Al-Mustaqbal University
College of Science
Intelligent medical systems Department
First Stage

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Lecture (5)

MATRICES AND DETERMINANTS

المادة : رياضيات

المرحلة : الاولى

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Matrices and Determinants

A matrix is a rectangular array of elements (scalars) from a field. The order, or size, of a matrix is specified by the number of rows and the number of columns, i.e. A an "m by n" matrix has m rows and n columns, and the element in the i *th* row and j *th* column is often denoted by a_{ij} :

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & .... & a_{1n} \\ a_{21} & a_{22} & .... & a_{2n} \\ .... & .... & .... & .... \\ .... & .... & .... & .... \\ a_{m1} & a_{m2} & .... & a_{mn} \end{bmatrix}$$

A vector is a matrix with a single row (or column) of n elements , i.e. the column vector is:-

$$A = \begin{bmatrix} a_1 \\ a_2 \\ . \\ . \\ a_n \end{bmatrix} \quad \textit{and row vector is} \quad A = [a_1 \quad a_2 \quad . \quad . \quad a_n]$$

The matrix is square if the number of rows and columns are equal (i.e. $m = n$) and the elements a_{ij} of a square matrix are called the main diagonal.

The identity matrix: $I = \begin{bmatrix} 1 & 0 & .... & 0 \\ 0 & 1 & .... & 0 \\ .... & .... & .... & .... \\ .... & .... & .... & .... \\ 0 & 0 & .... & 1 \end{bmatrix}$ is square matrix

with one in each main diagonal position and zeros else.

The diagonal matrix $D = \begin{bmatrix} a_1 & 0 & \cdots & 0 \\ 0 & a_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & a_n \end{bmatrix}$ has the elements $a_1, a_2, \ldots, a_n$ in its main diagonal position and zeros in all other locations, some of the a_i may be zero but not all.

A $n \times n$ triangular matrix has the pattern:-

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ 0 & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & a_{nn} \end{bmatrix} \quad or \quad \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

lower triangular matrix *upper triangular matrix*

The $m \times n$ null matrix:- $0 = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$ has zero in each of its positions.

Elementary operations with matrices and vectors

1. **Equality:-** Two $m \times n$ matrices and A and B are said to be equal if: $a_{ij} = b_{ij} \quad \forall$ *pairs of i and j.*

EX-1 – Find the values of x , y for the following matrix equation:

$$\begin{bmatrix} x - 2y & 0 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ -2 & x + y \end{bmatrix}$$

Sol. –

$$\left.\begin{array}{ll} x-2y=3 & \ldots.(1) \\ x+y=6 & \ldots.(2)*2 \end{array}\right\} \Rightarrow \begin{array}{c} x-2y=3 \\ \underline{2x+2y=12} \\ 3x=15 \Rightarrow \boxed{x=5} \end{array}$$

$$substitution\ \ x=5\ \ in(2) \Rightarrow 5+y=6 \Rightarrow \boxed{y=1}$$

2. <u>Addition:-</u> The sum of two matrices of like dimensions is the matrix of the sum of the corresponding elements. If:-

$$A=\begin{bmatrix} a_{11} & a_{12} & \ldots. & a_{1n} \\ a_{21} & a_{22} & \ldots. & a_{2n} \\ \ldots. & \ldots. & \ldots. & \ldots. \\ a_{m1} & a_{m2} & \ldots. & a_{mn} \end{bmatrix} \quad , \quad B=\begin{bmatrix} b_{11} & b_{12} & \ldots. & b_{1n} \\ b_{21} & b_{22} & \ldots. & b_{2n} \\ \ldots. & \ldots. & \ldots. & \ldots. \\ b_{m1} & b_{m2} & \ldots. & b_{mn} \end{bmatrix}$$

then

$$A\mp B=\begin{bmatrix} a_{11}\mp b_{11} & a_{12}\mp b_{12} & \ldots. & a_{1n}\mp b_{1n} \\ a_{21}\mp b_{21} & a_{22}\mp b_{22} & \ldots. & a_{2n}\mp b_{2n} \\ \ldots. & \ldots. & \ldots. & \ldots. \\ a_{m1}\mp b_{m1} & a_{m2}\mp b_{m2} & \ldots. & a_{mn}\mp b_{mn} \end{bmatrix}$$

thus:

1) $A+B = B+A$
2) $A+(B+C) = (A+B)+C$
3) $A-(B-C) = A-B+C$

***<u>EX-2</u>*- Find $A+B$ and $A-B$ if:-**

$$A=\begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & -2 \end{bmatrix} \quad , \quad B=\begin{bmatrix} 1 & -2 & 2 \\ 2 & 3 & -1 \end{bmatrix}$$

<u>Sol.</u>-

$$A+B=\begin{bmatrix} 2+1 & 1-2 & 3+2 \\ 1+2 & 0+3 & -2-1 \end{bmatrix}=\begin{bmatrix} 3 & -1 & 5 \\ 3 & 3 & -3 \end{bmatrix}$$

$$A-B=\begin{bmatrix} 2-1 & 1-(-2) & 3-2 \\ 1-2 & 0-(+3) & -2-(-1) \end{bmatrix}=\begin{bmatrix} 1 & 3 & 1 \\ -1 & -3 & -1 \end{bmatrix}$$

3. **Multiplication by a scalar:-** The matrix A is multiplied by the scalar C by multiplying each element of A by c:-

$$CA = \begin{bmatrix} ca_{11} & ca_{12} & \cdots & ca_{1n} \\ ca_{21} & ca_{22} & \cdots & ca_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ ca_{m1} & ca_{m2} & \cdots & ca_{mn} \end{bmatrix}$$

EX-3- Assume $A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 5 & -1 \end{bmatrix}$, find $3A$.

Sol.-

$$3A = \begin{bmatrix} 3*3 & 3*2 & 3*1 \\ 3*0 & 3*5 & 3*(-1) \end{bmatrix} = \begin{bmatrix} 9 & 6 & 3 \\ 0 & 15 & -3 \end{bmatrix}$$

4. **Matrix multiplication:-** For the matrix product AB to be defined it is necessary that the number of columns of A be equal to the number of rows of B. The dimensions of such matrices are said to be conformable. If A is of dimensions $m \times p$ and B is $p \times n$, then the $ij\ th$ element of the product $C=AB$ is computed as:-

$$C_{ij} = \sum_{k=1}^{p} a_{ik} b_{kj}$$

This is the sum of the products of corresponding elements in the $i\ th$ row of A and $j\ th$ column of B. The dimensions of AB are of course $m \times n$.

EX-4- Assume $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 5 & 4 \\ -1 & 1 & -1 \\ 0 & 2 & 0 \end{bmatrix}$ find AB.

Sol.-

$$AB = \begin{bmatrix} 1*6+2(-1)+3*0 & 1*5+2*1+3*2 & 1*4+2(-1)+3*0 \\ -1*6+0(-1)+1*0 & -1*5+0*1+1*2 & -1*4+0(-1)+1*0 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 13 & 2 \\ -6 & -3 & -4 \end{bmatrix}$$

Properties of multiplication:-

a) $A(B+C) = AB + AC$ *distributive law*

b) $A(BC) = (AB)C$ *associative law*

c) $AB \neq BA$ *commutative law does not hold*

d) $AI = IA = A$

EX-5- Assume $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ *and* $B = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$, verify that $AB \neq BA$.

Sol.-

$$AB = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 6 & 3 \end{bmatrix} \quad \& \quad BA = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 2 & 7 \end{bmatrix}$$

Hence $AB \neq BA$

5. Transpose of matrix:- Let A is any $m \times n$ matrix the transpose of A is $n \times m$ matrix A' formed by interchanging the role of rows and columns.

$$A' = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}' = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \cdots & \cdots & \cdots & \cdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{bmatrix}$$

If a matrix is square and equal to its transpose, it is said to be symmetric, then $a_{ij} = a_{ji}$ for all pairs of i and j.

Properties of transpose are:-

a) $(A \mp B)' = A' \mp B'$

b) $(AB)' = B'A'$

c) $(A')' = A$

EX-6- Assume $A = \begin{bmatrix} 3 & 2 & 5 \\ 2 & -1 & 4 \\ 5 & 4 & 0 \end{bmatrix}$ *and* $B = \begin{bmatrix} 4 & -1 & 0 \\ 5 & 4 & 3 \\ 2 & 1 & -1 \end{bmatrix}$, show that:-

1) A *is symmetric matrix*   2) $(A+B)' = A' + B'$

3) $(AB)' = B'A'$

Sol.-

$$1)\; A' = \begin{bmatrix} 3 & 2 & 5 \\ 2 & -1 & 4 \\ 5 & 4 & 0 \end{bmatrix}' = \begin{bmatrix} 3 & 2 & 5 \\ 2 & -1 & 4 \\ 5 & 4 & 0 \end{bmatrix} = A \Rightarrow A \text{ is a symmetric matrix.}$$

$$2)\; L.H.S. = (A+B)' = \begin{bmatrix} 7 & 1 & 5 \\ 7 & 3 & 7 \\ 7 & 5 & -1 \end{bmatrix}' = \begin{bmatrix} 7 & 7 & 7 \\ 1 & 3 & 5 \\ 5 & 7 & -1 \end{bmatrix}$$

$$R.H.S. = A' + B' = \begin{bmatrix} 3 & 2 & 5 \\ 2 & -1 & 4 \\ 5 & 4 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 5 & 2 \\ -1 & 4 & 1 \\ 0 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 7 & 7 & 7 \\ 1 & 3 & 5 \\ 5 & 7 & -1 \end{bmatrix} = L.H.S.$$

$$\therefore (A+B)' = A' + B'$$

$$3)\; L.H.S. = (AB)' = \begin{bmatrix} 32 & 10 & 1 \\ 11 & -2 & -7 \\ 40 & 11 & 12 \end{bmatrix}' = \begin{bmatrix} 32 & 11 & 40 \\ 10 & -2 & 11 \\ 1 & -7 & 12 \end{bmatrix}$$

$$R.H.S. = B'A' = \begin{bmatrix} 4 & 5 & 2 \\ -1 & 4 & 1 \\ 0 & 3 & -1 \end{bmatrix}\begin{bmatrix} 3 & 2 & 5 \\ 2 & -1 & 4 \\ 5 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 32 & 11 & 40 \\ 10 & -2 & 11 \\ 1 & -7 & 12 \end{bmatrix} = L.H.S.$$

$$\therefore (AB)' = B'A'$$