جامـــعة المــســتـقـبـل
AL MUSTAQBAL UNIVERSITY

كلية العلــوم

قـســـم علوم الذكاء الاصطناعي

المحاضرة الثانية

المادة : التحليل العددي
المرحلة : الثانية/ الكورس الاول
اسم الاستاذ: م.د. رياض حامد سلمان

Even and Odd Function

If $f(x) = f(-x)$, is called even function.

If $f(-x) = -f(x)$, is called odd function.

Fourier series of Even and Odd Function

If f(x) is even when { x^2, x^4, x^6… cosx, $\sin^2 x$,| f(x)|.

If f(x) is odd when { x, x^3, x^5,……, sinx.

(i) If f(x) is even then

$b_n = 0$

(ii) If f(x) is odd then

$a_0 = a_n = 0$.

Example 1

Find Fourier series of the function

$f(x) = x$, for $(-\Pi < x < \Pi)$.

Solution

Since $f(-x) = -x = -f(x)$, $\therefore$ the function is odd.

$\therefore\ a_0 = a_n = 0$.

$$b_n = \frac{1}{\Pi}\int_{-\Pi}^{\Pi} f(x)\sin nx\,dx = \quad b_n = \frac{1}{\Pi}\int_{-\Pi}^{\Pi} x\sin nx\,dx = \frac{2}{\Pi}\int_{0}^{\Pi} x\sin nx\,dx$$

$$= \frac{2}{\Pi}\left[\frac{-x\cos nx}{n} - \frac{-\sin nx}{n^2}\right]_0^{\Pi}$$

$$= \frac{2}{\Pi}\left[-\frac{-\Pi\cos n\Pi}{n}\right]$$

$$= -\frac{2}{n}\cos n\Pi$$

$$= -\frac{2}{n}(-1)^n$$

Then the series becomes:

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$f(x) = -2\sum_{n=1}^{\infty}(-1)^n\left(\frac{\sin nx}{n}\right)$$

$$f(x) = 2\left(\sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \dots\dots\right)$$

Example 2

Find Fourier series of the function

$$f(x) = \begin{cases} 1 & \text{if } 0 < x < \Pi. \\ 2 & \text{if } \Pi < x < 2\Pi. \end{cases}$$

Solution

$$a_0 = \frac{1}{2\Pi}\int_0^{2\Pi} f(x)\,dx = \frac{1}{2\Pi}\int_0^{\Pi} f(x)dx + \frac{1}{2\Pi}\int_{\Pi}^{2\Pi} f(x)\,dx$$

$$= \frac{1}{2\Pi}\int_0^{\Pi} dx \;+\; \frac{1}{2\Pi}\int_{\Pi}^{2\Pi} 2\,dx$$

$$= \frac{1}{2\Pi} x\Big]_0^{\Pi} \;+\; \frac{1}{\Pi} x\Big]_{\Pi}^{2\Pi}\, dx$$

$a_0 = 3/2.$

$$a_n = \frac{1}{\Pi}\int_0^{2\Pi} f(x)\cos nx\,dx = \frac{1}{\Pi}\int_0^{\Pi} f(x)\cos nx\,dx + \frac{1}{\Pi}\int_{\Pi}^{2\Pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\Pi}\int_0^{\Pi} \cos nx\,dx + \frac{1}{\Pi}\int_{\Pi}^{2\Pi} 2\cos nx\,dx\,,$$

$$= \frac{1}{\Pi}\frac{\sin nx}{n}\Big]_0^{\Pi} + \frac{1}{\Pi} 2\frac{\sin nx}{n}\Big]_{\Pi}^{2\Pi}$$

$$\frac{1}{\Pi}\int_0^{2\Pi} x\cos nx\,dx,$$

$$= \frac{1}{\Pi}\Big[x\frac{\sin nx}{n} - \frac{-\cos nx}{n^2}\Big]\Big]_0^{2\Pi} = 0.$$

$a_n = 0.$

$$b_n = \frac{1}{\Pi}\int_0^{\Pi} \sin nx\,dx + \frac{1}{\Pi}\int_{\Pi}^{2\Pi} 2\sin nx\,dx$$

$$= \frac{1}{\Pi}\frac{-\cos nx}{n}\Big]_0^{\Pi} + \frac{1}{\Pi}\,2\,\frac{-\cos nx}{n}\Big]_{\Pi}^{2\Pi},$$

$$= \frac{1}{\Pi}\Big[\frac{\cos n\Pi - 1}{n}\Big] = \frac{(-1)^n - 1}{n\Pi},$$

$$\therefore\; b_n = \frac{(-1)^n - 1}{n\Pi},$$

$$a_0 = 3/2\,,\; a_n = 0.\; b_0 = \frac{-2}{\Pi},\; b_1 = 0\; b_3 = \frac{-2}{3\Pi},$$

$$\therefore\; f(x) = 3/2 - \frac{2}{\Pi}\Big[\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \ldots\ldots\Big].$$

<u>Half-Range Series</u>

If we want find Fourier series on interval ($0< x<\Pi$), does not on all interval ($-\Pi < x< \Pi$), then we can find the Fourier series by :-

1- Fourier Cosine series or f(x) an even function as:-

$f(x) = a_0 + a_1\cos x + a_2\cos 2x + \ldots\ldots\ldots + a_n \cos nx.$

2- Fourier Sine series or f(x) an odd function as:-

$f(x) = b_1 \sin x + b_2 \sin 2x + \ldots\ldots\ldots\ldots\ldots\ldots + b_n \sin nx$

Such that

$$a_0 = \frac{1}{\Pi}\int_0^{\Pi} f(x)\,dx$$

$$a_n = \frac{2}{\Pi}\int_0^{\Pi} f(x)\cos nx\,dx\ ,\quad n = 1, 2, 3, \ldots.$$

$$b_n = \frac{2}{\Pi}\int_0^{\Pi} f(x)\sin nx\,dx.$$

Example 3

Find cosine Half-range series for the function defined as

$f(x) = x,\ \text{for } 0 < x < \Pi.$

Solution

Use the rule to find a_0 and a_n

$$a_0 = \frac{1}{\Pi}\int_0^{\Pi} f(x)\,dx = \frac{1}{\Pi}\int_0^{\Pi} x\,dx = \frac{1}{\Pi}\int_0^{\Pi} f(x)\,dx$$

$$a_0 = \frac{1}{2\Pi}x^2\Big]_0^{\Pi} = \frac{\Pi}{2}.$$

$$a_n = \frac{2}{\Pi}\int_0^{\Pi} f(x)\cos nx\,dx = \frac{2}{\Pi}\int_0^{\Pi} x\cos nx\,dx,$$

$$= \frac{2}{\Pi}\left[x\frac{\sin nx}{n} - \frac{-\cos nx}{n^2}\right]\Big]_0^{\Pi}$$

$$= \frac{2(\cos n\Pi - 1)}{\Pi n^2}$$

$$a_n = \begin{cases} 0 & \text{if n even.} \\ \dfrac{-4}{\Pi n^2} & \text{if n odd} \end{cases}$$

$$\therefore\ f(x) = \frac{\Pi}{2} - \frac{4}{\Pi}\left[\cos x + \frac{\cos 3x}{3} + \frac{\cos 5x}{5} + \dots\dots\right].$$

Example 4

Find sine Half-range series for the function defined as

$f(x) = x$, for $0 < x < \Pi$.

Solution

Use the rule to find b_n

$$b_n = \frac{2}{\Pi}\int_0^{\Pi} f(x)\sin nx\,dx = \quad b_n = \frac{2}{\Pi}\int_0^{\Pi} x\sin nx\,dx = \frac{2}{\Pi}\int_0^{\Pi} x\sin nx\,dx$$

$$= \frac{2}{\Pi}\left[\frac{-x\cos nx}{n} - \frac{-\sin nx}{n^2}\right]_0^{\Pi}$$

$$= \frac{2}{\Pi}\left[\frac{-\Pi\cos n\Pi}{n}\right]$$

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Then the series becomes:

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$f(x) = -2\sum_{n=1}^{\infty} (-1)^n \left(\frac{\sin nx}{n}\right)$$

$$f(x) = 2\left(\sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \frac{\sin 4x}{4} \dots\dots\right).$$