



# *Fuzzy Logic*

## *Lecture 4:*

# *Algebraic Operations on Fuzzy Set*



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# Algebraic Operations on Fuzzy Set

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## Algebraic Product

- The product of two fuzzy sets in the same universe of discourse is the new fuzzy set  $A.B$  with a membership function that equal product of the membership function of  $A$  and the membership function of  $B$ .

$$\mu_{A.B}(x) = \{\mu_A(x) \cdot \mu_B(x) \mid x \in A, x \in B\}$$

# Algebraic Operations on Fuzzy Set

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## Multiplying a fuzzy set by a crisp number

- When a fuzzy set  $A$  is multiplied by a crisp number  $a$ , then its membership function is given by

$$\mu_{a.A}(x) = a\mu_A(x)$$

# Algebraic Operations on Fuzzy Set

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## Cartesian product

- The Cartesian product of two fuzzy sets A & B is a fuzzy set C denoted by  $A \times B$  and defined as

$$C = A \times B = \mu_C(x) / (a, b) \mid a \in A, b \in B$$

$$\mu_C(C) = \min(\mu_{A(a)}, \mu_{B(b)})$$

# Algebraic Operations on Fuzzy Set

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## Algebraic Sum

The Algebraic sum of two fuzzy sets A & B is a fuzzy set C denoted by  $A + B$  and defined as

$$C = A + B = \mu_C(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)$$

# Algebraic Operations on Fuzzy Set

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## Bounded Sum

- The bounded sum of two fuzzy sets A and B in the universes X and Y with the membership functions  $\mu_A(x)$  and  $\mu_B(x)$  respectively is defined by

$$\begin{aligned} A \oplus B = \mu_{A \oplus B}(x) &= 1 \wedge (\mu_A(x) + \mu_B(y)) \\ &= \min(1, (\mu_A(x) + \mu_B(y))) \end{aligned}$$

where the “+” sign is an arithmetic operator.

# Algebraic Operations on Fuzzy Set

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## Bounded product

- The bounded product of two fuzzy sets  $A$  and  $B$  in the universes  $X$  and  $Y$  with membership functions  $\mu_A(x)$  and  $\mu_B(x)$  respectively is defined as

$$\begin{aligned} A \otimes B &= \mu_{A \otimes B}(x) = 0 \vee (\mu_A(x) + \mu_B(x) - 1) \\ &= \max(0, (\mu_A(x) + \mu_B(x) - 1)). \end{aligned}$$

# Algebraic Operations on Fuzzy Set

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## Bounded difference

- The bounded of difference of two fuzzy sets A and B is a fuzzy set C denoted by

$$C = A \ominus B = \mu_C(x) = \min(1, \mu_{A(x)} - \mu_{B(x)})$$

## Example 7

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- Let us consider two fuzzy sets

$$A = \{(1, 0.6), (2, 1.0), (3, 0.5), (4, 0.3), (5, 0.8)\}$$

$$B = \{(2, 0.5), (3, 0.7)\}$$

Find out the Algebraic product, Cartesian product, Algebraic sum, Bounded Sum, Bounded difference.

# Solution 7

Algebraic product is calculated using equation  $\mu_{A \cdot B}(x) = \{\mu_A(x) \cdot \mu_B(x) \mid x \in A, x \in B\}$

$$\mu_{A \cdot B}(2) = \{\mu_A(2) \cdot \mu_B(2) = 1.0 \cdot 0.5 = 0.05\}$$

$$\mu_{A \cdot B}(3) = \{\mu_A(3) \cdot \mu_B(3) = 0.5 \cdot 0.7 = 0.35\}$$

$$A \cdot B = \{(2, 0.05), (3, 0.35)\}$$

Cartesian product of fuzzy set A, & B is given by

$$C = A \times B = \mu_c(x)/(a, b) \mid a \in A, b \in B, \mu_c(c) = \min(\mu_{A(a)}, \mu_{B(b)})$$

$$\mu_c(c)/(a, b), \mu_c(c) = \min(\mu_{A(1)}, \mu_{B(2)}) = \min(0.6, 0.5) = 0.5$$

$$\mu_c(c)/(a, b), \mu_c(c) = \min(\mu_{A(1)}, \mu_{B(3)}) = \min(0.6, 0.7) = 0.6$$

$$\mu_c(c)/(a, b), \mu_c(c) = \min(\mu_{A(2)}, \mu_{B(2)}) = \min(1.0, 0.5) = 0.5$$

$$\mu_c(c)/(a, b), \mu_c(c) = \min(\mu_{A(2)}, \mu_{B(3)}) = \min(1.0, 0.7) = 0.7$$

$$\mu_c(c)/(a, b), \mu_c(c) = \min(\mu_{A(3)}, \mu_{B(2)}) = \min(0.5, 0.5) = 0.5$$

$$\mu_c(c)/(a, b), \mu_c(c) = \min(\mu_{A(3)}, \mu_{B(3)}) = \min(0.5, 0.7) = 0.5$$

$$\mu_c(c)/(a, b), \mu_c(c) = \min(\mu_{A(4)}, \mu_{B(2)}) = \min(0.3, 0.5) = 0.3$$

# Solution 7 cont.....

$$\mu_c(c)/(a, b), \mu_c(c) = \min(\mu_{A(4)}, \mu_{B(3)}) = \min(0.3, 0.7) = 0.3$$

$$\mu_c(c)/(a, b), \mu_c(c) = \min(\mu_{A(5)}, \mu_{B(2)}) = \min(0.8, 0.5) = 0.5$$

$$\mu_c(c)/(a, b), \mu_c(c) = \min(\mu_{A(5)}, \mu_{B(3)}) = \min(0.8, 0.7) = 0.7$$

$$C = A \times B = \{0.5/(1,2), 0.6/(1,3), 0.5/(2,2), 0.7/(2,3), 0.5/(3,2), 0.5/(3,3), 0.3/(4,2), 0.3/(4,3), 0.5/(5,2), 0.7/(5,3)\}$$

Algebraic Sum of two fuzzy sets A & B is a set C given by

$$C = A + B = \mu_c(x) = \mu_{A(x)} + \mu_{B(x)} - \mu_{A(x)} \cdot \mu_{B(x)}$$

$$\mu_c(1) = \mu_{A(1)} + \mu_{B(1)} - \mu_{A(1)} \cdot \mu_{B(1)} = 0.6 + 0.0 - 0.6 \cdot 0.0 = 0.6$$

$$\mu_c(2) = \mu_{A(2)} + \mu_{B(2)} - \mu_{A(2)} \cdot \mu_{B(2)} = 1.0 + 0.5 - 1.0 \cdot 0.5 = 1.0$$

$$\mu_c(3) = \mu_{A(3)} + \mu_{B(3)} - \mu_{A(3)} \cdot \mu_{B(3)} = 0.5 + 0.7 - 0.5 \cdot 0.7 = 0.85$$

$$\mu_c(4) = \mu_{A(4)} + \mu_{B(4)} - \mu_{A(4)} \cdot \mu_{B(4)} = 0.3 + 0.0 - 0.3 \cdot 0.0 = 0.3$$

$$\mu_c(5) = \mu_{A(5)} + \mu_{B(5)} - \mu_{A(5)} \cdot \mu_{B(5)} = 0.8 + 0.0 - 0.8 \cdot 0.0 = 0.8$$

$$C = A + B = \{(1,0.6), (2,1.0), (3,0.85), (4,0.3), (5,0.8)\}$$

# Solution 7 cont.....

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The Bounded Sum of two fuzzy sets A & B is a fuzzy set C given by

$$C = A \oplus B = \mu_c(x) = \min(1, \mu_A(x) + \mu_B(x))$$

$$\mu_{c(1)} = \min(1, \mu_{A(1)} + \mu_{B(1)}) = \min(1, 0.6 + 0.0) = 0.6$$

$$\mu_{c(2)} = \min(1, \mu_{A(2)} + \mu_{B(2)}) = \min(1, 1.0 + 0.5) = 1.0$$

$$\mu_{c(3)} = \min(1, \mu_{A(3)} + \mu_{B(3)}) = \min(1, 0.5 + 0.7) = 1.0$$

$$\mu_{c(4)} = \min(1, \mu_{A(4)} + \mu_{B(4)}) = \min(1, 0.3 + 0.0) = 0.3$$

$$\mu_{c(5)} = \min(1, \mu_{A(5)} + \mu_{B(5)}) = \min(1, 0.8 + 0.0) = 0.8$$

$$C = A \oplus B = \{(1, 0.6), (2, 1.0), (3, 1.0), (4, 0.3), (5, 0.8)\}$$

The Bounded difference of two fuzzy sets A & B is a fuzzy set C given by

$$C = A \ominus B = \mu_c(x) = \min(1, \mu_A(x) - \mu_B(x))$$

$$\mu_{c(1)} = \min(1, \mu_{A(1)} - \mu_{B(1)}) = \min(1, 0.6 - 0.0) = 0.6$$

$$\mu_{c(2)} = \min(1, \mu_{A(2)} - \mu_{B(2)}) = \min(1, 1.0 - 0.5) = 0.5$$

$$\mu_{c(3)} = \min(1, \mu_{A(3)} - \mu_{B(3)}) = \min(1, 0.5 - 0.7) = 0.0 \text{ (as } \mu_{A(3)} < \mu_{B(3)})$$

$$\mu_{c(4)} = \min(1, \mu_{A(4)} - \mu_{B(4)}) = \min(1, 0.3 - 0.0) = 0.3$$

$$\mu_{c(5)} = \min(1, \mu_{A(5)} - \mu_{B(5)}) = \min(1, 0.8 - 0.0) = 0.8$$

$$C = A \ominus B = \{(1, 0.6), (2, 0.5), (3, 0.0), (4, 0.3), (5, 0.8)\}$$

# Properties of Fuzzy sets

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- 1. Associative Property**
- 2. Commutative Property**
- 3. Distributive Property**
- 4. Idem Potency**
- 5. Identity**
- 6. Transitive**
- 7. Involution**
- 8. Demorgan's Law**

# Properties of Fuzzy sets

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## Associative Property (الخاصية المشتركة)

$$A \cup (B \cap C) = (A \cup B) \cap C$$

or

$$\max [\mu_A(X), \max\{\mu_B(Y), \mu_C(Z)\}] = \max[ \{\max \{\mu_A(x), \mu_B(Y)\}, \mu_C(Z) ]$$

and  $A \cap (B \cup C) = (A \cap B) \cup C$  or

$$\min [\mu_A(X), \min\{\mu_B(Y), \mu_C(Z)\}] = \min[ \{\min \{\mu_A(x), \mu_B(Y)\}, \mu_C(Z) ]$$

# Properties of Fuzzy sets

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## Commutative Property (خاصية التبديل)

$A \cup B = B \cup A$  or  $\max [\mu_A(x), \mu_B(y)] = \max [\mu_B(x), \mu_A(y)]$   
and  $A \cap B = B \cap A$  or  $\min [\mu_A(x), \mu_B(y)] = \min [\mu_B(x), \mu_A(y)]$ .

# Properties of Fuzzy sets

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## Distributive Property (خاصية التوزيع)

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \text{ or}$$
$$\max [\mu_A(x), \min(\mu_B(x), \mu_C(z))] = \min[\max \{\mu_A(x), \mu_B(y)\}, \max \{\mu_A(x), \mu_C(z)\}].$$

# Properties of Fuzzy sets

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## Idem Potency

$A \cup A = A$  means

$$\max \{ \mu_A(x), \mu_A(x) \} = \mu_A(x)$$

and  $A \cap A = A$  means

$$\min \{ \mu_A(x), \mu_A(x) \} = \mu_A(x).$$

# Properties of Fuzzy sets

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## Identity

$A \cup \phi = A$  means

$$\max \{ \mu_A(x), 0 \} = \mu_A(x)$$

$A \cap X = A$  means

$$\min \{ \mu_A(x), 1 \} = \mu_A(x)$$

$A \cap \phi = \phi$  means

$$\min \{ \mu_A(x), 0 \} = 0$$

and  $A \cup X = X$  means

$$\max \{ \mu_A(x), 1 \} = 1.$$

# Properties of Fuzzy sets

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## Transitive

If  $A \subseteq B$  and  $B \subseteq C$  Then  $A \subseteq C$

# Properties of Fuzzy sets

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**Involution(الالتفاف)**

$$A''=A$$

$$1-(1-\mu_A(x)) = \mu_A(x).$$

# Properties of Fuzzy sets

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## Demorgan's Law

$$\overline{A \cap B} = \overline{A} \cup \overline{B},$$

$$\overline{A \cup B} = \overline{A} \cap \overline{B}.$$

# Prove

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- Prove that the De Morgan's Laws hold for fuzzy sets that is

$$\overline{A \cap B} = \overline{A} \cup \overline{B},$$

$$\overline{A \cup B} = \overline{A} \cap \overline{B}.$$

# Solution

To show that equation (1) is true, we have to prove

$$1 - \max(\mu_{A(x)}, \mu_{B(x)}) = \min[(1 - \mu_{A(x)}), (1 - \mu_{B(x)})]$$

Considering a case that,  $\mu_{A(x)} > \mu_{B(x)}$

$\therefore$  L.H.S. of equation(3) is equal to  $[1 - \mu_{A(x)}]$ , and

R.H.S. of equation(3) is equal to  $[1 - \mu_{A(x)}]$

And similarly for the case  $\mu_{B(x)} > \mu_{A(x)}$

To show that equation (2) is true, we have to prove

$$1 - \min(\mu_{A(x)}, \mu_{B(x)}) = \max[(1 - \mu_{A(x)}), (1 - \mu_{B(x)})]$$

Considering a case that,  $\mu_{A(x)} > \mu_{B(x)}$

$\therefore$  L.H.S. of equation(4) is equal to  $[1 - \mu_{B(x)}]$ , and

R.H.S. of equation(4) is equal to  $[1 - \mu_{B(x)}]$

And similarly for the case  $\mu_{B(x)} > \mu_{A(x)}$

# Solve

Example 5. Show that  $\mu_A(x) \wedge \mu_B(x) = \mu_B(x) \ominus (\mu_B(x) \ominus \mu_A(x))$

Taking L.H.S.

$$\mu_A(x) \wedge \mu_B(x) = \begin{cases} \mu_B(x), & \text{for } \mu_A(x) \geq \mu_B(x) \\ \mu_A(x), & \text{for } \mu_A(x) < \mu_B(x) \end{cases} \quad \dots(1)$$

Solving for R.H.S. under the conditions

(a)  $\mu_A(x) > \mu_B(x)$

$$\Rightarrow \mu_B(x) \ominus 0$$

$$\Rightarrow \mu_B(x)$$

$$\because \mu_B(x) \ominus \mu_A(x) = \min(1, \mu_B(x) - \mu_A(x)) = \min(1, 0) = 0$$

... (2)

(b)  $\mu_B(x) > \mu_A(x)$

$$\Rightarrow \mu_B(x) \ominus (\mu_B(x) - \mu_A(x))$$

$$\Rightarrow \mu_B(x) - (\mu_B(x) - \mu_A(x))$$

$$\Rightarrow \mu_A(x)$$

$$\because \mu_B(x) \ominus \mu_A(x) = \min(1, \mu_B(x) - \mu_A(x)) = \mu_B(x) - \mu_A(x)$$

... (3)

From equations (1), (2) and (3) one may conclude that

$$\mu_A(x) \wedge \mu_B(x) = \mu_B(x) \ominus (\mu_B(x) \ominus \mu_A(x))$$

# Solve

Example 6. Show that  $\mu_{A(x)} \vee \mu_{B(x)} = \mu_{A(x)} + (\mu_{B(x)} \ominus \mu_{A(x)})$

Taking L.H.S.

$$\mu_{A(x)} \vee \mu_{B(x)} = \begin{cases} \mu_{A(x)}, & \text{for } \mu_{A(x)} \geq \mu_{B(x)} \\ \mu_{B(x)}, & \text{for } \mu_{B(x)} > \mu_{A(x)} \end{cases} \quad \dots(1)$$

Solving for R.H.S. under the conditions

(a)  $\mu_{A(x)} \geq \mu_{B(x)}$

$$\Rightarrow \mu_{A(x)} + (\mu_{B(x)} \ominus \mu_{A(x)})$$

$$\Rightarrow \mu_{A(x)} + 0$$

$$\Rightarrow \mu_{A(x)}$$

$$\because \mu_{B(x)} \ominus \mu_{A(x)} = \min(1, \mu_{B(x)} - \mu_{A(x)}) = \min(1, 0) = 0 \quad \dots(2)$$

(b)  $\mu_{B(x)} > \mu_{A(x)}$

$$\Rightarrow \mu_{A(x)} + (\mu_{B(x)} \ominus \mu_{A(x)})$$

$$\Rightarrow \mu_{A(x)} + (\mu_{B(x)} - \mu_{A(x)})$$

$$\Rightarrow \mu_{B(x)}$$

$$\because \mu_{B(x)} \ominus \mu_{A(x)} = \min(1, \mu_{B(x)} - \mu_{A(x)}) = \mu_{B(x)} - \mu_{A(x)} \quad \dots(3)$$

From (1), (2) and (3) one may conclude that

$$\mu_{A(x)} \vee \mu_{B(x)} = \mu_{A(x)} + (\mu_{B(x)} \ominus \mu_{A(x)})$$

# Solve

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1. For the given fuzzy set

$$A_{\sim} = \left\{ \frac{1}{1.0} + \frac{0.65}{1.5} + \frac{0.4}{2.0} + \frac{0.35}{2.5} + \frac{0}{3.0} \right\},$$

$$B_{\sim} = \left\{ \frac{0}{1.0} + \frac{0.25}{1.5} + \frac{0.6}{2.0} + \frac{0.25}{2.5} + \frac{1}{3.0} \right\},$$

$$C_{\sim} = \left\{ \frac{0.5}{1.0} + \frac{0.25}{1.5} + \frac{0}{2.0} + \frac{0.25}{2.5} + \frac{0.5}{3.0} \right\}.$$

Prove the associative and the distributive property for the above given sets.

# Solution

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- To prove associative property

$$1. \underset{\sim}{A} \cup \left( \underset{\sim}{B} \cup \underset{\sim}{C} \right) = \left( \underset{\sim}{A} \cup \underset{\sim}{B} \right) \cup \underset{\sim}{C}$$

LHS

$$\underset{\sim}{A} \cup \left( \underset{\sim}{B} \cup \underset{\sim}{C} \right)$$

$$\left( \underset{\sim}{B} \cup \underset{\sim}{C} \right) = \left\{ \frac{0.5}{1.0} + \frac{0.25}{1.5} + \frac{0.6}{2.0} + \frac{0.25}{2.5} + \frac{1}{3.0} \right\}$$

$$\underset{\sim}{A} \cup \left( \underset{\sim}{B} \cup \underset{\sim}{C} \right) = \left\{ \frac{1}{1.0} + \frac{0.65}{1.5} + \frac{0.6}{2.0} + \frac{0.35}{2.5} + \frac{1}{3.0} \right\}$$

RHS

$$\left( \underset{\sim}{A} \cup \underset{\sim}{B} \right) \cup \underset{\sim}{C}$$

$$\left( \underset{\sim}{A} \cup \underset{\sim}{B} \right) = \left\{ \frac{1}{1.0} + \frac{0.65}{1.5} + \frac{0.6}{2.0} + \frac{0.35}{2.5} + \frac{1}{3.0} \right\}$$

$$\left( \underset{\sim}{A} \cup \underset{\sim}{B} \right) \cup \underset{\sim}{C} = \left\{ \frac{1}{1.0} + \frac{0.65}{1.5} + \frac{0.6}{2.0} + \frac{0.35}{2.5} + \frac{1}{3.0} \right\}$$

# Solution

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- To prove distribute property

$$2. \left( \underset{\sim}{A} \cap \left( \underset{\sim}{B} \cup \underset{\sim}{C} \right) \right) = \left( \underset{\sim}{A} \cap \underset{\sim}{B} \right) \cup \left( \underset{\sim}{A} \cap \underset{\sim}{C} \right)$$

LHS

$$\begin{aligned} & \underset{\sim}{A} \cap \left( \underset{\sim}{B} \cup \underset{\sim}{C} \right) \\ & \left( \underset{\sim}{B} \cup \underset{\sim}{C} \right) = \left\{ \frac{0.5}{1.5} + \frac{0.25}{1.5} + \frac{0.6}{2.0} + \frac{0.25}{2.5} + \frac{1}{3.0} \right\} \\ & \underset{\sim}{A} \cap \left( \underset{\sim}{B} \cup \underset{\sim}{C} \right) = \left\{ \frac{0.5}{1.0} + \frac{0.25}{1.5} + \frac{0.4}{2.0} + \frac{0.25}{2.5} + \frac{0}{3.0} \right\} \quad (2.11) \end{aligned}$$

RHS

$$\begin{aligned} & \left( \underset{\sim}{A} \cap \underset{\sim}{B} \right) = \left\{ \frac{0}{1.0} + \frac{0.25}{1.5} + \frac{0.4}{2.0} + \frac{0.25}{2.5} + \frac{0}{3.0} \right\} \\ & \left( \underset{\sim}{A} \cap \underset{\sim}{C} \right) = \left\{ \frac{0.5}{1.0} + \frac{0.25}{1.5} + \frac{0}{2.0} + \frac{0.25}{2.5} + \frac{0}{3.0} \right\} \\ & \left( \underset{\sim}{A} \cap \underset{\sim}{B} \right) \cup \left( \underset{\sim}{A} \cap \underset{\sim}{C} \right) = \left\{ \frac{0.5}{1.0} + \frac{0.25}{1.5} + \frac{0.4}{2.0} + \frac{0.25}{2.5} + \frac{0}{3.0} \right\} \quad (2.12) \end{aligned}$$

# Solve

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**Example 2.10.** Consider the following fuzzy sets

$$A = \left\{ \frac{0.8}{10} + \frac{0.3}{15} + \frac{0.6}{20} + \frac{0.2}{25} \right\},$$

$$B = \left\{ \frac{0.4}{10} + \frac{0.2}{15} + \frac{0.9}{20} + \frac{0.1}{25} \right\}.$$

Calculate the Demorgan's law  $\overline{A \cup B} = \bar{A} \cap \bar{B}$ , and  $\overline{A \cap B} = \bar{A} \cup \bar{B}$

# Solve

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3. Consider two fuzzy sets  $A_{\sim}$  and  $B_{\sim}$  find Complement, Union, Intersection, Difference, and De Morgan's law.

$$A_{\sim} = \left\{ \frac{1}{2} + \frac{0.5}{3} + \frac{0.6}{4} + \frac{0.2}{5} + \frac{0.6}{6} \right\},$$
$$B_{\sim} = \left\{ \frac{0.5}{2} + \frac{0.8}{3} + \frac{0.4}{4} + \frac{0.7}{5} + \frac{0.3}{6} \right\}.$$

# Solve

4. Given the classical sets,

$$A = \{9, 5, 6, 8, 10\} \quad B = \{1, 2, 3, 7, 9\} \quad C = \{1, 0\}$$

Prove the classical set properties associativity and distributivity.

# Solve

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**5. Consider,  $X = \{a, b, c, d, e, f, g, h\}$ .** and the set *A* is defined as  $\{a, d, f\}$ . So for this classical set *prove the identity* property.

# Solution

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*Solution.* Given,  $X = \{a, b, c, d, e, f, g, h\}$ ,

$$A = \{a, d, f\}.$$

The identity property is given as

1.  $A \cup \phi = A.$

2.  $A \cap \phi = A.$

$\phi$  is going to be a null set, hence it is clearly understood, that,  $A \cup \phi \neq A \cap \phi$  will give as the same set  $A$ .

3.  $A \cap X = A.$

$$A \cap X = \{a, d, f\},$$

$$A = \{a, d, f\}.$$

Hence,  $A \cap X = A.$

4.  $A \cup X = X,$

$$A \cup X = \{a, b, c, d, e, f, g, h\},$$

$$X = \{a, b, c, d, e, f, g, h\}.$$

Hence,  $A \cup X = X.$

This identity property is proved.

# Homework

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1. Determine all possible  $\alpha$ -level sets of  $A = \{(1, 0.6), (2, 0.7), (3, 0.5), (4, 0.3), (5, 0.8)\}$ .
2. Determine all possible strong  $\alpha$ -level sets of  $A = \{(1, 0.6), (2, 0.5), (3, 0.6), (4, 0.3), (5, 0.8), (6, 0.5), (7, 0.8)\}$ .
3. Determine Support, and crossover point for set  $A = \{(1, 0.6), (2, 0.4), (3, 0.7), (4, 0.5), (5, 0.8), (6, 0.4), (7, 0.9), (8, 0.0)\}$ .
4. Let  $A = \{(1, 0.3), (3, 0.8), (4, 0.6), (5, 0.7), (6, 0.6)\}$  and  $B = \{(1, 0.2), (3, 0.4), (4, 0.5), (5, 0.6), (6, 0.4)\}$ ; determine
  - (i)  $A'$ ,
  - (ii)  $B'$ ,
  - (iii)  $A - B$ ,
  - (iv)  $B - A$ .
5. Given  $A = \{(1, 0.2), (2, 0.4), (3, 0.5), (4, 0.8)\}$  and  $B = \{(1, 0.2), (3, 0.4), (4, 0.5), (5, 0.6), (6, 0.4)\}$ , determine
  - (i)  $(A \cap B)'$ ,
  - (ii)  $(A \cap B)'$ ,
  - (iii)  $A' \cap B'$ ,
  - (iv)  $B' \cap A'$ .

# Homework

6. Given  $A = \{(2, 0.4), (3, 0.6), (5, 0.5)\}$  and  $B = \{(1, 0.2), (3, 0.4)\}$ , determine the Cartesian product of two sets *i.e.*,  $A \times B$ .
7. Given  $A = \{(1, 0.2), (2, 0.4), (3, 0.5), (4, 0.8)\}$  and  $B = \{(1, 0.5), (2, 0.4), (3, 0.6), (4, 0.6), (6, 0.4)\}$ , determine
- (i) Algebraic product,
  - (ii)  $CON(A)$
  - (iii)  $CON(B)$
  - (iv)  $DIL(A)$
  - (v)  $DIL(B)$
8. Given  $A = \{(1, 0.6), (2, 0.7), (3, 0.5), (4, 0.3), (5, 0.8)\}$ , and  $B = \{(1, 0.4), (2, 0.5), (3, 0.6), (4, 0.5), (5, 0.4), (6, 0.5), (7, 0.8)\}$ . Represent Graphically Fuzzy Set A, B and their complements.
9. Determine graphically union and intersection of fuzzy sets whose values are  $A = \{(1, 0.6), (2, 0.5), (3, 0.6), (4, 0.3), (5, 0.8), (6, 0.5), (7, 0.8)\}$ , and  $B = \{(1, 0.4), (2, 0.3), (3, 0.2), (4, 0.7), (5, 0.4), (6, 0.8), (7, 0.9), (8, 1.0)\}$
10. Given  $A = \{(2, 0.4), (3, 0.6), (5, 0.5), (6, 0.7), (7, 0.9)\}$  and  $B = \{(2, 0.5), (3, 0.7), (5, 1.0), (6, 0.5), (7, 0.8)\}$  determine the Algebraic sum & Bounded sum of the two sets.
11. Given  $A = \{(1, 0.2), (2, 0.4), (3, 0.5), (4, 0.8)\}$  and  $B = \{(1, 0.5), (2, 0.4), (3, 0.6), (4, 0.6), (6, 0.4)\}$ , determine
- (i)  $A - B$ ,
  - (ii)  $B - A$ ,
  - (iii)  $A \ominus B$ , and
  - (iv)  $B \ominus A$ .

# *Thank you...*

## *Any questions??*



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