyam.thair.ahmed@uomus.edu.iq](mailto:reyam.thair.ahmed@uomus.edu.iq)