College of Sciences
Artifactual intelligent science
Department

Lecture 5:

Fuzzy Numbers

Subject: Fuzzy Logic

Level: Second

Semester: second

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Google Class Room

Why Fuzzy Numbers?

Many real-world values are not perfectly exact. People often say "about", "around", or "approximately". A fuzzy number represents this imprecision while keeping the most likely value.

Real-life examples:

- Arrival time: "The trip takes about 20 minutes."
- Price: "This product costs around $50."
- Temperature: "The patient's temperature is approximately 38°C."
- Workload: "The task needs about 10 hours."

What is a Fuzzy Number?

A fuzzy number is a **special fuzzy set** on the real line $\mathbb{R}$ used to represent an imprecise numeric quantity like "about 20". Instead of giving a single value, we describe a range of possible values and how plausible each value is.

Typical properties (informal):

- Normal: there is at least one value with membership 1 (the most plausible value).
- Convex: the shape has one "hump" (no dips in the middle).
- Bounded support: outside a certain range, membership becomes 0.

Membership Function μ(x)

For a fuzzy number $\tilde{A}$, the membership function $\mu\tilde{A}(x)$ returns a value between 0 and 1:

- $\mu(x) = 1 \rightarrow$ x is the best / most plausible value
- $\mu(x) = 0 \rightarrow$ x is not plausible (outside the range)
- $0 < \mu(x) < 1 \rightarrow$ x is partially plausible

Triangular Fuzzy Number (TFN)

A Triangular Fuzzy Number (TFN) is a way to represent an approximate value instead of one exact number. In real life we often say "about", "around", or "approximately". For example, instead of saying:

"Arrival time is exactly 20 minutes,"

we say:

"Arrival time is **about 20 minutes**."
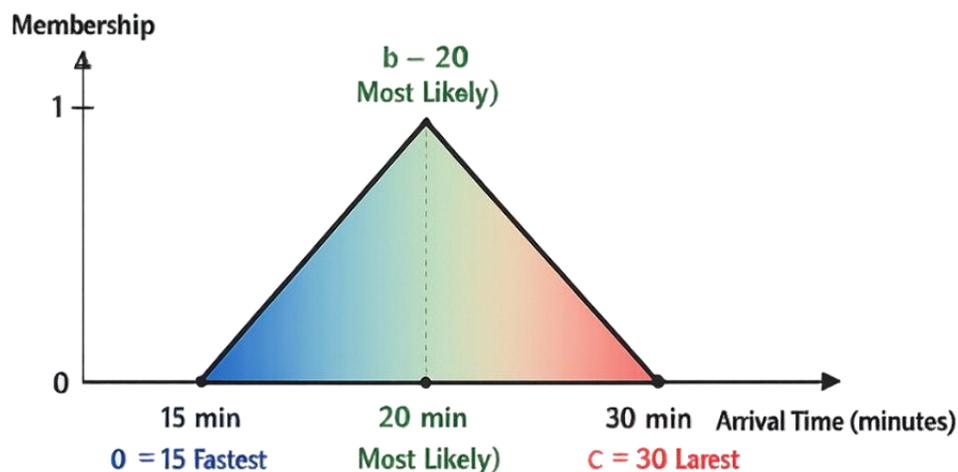
A Triangular Fuzzy Number is written as:

$$\tilde{A} = (a, b, c) \quad \text{with} \quad a \leq b \leq c$$

Interpretation:

- a: the smallest possible value (membership 0)
- b: the most likely value (membership 1)
- c: the largest possible value (membership 0)

Real-life model example (arrival time):

- Arrival time ≈ (15, 20, 30) minutes
- Meaning: it can be as fast as 15, usually about 20, and can be as late as 30.

TFN Membership Function (Piecewise)

For Ã = (a, b, c), the membership function is defined in 4 cases:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < a \\ \dfrac{x-a}{b-a} & a \leq x \leq b \\ \dfrac{c-x}{c-b} & b \leq x \leq c \\ 0 & x > c \end{cases}$$

- If $x < a \rightarrow \mu(x) = 0$
- If $a \leq x \leq b \rightarrow \mu(x)$ rises linearly from 0 to 1
- If $b \leq x \leq c \rightarrow \mu(x)$ falls linearly from 1 to 0
- If $x > c \rightarrow \mu(x) = 0$

Think of it as a triangle: rising edge (a→b) and falling edge (b→c).

Worked Membership Examples (Step-by-step)

Let Ã = (15, 20, 30).

Example 1: Compute μ(18)

- 18 is between 15 and 20 (rising part).
- $\mu(18) = (18 - 15) / (20 - 15) = 3/5 = 0.6$
- Interpretation: 18 minutes is fairly plausible (0.6).

Example 2: Compute μ(26)

- 26 is between 20 and 30 (falling part).
- $\mu(26) = (30 - 26) / (30 - 20) = 4/10 = 0.4$
- Interpretation: 26 minutes is possible but less plausible than 18.

α-cuts (Alpha-cuts) — Confidence Slices

An α-cut keeps only the values whose membership is at least α. It answers: "Which values are acceptable if we require at least α confidence?"

Definition:

$$\tilde{A}\alpha = \{\, x \mid \mu\tilde{A}(x) \geq \alpha \,\}\ , \text{ where } 0 < \alpha \leq 1$$

For fuzzy numbers, $\tilde{A}\alpha$ becomes an interval: $\tilde{A}\alpha = [AL(\alpha), AU(\alpha)]$.

TFN α-cut formula:

$$\text{If } \tilde{A} = (a, b, c), \text{ then: } \tilde{A}\alpha = [\, a + \alpha(b - a)\ ,\ c - \alpha(c - b)\,]$$
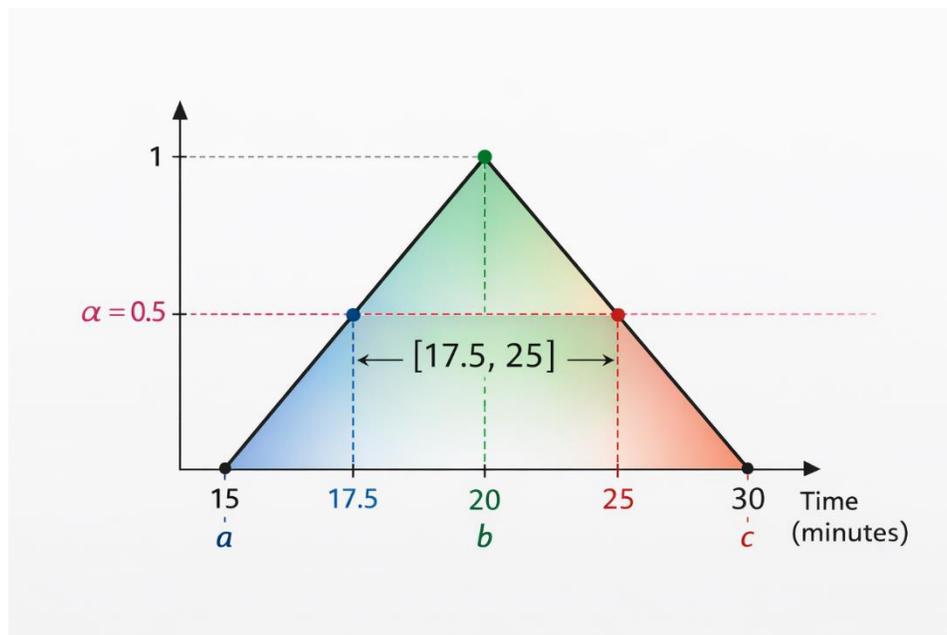
Example: $\tilde{A} = (15, 20, 30)$, $\alpha = 0.5$

$AL = 15 + 0.5(20 - 15) = 17.5$
$AU = 30 - 0.5(30 - 20) = 25$
So $\tilde{A}0.5 = [17.5, 25]$

Meaning: with confidence ≥ 0.5, time is likely between 17.5 and 25 minutes.

Basic Operations on TFNs (Student-friendly rules)

For TFNs, we often use simple rules that are easy to apply in class.

Operation	Rule ($\tilde{A}$=(a,b,c), $\tilde{B}$=(d,e,f))	When to use
Addition	$\tilde{A} + \tilde{B} = (a+d, \ b+e, \ c+f)$	Always safe for TFNs
Subtraction	$\tilde{A} - \tilde{B} = (a-f, \ b-e, \ c-d)$	Common in budgeting/differences
Multiplication	$\tilde{A} \times \tilde{B} = (a \cdot d, \ b \cdot e, \ c \cdot f)$	Use when all values are positive

Real-life Examples

Example A — Total time = travel + parking delay

Travel time: (15, 20, 30)
Parking delay: (5, 10, 15)
Total: (20, 30, 45)
Meaning: usually 30 minutes, but it can be between 20 and 45.

Example B — Remaining money = budget − expenses

Budget: (900, 1000, 1200)
Expenses: (600, 700, 900)
Remaining: (0, 300, 600)
Meaning: maybe 0 in worst case, usually ~300, could be up to ~600.

Example C — Cost = price × quantity (positive values)

Price per unit: (4, 5, 6)
Quantity: (8, 10, 12)
Cost: (32, 50, 72)
Meaning: likely around 50, but can range 32–72.

In-class Activities (10–15 minutes)

- Activity 1 (individual): For Ã = (10, 15, 20), compute $\mu(12)$ and $\mu(18)$.
- Activity 2 (pairs): For Ã = (15, 20, 30), compute $Ã0.75$ and interpret it.

Homework

1) Model "about 50" as a TFN. Explain your choice of (a, b, c).
2) Let Ã=(2,4,6) and B̃=(1,2,5): compute Ã+B̃ and Ã−B̃.
3) For Ã=(15,20,30), compute $Ã0.5$ and interpret it in words.