جامعة المستقبل
AL MUSTAQBAL UNIVERSITY

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Lecture (1)

FUNCTIONS AND THEIR GRAPHS

المادة : رياضيات

المرحلة : الاولى

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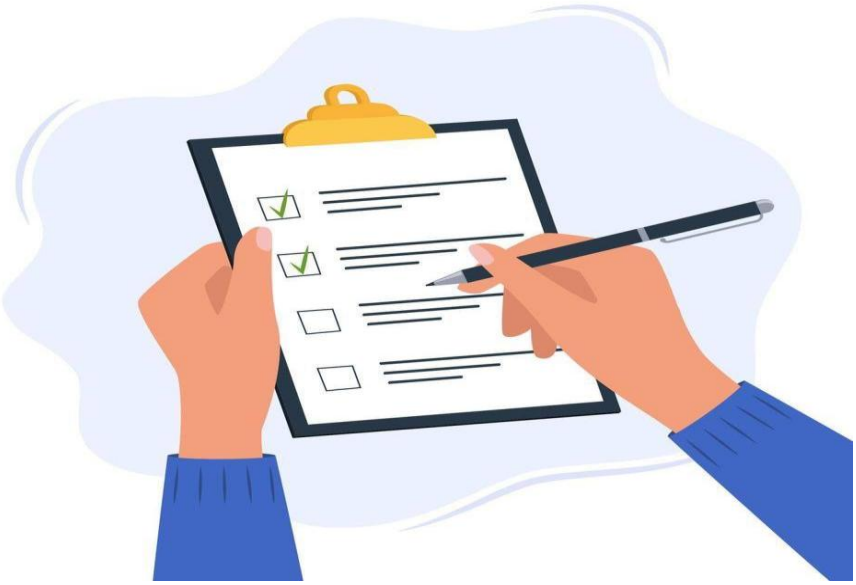

The General Aim

The general aim of a function is to describe the relationship between two quantities, where one depends on the other. It allows us to analyze, predict, and model real-life and mathematical situations accurately.

The Behavioral objectives

By the end of the lecture, the student will be able to:

- ✓ Define a function and identify its domain and range correctly.
- ✓ Distinguish between relations and functions using tables, formulas, and graphs.
- ✓ Evaluate a function for given values of the independent variable.
- ✓ Plot the graph of a given function accurately on the Cartesian plane.
- ✓ Interpret the graph of a function to describe relationships between variables.
- ✓ Determine whether a graph represents a function using the vertical line test.

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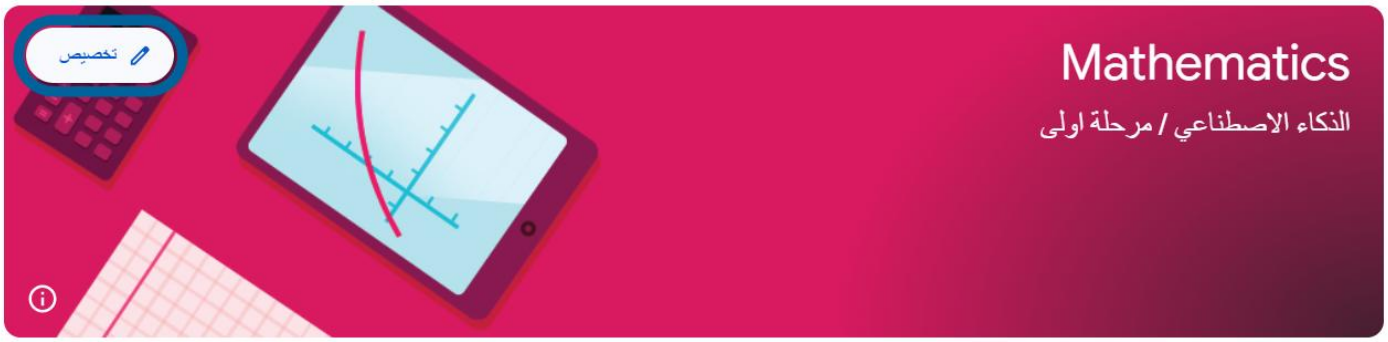

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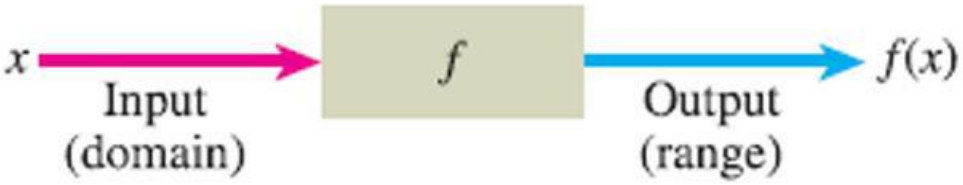
I. What is Function?

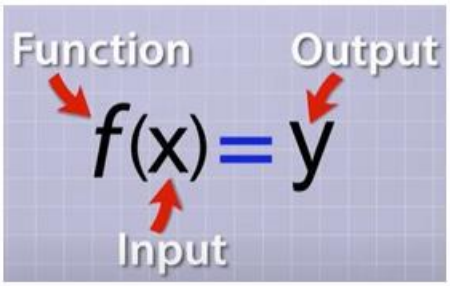
- Functions are a tool for describing the real world in mathematical terms.
- A function can be represented by an equation, a graph, a numerical table, or a verbal description.

A function takes an **input** (usually called x) and produces an **output** (usually called y).

We write it as:

$$y = f(x)$$





II. The Domain and Range

A function f consists of a set of inputs, a set of outputs, and a rule for assigning each input to exactly one output. The set of inputs is called Domain of the function. The set of output is called the Range of the function.

Example

For the function $f(x) = 3x^2 + 2x - 1$, evaluate

a. $f(-2)$

b. $f(\sqrt{2})$

c. $f(a+h)$

Solution

Substitute the given value for x in the formula for $f(x)$.

a. $f(-2) = 3(-2)^2 + 2(-2) - 1 = 12 - 4 - 1 = 7$

b. $f(\sqrt{2}) = 3(\sqrt{2})^2 + 2\sqrt{2} - 1 = 6 + 2\sqrt{2} - 1 = 5 + 2\sqrt{2}$

c. $$f(a+h) = 3(a+h)^2 + 2(a+h) - 1 = 3\left(a^2 + 2ah + h^2\right) + 2a + 2h - 1$$
$$= 3a^2 + 6ah + 3h^2 + 2a + 2h - 1$$

Example: If $f(x) = 2x^2 - 5x + 1$ and $h \neq 0$, evaluate $\dfrac{f(a+h) - f(a)}{h}$

Solution: We first evaluate $f(a+h)$ by replacing x by $a+h$ in the expression for $f(x)$:

$$f(a+h) = 2(a+h)^2 - 5(a+h) + 1$$
$$= 2(a^2 + 2ah + h^2) - 5(a+h) + 1$$
$$= 2a^2 + 4ah + 2h^2 - 5a - 5h + 1$$

Then we substitute into the given expression and simplify:

$$\frac{f(a+h)-f(a)}{h} = \frac{(2a^2+4ah+2h^2-5a-5h+1)-(2a^2-5a+1)}{h}$$

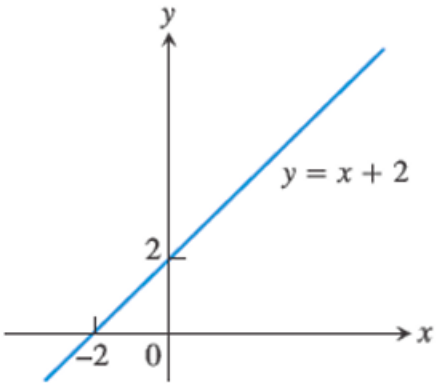
$$= \frac{2a^2+4ah+2h^2-5a-5h+1-2a^2+5a-1}{h}$$

$$= \frac{4ah+2h^2-5h}{h} = 4a+2h-5$$

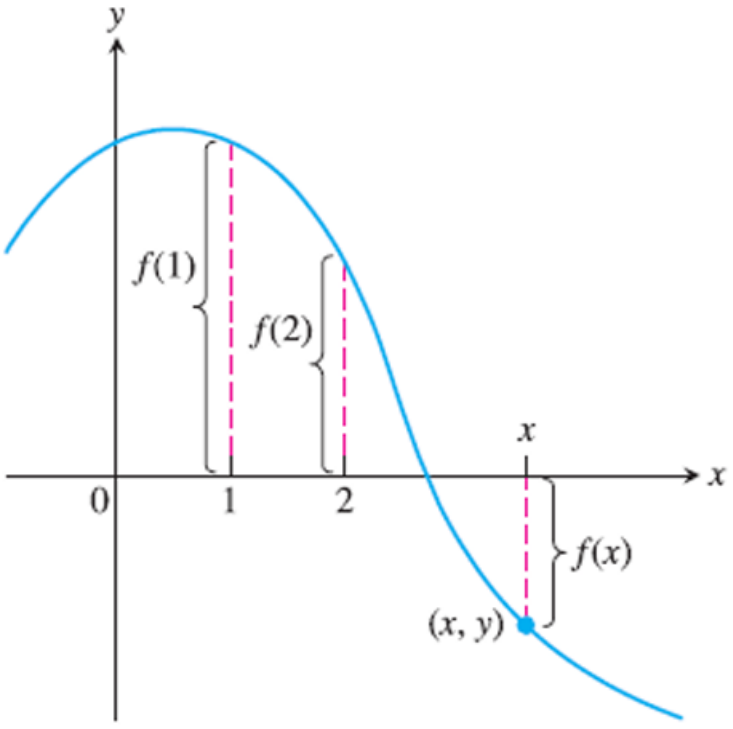
III. Graphs of Function

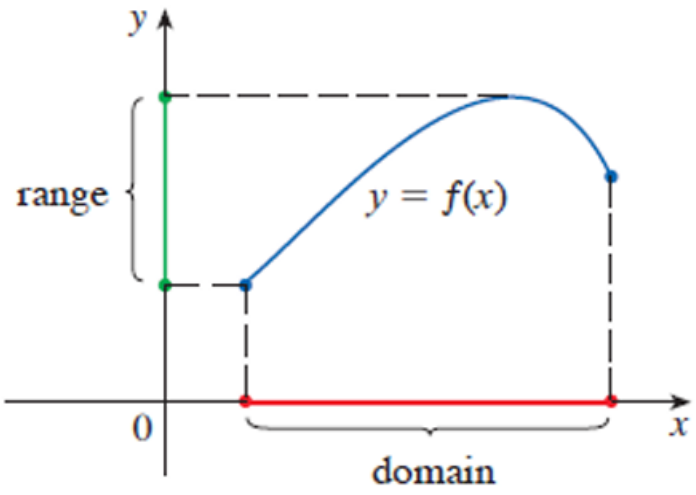
If f is a function with domain D, its graph consists of the points in the Cartesian plane whose coordinates are the input-output pairs for f. In set notation. the graph is:

$$\{(x, f(x)) \mid x \in D\}$$



The graph of $f(x) = x + 2$ is the set of points (x, y) for which y has the value $x + 2$.

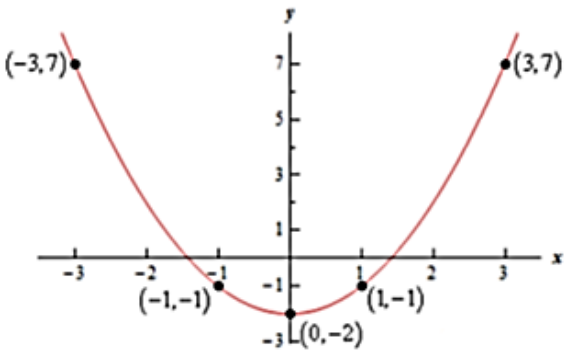




Example 1: Sketch the graph of the following function.

$f(x) = x^2 - 2$

x	$f(x)$	(x, y)
-3	7	$(-3, 7)$
-1	-1	$(-1, -1)$
0	-2	$(0, -2)$
1	-1	$(1, -1)$
3	7	$(3, 7)$

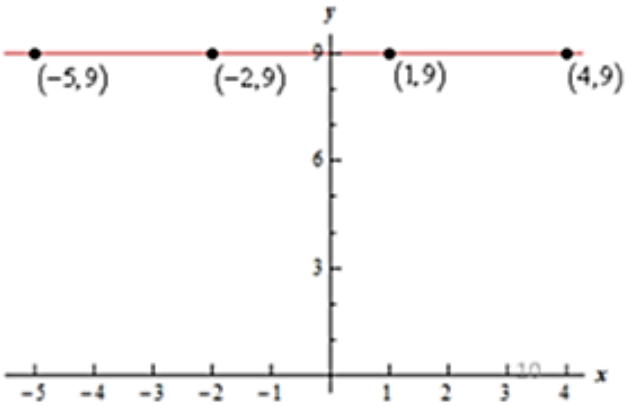


Domain is $(-\infty, \infty)$ *and*
Range is $[-2, \infty)$

Example 2: Sketch the graph of the function:

$f(x) = 9$

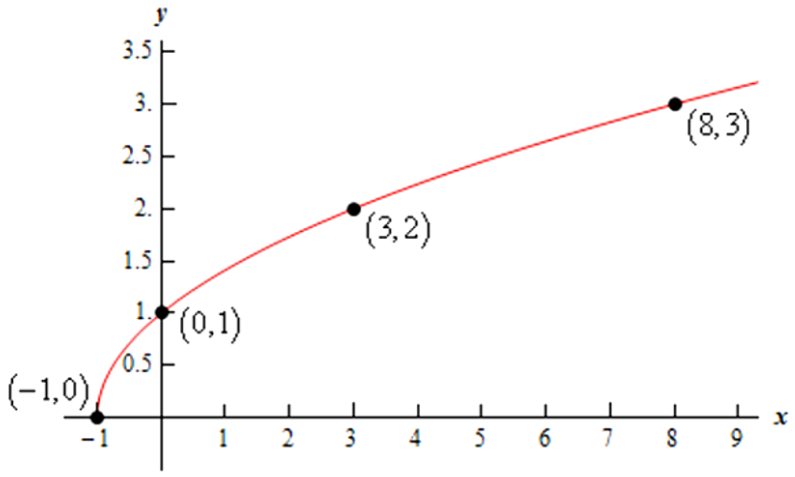
x	$f(x)$	(x, y)
-5	9	$(-5, 9)$
-2	9	$(-2, 9)$
1	9	$(1, 9)$
4	9	$(4, 9)$



Example 3: graph of the following function.

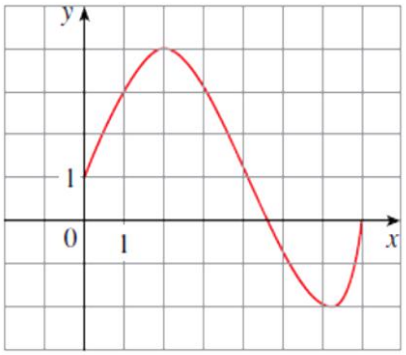
$f(x) = \sqrt{x+1}$

x	$f(x)$	(x, y)
-1	0	$(-1, 0)$
0	1	$(0, 1)$
3	2	$(3, 2)$
8	3	$(8, 3)$



Domain is $[-1, \infty)$ *and*
Range is $[0, \infty)$

EXAMPLE // The graph of a function f is shown in Figure shown.



(a) Find the values of $f(1)$ and $f(5)$.

(b) What are the domain and range of f?

SOLUTION

(a) We see from Figure the value of f at 1 is $f(1) = 3$.
(the point on the graph that lies above $x = 1$ is 3 units above the x-axis.)

When $x = 5$, the graph lies about 0.7 units below the x-axis,
so we estimate that $f(5) \approx -0.7$.

(b) We see that $f(x)$ is defined when $0 \leqslant x \leqslant 7$, so the domain of f is the closed interval $[0, 7]$. Notice that f takes on all values from -2 to 4, so the range of f is

$$\{y \mid -2 \leqslant y \leqslant 4\} = [-2, 4]$$

Example: Find the Domains and Ranges of each of all of the following

(a) $y = x^3 \quad -5 \leq x < 4$ (b) $y = x^4$ (c) $y = \dfrac{1}{(x-1)(x+2)} \quad 0 \leq x \leq 6$

Solution

(a) $y = x^3 \quad -5 \leq x < 4$
domain $-5 \leq x < 4$, range $-125 \leq y < 64$

(b) $y = x^4$
domain $-\infty < x < \infty$, range $0 \leq y < \infty$

(c) $y = \dfrac{1}{(x-1)(x+2)}, \quad 0 \leq x \leq 6$
domain $0 \leq x < 1$ and $1 < x \leq 6$,
range $-\infty < y \leq -0{\cdot}5, \; 0{\cdot}25 \leq y < \infty$

$(x-1)(x+2) \geq 0$
$x^2 + x - 2 \geq 0$

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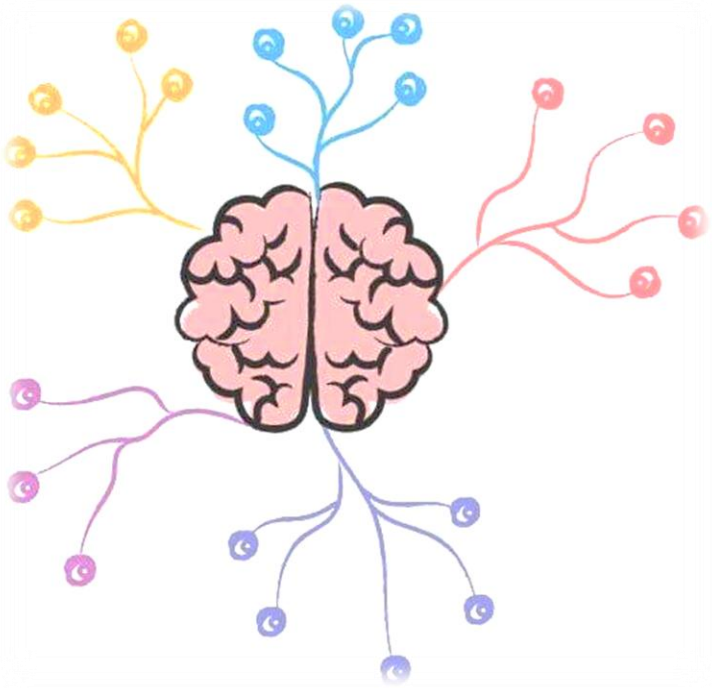
TASK:

- Sketch the following functions:

1. $f(x) = 2x - 3$
2. $f(x) = \frac{1}{2}x^2 - 4$



Note: The Answer must be sent to the Google Classroom

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