



Department of biology

((Biophysics))

1Stage

LEC 5

Scalar Quantities and vector

By

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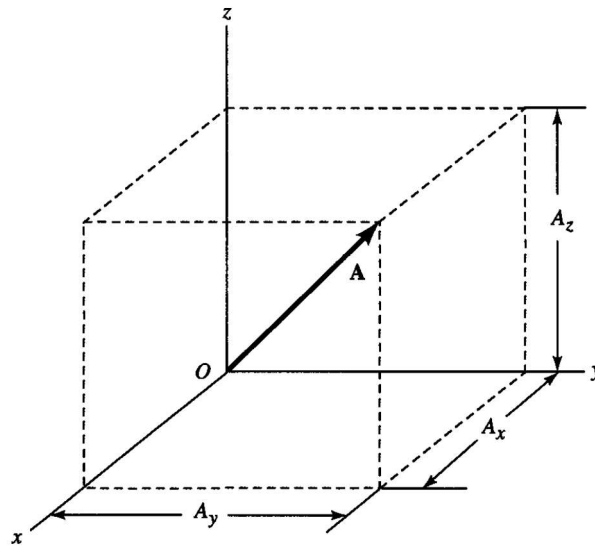
1.1 Physical Quantities

The motion of dynamical systems is typically described in terms of two basic quantities: scalars and vectors.

1. **A Scalar Quantities** is a physical quantity that has magnitude only, such as the mass of an object.

2. **Vector Quantitie** has both magnitude and direction, such as the displacement, velocity, acceleration, and force.

The scalar quantity is represented by the symbol (A), we denote vector quantities simply by $\vec{A}$, a given vector $\vec{A}$, is specified by stating its magnitude and its direction relative to some arbitrarily chosen coordinate system. It is represented diagrammatically as a directed line segment, as shown in three-dimensional space in Figure 1.



The Vector $\vec{A} = iA_x + jA_y + kA_z$ means that there vector (A) is expressed on the right in terms of its components in a particular Coordinate system.

1.2 The Definitions and Rules

1. Unit Vector

The unit vector ($\hat{u}_A$) in the direction of the vector (A) is defined as follows:

$$\hat{u}_A = \frac{\vec{A}}{|\vec{A}|} \quad (1-1)$$

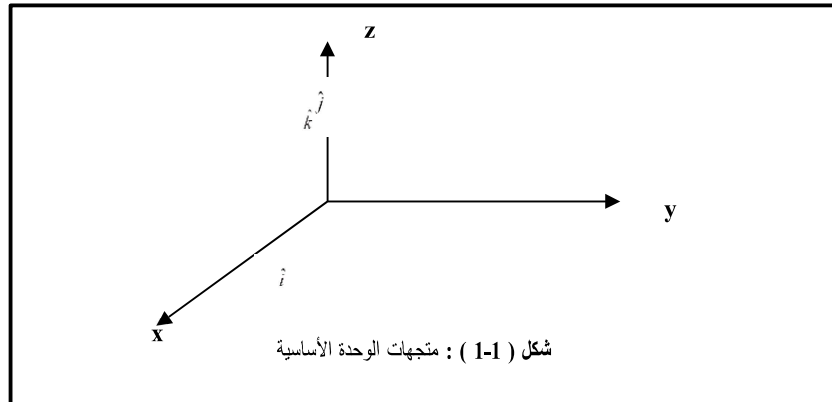
$\hat{u}_A$: The unit vector is in the direction of vector (A)

A : The vector

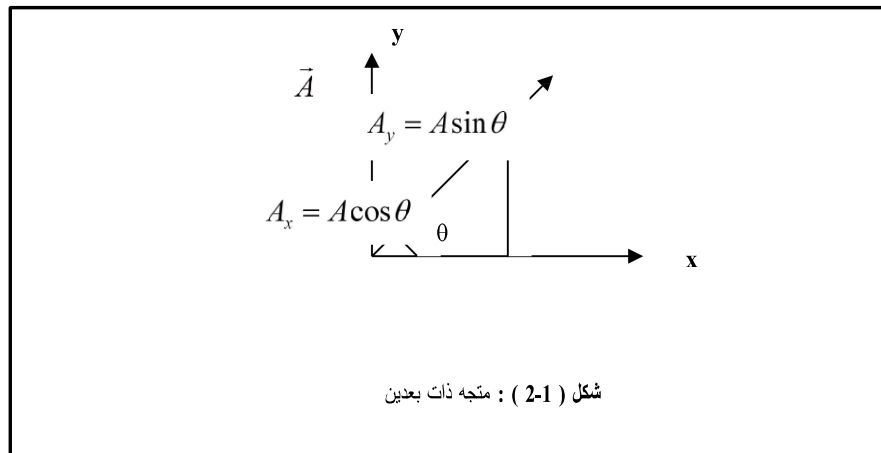
$|\vec{A}|$: The magnitude of the vector

2. (Basic Unit Vectors) $\hat{i}, \hat{j}, \hat{k}$

They are vectors of unit magnitude and work in the positive directions of the axes (x, y, z), respectively, as in Figure (1-1). Therefore, these three vectors are perpendicular.



To find the magnitude of the vector in the case of a two-dimensional vector, as in Figure (1-2)



From Figure (2-1) it is clear to us that:

$$|\vec{A}| = \vec{A} = \sqrt{A_x^2 + A_y^2} \dots (2-1)$$

$$A_x = A \cos \theta$$

$$A_y = A \sin \theta$$

θ = It is the angle that the resultant makes with the positive (x) axis, and is calculated from the following equation:

$$\theta = \tan^{-1} \frac{A_y}{A_x} \dots (3-1)$$

Now this can be generalized to a vector in space (with three dimensions) as follows:

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2} \dots (4-1)$$

Example: If $\vec{A} = 3\hat{i} + 4\hat{j}$

1- Calculate the magnitude of the vector (A)?

2- What is the unit vector in the direction (A)

Solution

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2} \dots (2-1)$$

$$A_x = 3$$

$$A_y = 4$$

$$|\vec{A}| = \sqrt{(3)^2 + (4)^2}$$

$$|\vec{A}| = \sqrt{9 + 16}$$

$$\text{units} |\vec{A}| = 5 \quad (A) \text{ Vector magnitude}$$



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$$\hat{u}_A = \frac{A \dots (1-1)}{|A|}$$

$$\hat{u}_A = \frac{1}{5}(3\hat{i} + 4\hat{j})$$

$$\hat{u}_A = \frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}$$

$$\boxed{\hat{u}_A = 0.6\hat{i} + 0.8\hat{j}} \text{ (} A \text{) Unit vector in one direction}$$

3. Addition and Subtraction of Vectors

$$\boxed{A = A_x\hat{i} + A_y\hat{j} \dots (5-1a)}$$

with three dimensions, the vector (A) can be written in the following form:

$$\boxed{A = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \dots (5-1b)}$$

For vector ($\vec{B}$)

$$\boxed{B = B_x\hat{i} + B_y\hat{j} + B_z\hat{k} \dots (6-1)}$$

Therefore, from equations (1-5 b) and (1-6), the equation for summing the vectors (A) and (B) can be written as follows:

$$\boxed{A + B = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k} \dots (7-1)}$$

The equation for subtracting the vectors (A) and ($\vec{B}$) is as follows:

$$\boxed{A - B = A + (-B) = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j} + (A_z - B_z)\hat{k} \dots (8-1)}$$

4. Equality of Vectors

$$\vec{A} = \vec{B}$$

$$A_x = B_x$$

$$A_y = B_y$$

$$A_z = B_z$$

5. The null vector

The vector $0 = (0, 0, 0)$ is called the null vector. The direction of the null vector is undefined. From (4) it follows that $A - A = 0$. Because there can be no confusion when the null vector is denoted by a zero, we shall hereafter use the notation $0=0$.

6. Multiplication of Vectors

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta \dots (9-1)$$

$|\vec{A}|$: Vector magnitude. $\vec{A}$

$|\vec{B}|$: Vector magnitude. $\vec{B}$

θ : The smallest angle between the two vectors $\vec{A}$ and $\vec{B}$ their extension is

$$\theta = \cos^{-1} \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} \dots (10-1)$$

$$\vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$\vec{A} \cdot \vec{B} = A_x B_x \hat{i} \cdot \hat{i} + A_x B_y \hat{i} \cdot \hat{j} + A_x B_z \hat{i} \cdot \hat{k} + A_y B_x \hat{j} \cdot \hat{i} + A_y B_y \hat{j} \cdot \hat{j} + A_y B_z \hat{j} \cdot \hat{k} + A_z B_x \hat{k} \cdot \hat{i} + A_z B_y \hat{k} \cdot \hat{j} + A_z B_z \hat{k} \cdot \hat{k} \dots (11-1)$$

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$\hat{i} \cdot \hat{j} = \hat{i} \cdot \hat{k} = \hat{j} \cdot \hat{i} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = \hat{k} \cdot \hat{j} = 0$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z \dots (12-1)$$

Example/ $\vec{A} = \hat{i} + 2\hat{j} - 2\hat{k}$
 $\vec{B} = 3\hat{i} - 4\hat{k}$

Calculate:

- 1- The angle between the two vectors $\vec{A}$ and $\vec{B}$?
- 2- The angle between the vector $\vec{A}$ and the positive (x) axis?

Solution/

$$\theta = \cos^{-1} \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A} \cdot \vec{B} = (1)(3) + (2)(0) + (-2)(-4)$$

$$\boxed{\vec{A} \cdot \vec{B} = 11}$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$|\vec{A}| = \sqrt{(1)^2 + (2)^2 + (-2)^2}$$

$$\boxed{|\vec{A}| = 3 \text{ units}}$$

$$|\vec{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

$$|\vec{B}| = \sqrt{(3)^2 + (0)^2 + (-4)^2}$$

$$\boxed{|\vec{B}| = 5 \text{ units}}$$

$$\boxed{\therefore |\vec{A}| |\vec{B}| = 15 \text{ units}}$$

$$\theta = \cos^{-1} \frac{11}{15}$$

$$\boxed{\theta = 42.8^\circ}$$

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$$\vec{A} \cdot \hat{i} = |\vec{A}| |\hat{i}| \cos \theta$$

$$\theta = \cos^{-1} \frac{\vec{A} \cdot \hat{i}}{|\vec{A}| |\hat{i}|}$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z \dots (12-1)$$

$$\vec{A} \cdot \hat{i} = (1)(1) + (2)(0) + (-2)(0)$$

$$\boxed{\vec{A} \cdot \hat{i} = 1}$$

$$\boxed{|\vec{A}| = 3}$$

$$\theta = \cos^{-1} \frac{1}{3} \quad \boxed{\theta = 70.5^\circ}$$

7. Vector product

$$|\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}| \sin \theta \quad (13-1)$$

$|\vec{A}|$: the magnitude of the vector $\vec{A}$

$|\vec{B}|$: the magnitude of the vector $\vec{B}$

θ : The smallest angle between two vectors $\vec{A}$ and $\vec{B}$ their extension

$$\vec{A} \times \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$\vec{A} \times \vec{B} = A_x \hat{i} \times B_y \hat{j} + A_x \hat{i} \times B_z \hat{k} + A_y \hat{j} \times B_x \hat{i} + A_y \hat{j} \times B_z \hat{k} + A_z \hat{k} \times B_x \hat{i} + A_z \hat{k} \times B_y \hat{j} \quad (14-1)$$

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

while

$$\hat{k} \times \hat{i} = \hat{j} \quad \hat{j} \times \hat{k} = \hat{i} \quad \hat{i} \times \hat{j} = \hat{k}$$

and

$$\hat{j} \times \hat{i} = -\hat{k} \quad \hat{k} \times \hat{j} = -\hat{i} \quad \hat{i} \times \hat{k} = -\hat{j}$$

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k} \quad (15-1)$$

Note: It becomes clear to us that in the case of cross multiplication:

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

Example

$$\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k}$$

$$\vec{B} = \hat{i} - 2\hat{j} + 2\hat{k}$$

Calculate all of the following:

1 $2\vec{A} - 3\vec{B}$?

$$2\vec{A} - 3\vec{B} = 2(2\hat{i} + 3\hat{j} + \hat{k}) - 3(\hat{i} - 2\hat{j} + 2\hat{k})$$

$$2\vec{A} - 3\vec{B} = 4\hat{i} + 6\hat{j} + 2\hat{k} - 3\hat{i} + 6\hat{j} - 6\hat{k}$$

$$2\vec{A} - 3\vec{B} = \hat{i} + 12\hat{j} - 4\hat{k}$$

2 $\vec{A}$ and $\vec{B}$?

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$|\vec{A}| = \sqrt{(2)^2 + (3)^2 + (1)^2}$$

$$\therefore |\vec{A}| = \sqrt{14} \text{units}$$

$$|\vec{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

$$|\vec{B}| = \sqrt{(1)^2 + (-2)^2 + (2)^2}$$

$$\therefore |\vec{B}| = 3 \text{units}$$

3] The angle between the two vectors $\vec{A}$ & $\vec{B}$?

$$\theta = \cos^{-1} \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} \dots (10-1)$$

$$\theta = \cos^{-1} \frac{A_x B_x + A_y B_y + A_z B_z}{|\vec{A}| |\vec{B}|}$$

$$\theta = \cos^{-1} \frac{(2)(1) + (3)(-2) + (1)(2)}{3\sqrt{14}}$$

$$\theta = \cos^{-1} \frac{-2}{3\sqrt{14}}$$

$$\therefore \theta = 100.3^\circ$$

4] $\vec{A} \times \vec{B}$?

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k} \dots (15-1)$$

$$\vec{A} \times \vec{B} = [(3)(2) - (1)(-2)] \hat{i} + [(1)(1) - (2)(2)] \hat{j} + [(2)(-2) - (3)(1)] \hat{k}$$

$$\vec{A} \times \vec{B} = 8\hat{i} - 3\hat{j} - 7\hat{k}$$

5] Unit vector in direction $\vec{A} \times \vec{B}$?

$$\hat{u}_{\vec{A} \times \vec{B}} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

$$\hat{u}_{\vec{A} \times \vec{B}} = \frac{8\hat{i} - 3\hat{j} - 7\hat{k}}{\sqrt{(8)^2 + (-3)^2 + (-7)^2}}$$

6 Unit vector in direction $\vec{B \times A}$:

$$\hat{u}_{A \times B} = \frac{8\hat{i} - 3\hat{j} - 7\hat{k}}{\sqrt{122}}$$

$$\vec{A \times B} = -\vec{B \times A}$$

$$\hat{u}_{A \times B} = -\hat{u}_{B \times A}$$

$$\hat{u}_{B \times A} = -\frac{8\hat{i} - 3\hat{j} - 7\hat{k}}{\sqrt{122}}$$

EX 3:- Find $\vec{A} + \vec{B}$ if $\vec{A} = 2\hat{i} + 2\hat{j}$ (B)

and then find the magnitude of $\vec{A} + \vec{B}$
 $\vec{B} = 2\hat{i} - 4\hat{j}$

Sol// $\vec{A} + \vec{B} = (2\hat{i} + 2\hat{j}) + (2\hat{i} - 4\hat{j})$
 $= (2+2)\hat{i} + (2-4)\hat{j}$
 $= 4\hat{i} - 2\hat{j}$
 $\begin{matrix} A_x+B_x & A_y+B_y \end{matrix}$ ليحار فينتج $\vec{A} + \vec{B}$

$$|\vec{A} + \vec{B}| = \sqrt{(A_x+B_x)^2 + (A_y+B_y)^2}$$
$$= \sqrt{(4)^2 + (-2)^2} = \sqrt{16+4} = \sqrt{20}$$
$$= 4.5$$

EX 4:- If $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = -\hat{i} + 2\hat{j}$
Find ① $\vec{A} \cdot \vec{B}$ ② angle θ between $\vec{A}$ and $\vec{B}$

Sol// ① $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$ احاطون الاول للفرج يوم ويجوز ان

$$\vec{A} \cdot \vec{B} = (2\hat{i} + 3\hat{j}) \cdot (-\hat{i} + 2\hat{j})$$
$$= (2\hat{i} \cdot -\hat{i}) + (3\hat{j} \cdot 2\hat{j})$$
$$= -2 + 6 = 4$$

$$\boxed{\vec{A} \cdot \vec{B} = 4}$$

$$\begin{aligned} \hat{i} \cdot \hat{i} &= \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1 \\ \hat{i} \cdot \hat{j} &= \hat{i} \cdot \hat{k} = \hat{j} \cdot \hat{k} = 0 \end{aligned}$$

④ تحديد الزاوية بين المتجهين باستخدام قانون الجيب
العدد الثاني

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

قانون الجيب
يوجد زاوية

$$\vec{A} = \underbrace{2}_{\vec{A}_x} \hat{i} + \underbrace{3}_{\vec{A}_y} \hat{j}$$

نوجد قيمة المتجه $\vec{A}$

$$A = |\vec{A}| = A = \sqrt{A_x^2 + A_y^2} = \sqrt{(2)^2 + (3)^2} = \sqrt{4+9} = \sqrt{13}$$

$$\vec{B} = \underbrace{-1}_{\vec{B}_x} \hat{i} + \underbrace{2}_{\vec{B}_y} \hat{j}$$

نوجد قيمة المتجه $\vec{B}$

$$B = |\vec{B}| = B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-1)^2 + (2)^2} = \sqrt{1+4} = \sqrt{5}$$

$$\vec{A} \cdot \vec{B} = 4$$

من المطلوب الدول

$$\therefore \vec{A} \cdot \vec{B} = AB \cos \theta$$

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{AB} \Rightarrow \cos \theta = \frac{4}{\sqrt{13}\sqrt{5}}$$

$$\cos \theta = \frac{4}{8.06} \Rightarrow \cos \theta = 0.49$$

$$\theta = \cos^{-1} 0.49$$

$$\therefore \theta = 60.6$$

EX5:- If $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = -\hat{i} + 2\hat{j}$ (5)

Find $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} \\ A_x & A_y \\ B_x & B_y \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} \\ 2 & 3 \\ -1 & 2 \end{vmatrix}$$

$$\begin{aligned} \hat{i} \times \hat{i} &= \hat{j} \times \hat{j} = \hat{k} \times \hat{k} \\ &= 0 \\ \hat{i} \times \hat{j} &= \hat{k} \\ \hat{j} \times \hat{k} &= \hat{i} \\ \hat{k} \times \hat{i} &= \hat{j} \end{aligned}$$

$$\vec{A} \times \vec{B} = \hat{k} (A_x B_y - A_y B_x)$$

$$= \hat{k} (2 \times 2 - (3 \times -1)) = \hat{k} (4 + 3) = 7\hat{k}$$

$$-\vec{B} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} \\ B_x & B_y \\ A_x & A_y \end{vmatrix} = \hat{k} (B_x A_y - B_y A_x)$$

$$= \hat{k} ((-1 \times 3) - 2 \times 2)$$

$$= \hat{k} (-3 - 4)$$

$$= -7\hat{k}$$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

$$7\hat{k} = -7\hat{k}$$