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Numerical Analysis

Practical

MATLAB

Lecture 2

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الرياضيات المتقدمة: المرحلة الثانية

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1 Solving Linear Systems of Equations in Matlab

1.1 Linear Systems of Equations

A system of linear equations consists of two or more linear equations involving the same set of variables. These systems can be solved using various methods, including graphing, substitution, elimination, and matrix operations, etc.

A typical linear system of equations can be written as:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n$$

1.2 Solving Linear Systems Using Substitution Method

The **Substitution Method** involves solving one equation for one variable and substituting that value into the other equations to solve for the remaining variables.

Example 1.1. Consider the following system of equations:

$$x + y = 10$$
$$x - y = 4$$

1. Solve the first equation for x:

$$x = 10 - y$$

2. Substitute $x = 10 - y$ into the second equation:

$$(10 - y) - y = 4$$

Simplifying:

$$10 - 2y = 4 \quad \Rightarrow \quad 2y = 6 \quad \Rightarrow \quad y = 3$$

3. Substitute $y = 3$ into the first equation to find x:

$$x + 3 = 10 \quad \Rightarrow \quad x = 7 \Rightarrow y = 3$$

```
% Define symbolic variables
syms x y

% Define the system of equations
eq1 = x + y == 10;
eq2 = x - y == 4;

% Solve the first equation for x
Sx = solve(eq1, x);

% Substitute x into the second equation
eq3 = subs(eq2, x, Sx);

% Solve for y
Sy = solve(eq3, y);

% Substitute y back into the first equation to find x
SFx = subs(Sx, y, Sy);

% Display the solution
disp('x = '), disp(SFx)
disp('y = '), disp(Sy)

%Output x=7, y=3
```

1.2.1 Homework of Substitution Method

Solve the following system of equations using the substitution method:

$$x + 2y + 3z = 12$$

$$4x - y + 2z = 5$$

$$-2x + 3y - z = 3$$

1.3 Solving Linear Equations Using Matrix Algebra

You can represent the same system of equations in matrix form:

$$A \cdot x = b$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

Where:

- A is the coefficient matrix,
- x is the vector of unknowns,
- b is the vector of constants.

Method 1: Using the Backslash Operator (\)

In MATLAB, the backslash operator (\) provides an efficient method to solve linear systems of the form:

$$A \cdot x = b$$

The syntax for solving this in MATLAB is:

```
x = A \ b;
```

Example 1.2. Consider the system of linear equations:

$$2x + 3y = 5$$

$$4x + 7y = 10$$

In matrix form, this is represented as:

$$\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

```
A = [2 3; 4 7];  % Coefficient matrix
b = [5; 10];     % Right-hand side vector

S = A \ b;       % Solve the system using the backslash operator
  x=S(1) % Display the solution
  y=S(2)
%Output: x = 2.5, y=0
```

Method 2: Using Inverse of Matrix

If A is invertible, the solution is:

$$x = A^{-1} \cdot b$$

Example 1.3. Consider the system of linear equations:

$$2x + 3y = 5$$

$$4x + 7y = 10$$

$$\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

$$x = A^{-1} \cdot b$$

```
A = [2 3; 4 7];  % Coefficient matrix
b = [5; 10];     % Right-hand side vector

S = inv(A)*b;        % Solve the system using inverse of matrix
x=S(1) % Display the solution
y=S(2)

%Output: x = 2.5 y=0
```

1.3.1 Homework of Solving Linear Equations Using Matrix Algebra

Solve the following system of equations using the Matrix Algebra:

$$2x + 3y - z + 4w = 14$$

$$3x - y + 2z + w = 5$$

$$x + 4y + 5z - 2w = 10$$

$$5x + 2y + z + 3w = 12$$

1.4 LU Decomposition

LU Decomposition is a method that factors a matrix A into the product of a lower triangular matrix L and an upper triangular matrix U. This decomposition can be used to solve the linear system $Ax = b$.

Steps to Solve a Linear System using LU Decomposition

1. Decompose the matrix A into L and U:

$$A = LU$$

2. Forward Substitution: Solve $Ly = b$ for y.

3. Backward Substitution: Solve $Ux = y$ for x.

Example 1.4. Consider the linear system:

$$2x + 3y = 8$$
$$4x + 9y = 18$$

In matrix form, this can be written as:

$$Ax = b$$

where

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 9 \end{bmatrix}, \quad x = \begin{bmatrix} x \\ y \end{bmatrix}, \quad b = \begin{bmatrix} 8 \\ 18 \end{bmatrix}$$

```
clear
% Define the coefficient matrix A and right-hand side vector b
A = [2 3; 4 9]
b = [8; 18]

% Perform LU decomposition
[L, U] = lu(A)

% Solve for y in the equation Ly = b using forward substitution
SL = L \ b
```

```
% Solve for x in the equation Ux = y using backward substitution
S = U \ SL
x=S(1)
y=S(2)
```

1.4.1 Homework of LU Decomposition

Solve the following system of equations using the LU Decomposition:

$$6x + 7y + 8z = 30 \quad (1)$$

$$2x - 3y + 4z = -2 \quad (2)$$

$$5x + 2y - z = 9 \quad (3)$$

$$-x + y + 2z = 1 \quad (4)$$