



جامعة المستقبل
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Lecture:(3)

Sets of numbers, Finite sets and counting principle

Subject: Discrete Structures

First Stage: Semester II

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Set of numbers:

Several sets are used so often, they are given special symbols.

\mathbb{N} = the set of natural numbers or positive integers

$$\mathbb{N} = \{0, 1, 2, 3, \dots\}$$

\mathbb{Z} = the set of all integers: $\dots, -2, -1, 0, 1, 2, \dots$

$$\mathbb{Z} = \mathbb{N} \cup \{\dots, -2, -1\}$$

\mathbb{Q} = the set of rational numbers

$$\mathbb{Q} = \mathbb{Z} \cup \{\dots, -1/3, -1/2, 1/2, 1/3, \dots, 2/3, 2/5, \dots\}$$

Where $\mathbb{Q} = \{a/b : a, b \in \mathbb{Z}, b \neq 0\}$

\mathbb{R} = the set of real numbers

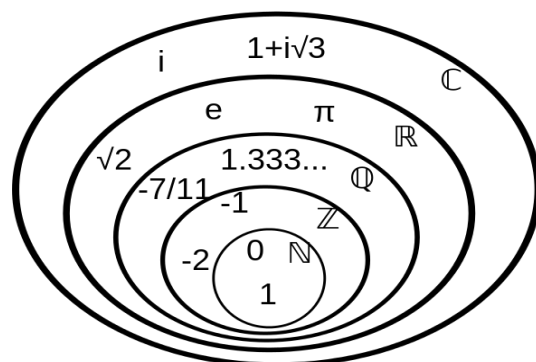
$$\mathbb{R} = \mathbb{Q} \cup \{\dots, -\pi, -\sqrt{2}, \sqrt{2}, \pi, \dots\}$$

\mathbb{C} = the set of complex numbers

$$\mathbb{C} = \mathbb{R} \cup \{i, 1+i, 1-i, \sqrt{2} + \pi i, \dots\}$$

Where $\mathbb{C} = \{x + iy ; x, y \in \mathbb{R}; i = \sqrt{-1}\}$

Observe that $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$.





Finite Sets and Counting Principle:

A set is said to be finite if it contains exactly m distinct elements, where m denotes some nonnegative integer. Otherwise, a set is said to be infinite.

For example:

- The empty set \emptyset and the set of letters of English alphabet are finite sets,
- The set of even positive integers, $\{2,4,6,\dots\}$, is infinite.
- If a set A is finite, we let $n(A)$ or $\#(A)$ denote the number of elements of A .

Example:

If $A = \{1,2,a,w\}$ then

$$n(A) = \#(A) = |A| = 4$$

Lemma: If A and B are finite sets and disjoint Then $A \cup B$ is finite set and:

$$n(A \cup B) = n(A) + n(B)$$

Theorem (Inclusion–Exclusion Principle): Suppose A and B are finite sets. Then

$A \cup B$ and $A \cap B$ are finite and

$$|A \cup B| = |A| + |B| - |A \cap B|$$

That is, we find the number of elements in A or B (or both) by first adding $n(A)$ and $n(B)$ (inclusion) and then subtracting $n(A \cap B)$ (exclusion) since its elements were counted twice.

We can apply this result to obtain a similar formula for three sets:



Corollary:

If A, B, C are finite sets then

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

Example (1) :

$$A = \{1, 2, 3\}$$

$$B = \{3, 4\}$$

$$C = \{5, 6\}$$

$$A \cup B \cup C = \{1, 2, 3, 4, 5, 6\}$$

$$|A \cup B \cup C| = 6$$

$$|A| = 3 \quad , \quad |B| = 2 \quad , \quad |C| = 2$$

$$A \cap B = \{3\} \quad , \quad |A \cap B| = 1$$

$$A \cap C = \{ \} \quad , \quad |A \cap C| = 0$$

$$B \cap C = \{ \} \quad , \quad |B \cap C| = 0$$

$$A \cap B \cap C = \{ \} \quad , \quad |A \cap B \cap C| = 0$$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

$$|A \cup B \cup C| = 3 + 2 + 2 - 1 - 0 - 0 + 0 = 6$$

Example (2):

Suppose a list A contains the 30 students in a mathematics class, and a list B contains the 35 students in an English class, and suppose there are 20 names on both lists. Find the number of students:

- (a) only on list A
- (b) only on list B
- (c) on list $A \cup B$



Solution:

(a) List A has 30 names and
20 are on list B;
hence $30 - 20 = 10$ names are only on list A.

(b) Similarly, $35 - 20 = 15$ are only on list B.

(c) We seek $n(A \cup B)$. By inclusion–exclusion,

$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 30 + 35 - 20 = 45. \end{aligned}$$

Example (3):

Suppose that 100 of 120 computer science students at a college take at least one of languages: French, German, and Russian:

65 study French (F).

45 study German (G).

42 study Russian (R).

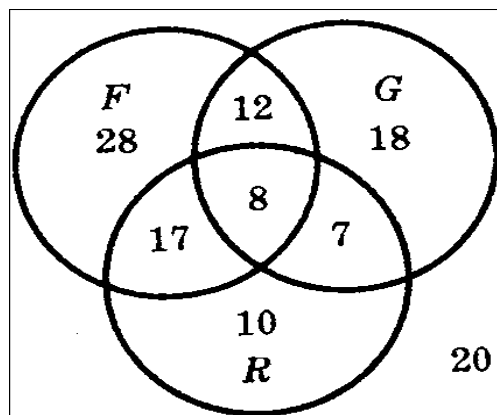
20 study French & German $F \cap G$.

25 study French & Russian $F \cap R$.

15 study German & Russian $G \cap R$.

Find the number of students who study:

- 1) All three languages ($F \cap G \cap R$)
- 2) The number of students in each of the eight regions of the Venn diagram





Solution:

$$|F \cup G \cup R| = |F| + |G| + |R| - |F \cap G| - |F \cap R| - |G \cap R| + |F \cap G \cap R|$$

$$100 = 65 + 45 + 42 - 20 - 25 - 15 + |F \cap G \cap R|$$

$$100 = 92 + |F \cap G \cap R|$$

$\therefore |F \cap G \cap R| = 8$ students study the 3 languages

$$20 - 8 = 12 \quad (F \cap G) - R$$

$$25 - 8 = 17 \quad (F \cap R) - G$$

$$15 - 8 = 7 \quad (G \cap R) - F$$

$65 - 12 - 8 - 17 = 28$ students study French only

$45 - 12 - 8 - 7 = 18$ students study German only

$42 - 17 - 8 - 7 = 10$ students study Russian only

$120 - 100 = 20$ students do not study any language



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