



جامعة المستقبل
AL MUSTAQL UNIVERSITY

كلية العلوم
قسم الانظمة الطبية الذكية

Lecture (1)

FUNCTIONS AND THEIR GRAPHS

المادة : رياضيات

المرحلة : الاولى

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- The General Aim
- The Behavioral objectives
- What is Function?
- The Domain and Range
- Graphs of Function



The General Aim



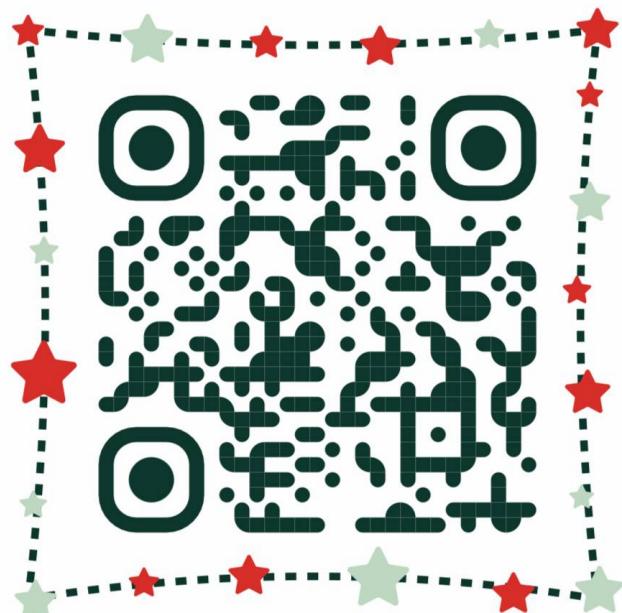
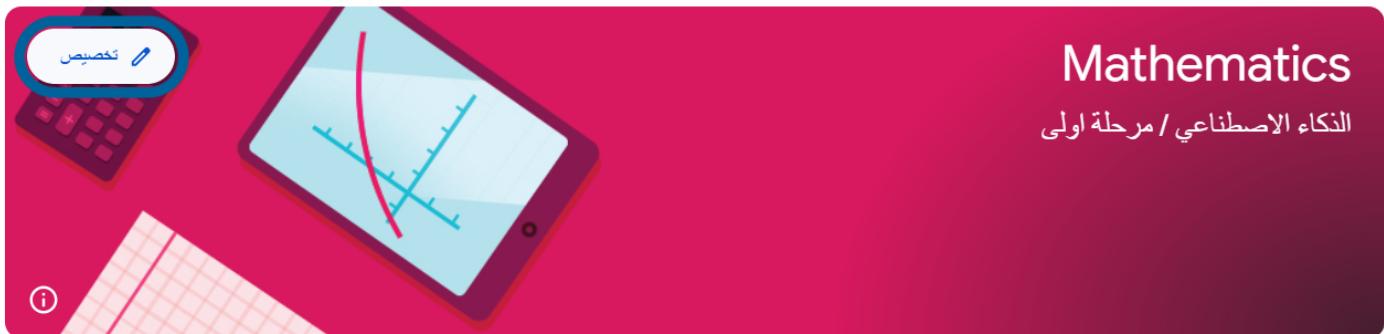
The general aim of a function is to describe the relationship between two quantities, where one depends on the other. It allows us to analyze, predict, and model real-life and mathematical situations accurately.

The Behavioral objectives

By the end of the lecture, the student will be able to:

- ✓ Define a function and identify its domain and range correctly.
- ✓ Distinguish between relations and functions using tables, formulas, and graphs.
- ✓ Evaluate a function for given values of the independent variable.
- ✓ Plot the graph of a given function accurately on the Cartesian plane.
- ✓ Interpret the graph of a function to describe relationships between variables.
- ✓ Determine whether a graph represents a function using the vertical line test.





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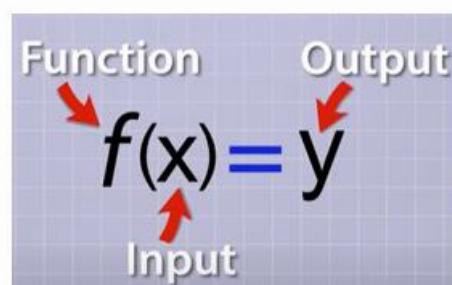
I. What is a Function?



- Functions are a tool for describing the real world in mathematical terms.
- A function can be represented by an equation, a graph, a numerical table, or a verbal description.

A function takes an **input** (usually called x) and produces an **output** (usually called $f(x)$).

We write it as:



II. The Domain and Range

A function f consists of a set of **inputs**, a set of **outputs**, and a rule for assigning each input to exactly one output. The set of inputs is called **Domain** of the function. The set of output is called the **Range** of the function.



Example

For the function $f(x) = 3x^2 + 2x - 1$, evaluate

- $f(-2)$
- $f(\sqrt{2})$
- $f(a + h)$

Solution

Substitute the given value for x in the formula for $f(x)$.

- $f(-2) = 3(-2)^2 + 2(-2) - 1 = 12 - 4 - 1 = 7$
- $f(\sqrt{2}) = 3(\sqrt{2})^2 + 2\sqrt{2} - 1 = 6 + 2\sqrt{2} - 1 = 5 + 2\sqrt{2}$
- $$\begin{aligned}f(a + h) &= 3(a + h)^2 + 2(a + h) - 1 = 3(a^2 + 2ah + h^2) + 2a + 2h - 1 \\&= 3a^2 + 6ah + 3h^2 + 2a + 2h - 1\end{aligned}$$

Example: If $f(x) = 2x^2 - 5x + 1$ and $h \neq 0$, evaluate $\frac{f(a + h) - f(a)}{h}$

Solution: We first evaluate $f(a + h)$ by replacing x by $a + h$ in the expression for $f(x)$:

$$\begin{aligned}f(a + h) &= 2(a + h)^2 - 5(a + h) + 1 \\&= 2(a^2 + 2ah + h^2) - 5(a + h) + 1 \\&= 2a^2 + 4ah + 2h^2 - 5a - 5h + 1\end{aligned}$$



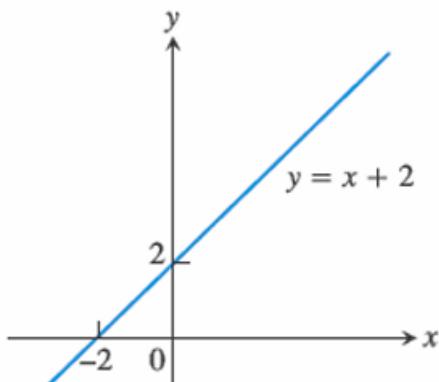
Then we substitute into the given expression and simplify:

$$\begin{aligned}\frac{f(a+h) - f(a)}{h} &= \frac{(2a^2 + 4ah + 2h^2 - 5a - 5h + 1) - (2a^2 - 5a + 1)}{h} \\ &= \frac{2a^2 + 4ah + 2h^2 - 5a - 5h + 1 - 2a^2 + 5a - 1}{h} \\ &= \frac{4ah + 2h^2 - 5h}{h} = 4a + 2h - 5\end{aligned}$$

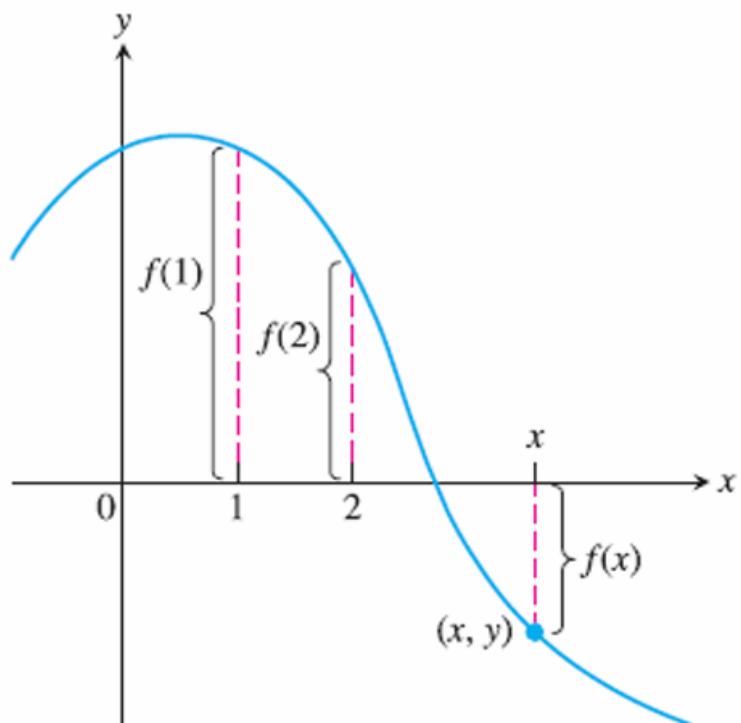
III. Graphs of Function

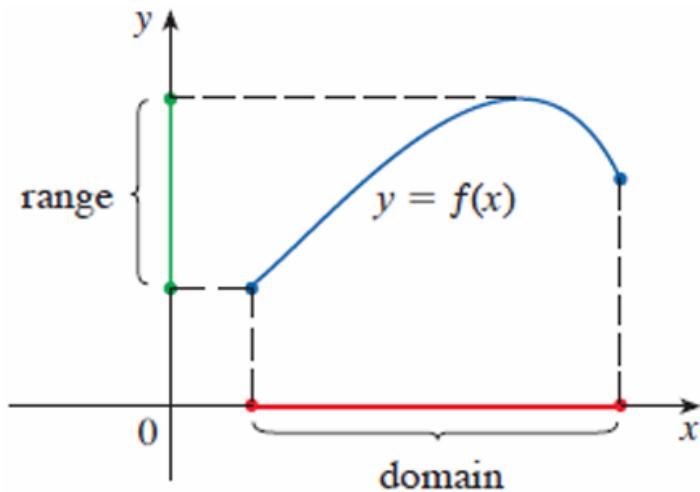
If f is a function with domain D , its graph consists of the points in the Cartesian plane whose coordinates are the input-output pairs for f . In set notation, the graph is:

$$\{(x, f(x)) \mid x \in D\}$$



The graph of $f(x) = x + 2$ is the set of points (x, y) for which y has the value $x + 2$.

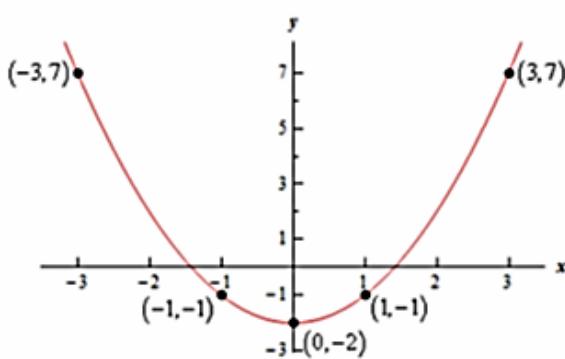




Example 1: Sketch the graph of the following function.

$$f(x) = x^2 - 2$$

x	$f(x)$	(x, y)
-3	7	(-3, 7)
-1	-1	(-1, -1)
0	-2	(0, -2)
1	-1	(1, -1)
3	7	(3, 7)



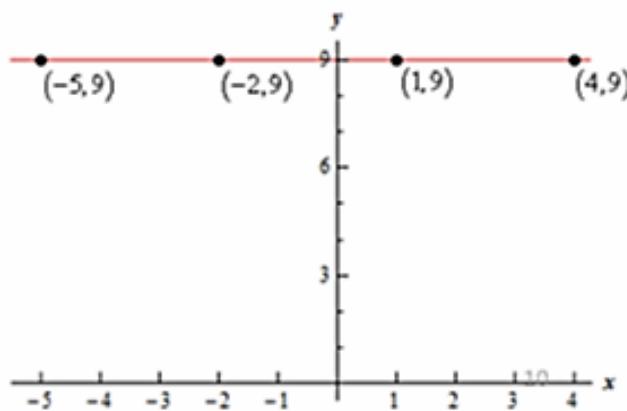
Domain is $(-\infty, \infty)$ and
Range is $[-2, \infty)$



Example 2: Sketch the graph of the function:

$$f(x) = 9$$

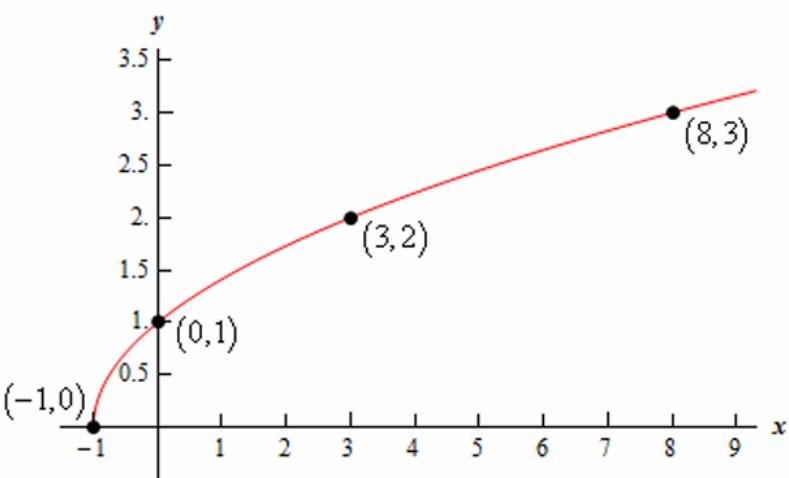
x	$f(x)$	(x, y)
-5	9	(-5, 9)
-2	9	(-2, 9)
1	9	(1, 9)
4	9	(4, 9)



Example 3: graph of the following function.

$$f(x) = \sqrt{x + 1}$$

x	$f(x)$	(x, y)
-1	0	(-1, 0)
0	1	(0, 1)
3	2	(3, 2)
8	3	(8, 3)

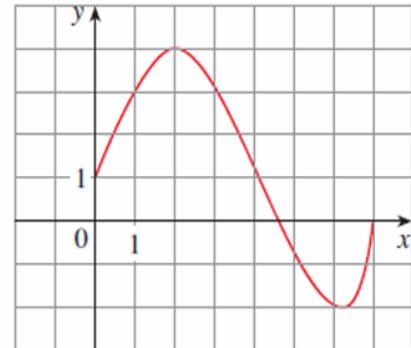


Domain is $[-1, \infty)$ and
Range is $[0, \infty)$



EXAMPLE // The graph of a function f is shown in Figure shown.

- Find the values of $f(1)$ and $f(5)$.
- What are the domain and range of f ?



SOLUTION

(a) We see from Figure the value of f at 1 is $f(1) = 3$.
(the point on the graph that lies above $x = 1$ is 3 units above the x -axis.)
When $x = 5$, the graph lies about 0.7 units below the x -axis,
so we estimate that $f(5) \approx -0.7$.

(b) We see that $f(x)$ is defined when $0 \leq x \leq 7$, so the domain of f is the closed interval $[0, 7]$. Notice that f takes on all values from -2 to 4 , so the range of f is

$$\{y \mid -2 \leq y \leq 4\} = [-2, 4]$$

Example: Find the Domains and Ranges of each of all of the following

(a) $y = x^3 \quad -5 \leq x < 4$ (b) $y = x^4$ (c) $y = \frac{1}{(x-1)(x+2)} \quad 0 \leq x \leq 6$

Solution

(a) $y = x^3 \quad -5 \leq x < 4$
domain $-5 \leq x < 4$, range $-125 \leq y < 64$

(b) $y = x^4$
domain $-\infty < x < \infty$, range $0 \leq y < \infty$

(c) $y = \frac{1}{(x-1)(x+2)}, \quad 0 \leq x \leq 6$

domain $0 \leq x < 1$ and $1 < x \leq 6$,
range $-\infty < y \leq -0.5, 0.25 \leq y < \infty$

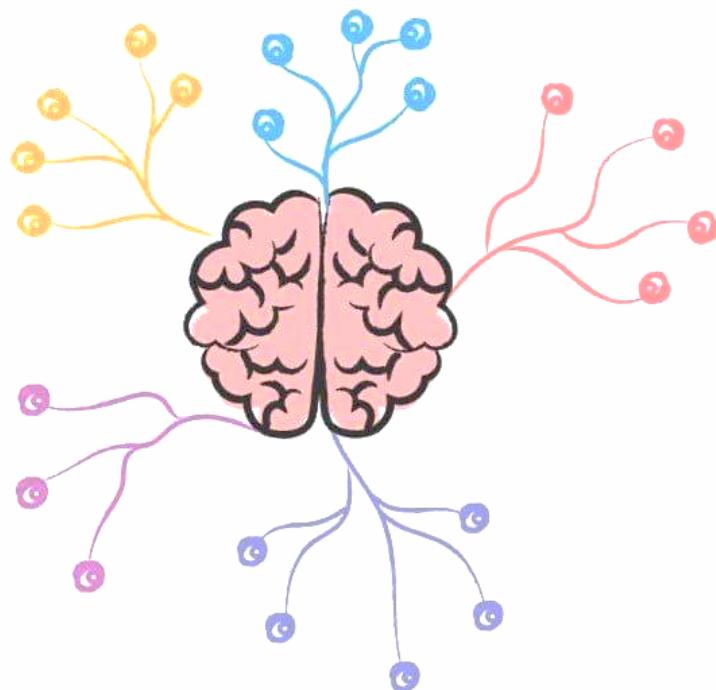
$$\begin{aligned} (x-1)(x+2) &\geq 0 \\ x^2 + x - 2 &\geq 0 \end{aligned}$$



TASK:

- Sketch the following functions:

1.
2.



Note: The Answer must be sent to the Google Classroom



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