كـلـيـة الـعـلـوم

قـسـم الـذكـاء الاصـطـنـاعي

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Subject : Statistical Symbols

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Lecture: (2)

Statistical Symbols

The Summation Symbol

The summation symbol is denoted by $\sum$ and is used to represent the addition of a set of values. Mathematically, it can be expressed as the sum of the values of the variable YYY starting from the first observation to the last observation, that is:

$$\sum_{i=1}^{n} Y_i = Y_1 + Y_2 + \cdots + Y_n$$

For simplicity, it is often written as: $\sum Y_i$

Some Examples:

The partial sum of the observations 3, 4, and 5 is given by

$$\sum_{i=1}^{3} Y_i = Y_1 + Y_2 + Y_3$$

The **sum of squares** of all observations is denoted by: $\sum_{i=1}^{n} Y_i^2$

$$\sum_{i=1}^{n} Y_i^2 = Y_1^2 + Y_2^2 + \cdots + Y_n^2$$

The square of the sum of the observations is given by: $\left(\sum_{i=1}^{n} Y_i\right)^2$

$$\left(\sum Y_i\right)^2 = (Y_1 + Y_2 + \cdots + Y_n)^2$$

It should be noted that:

$$\left(\sum_{i=1}^{n} Y_i^2\right) \neq \left(\sum_{i=1}^{n} Y_i\right)^2$$

Sum of Products of Two Variables:

The sum of the products of two variables X and Y is expressed as: $\sum X_i Y_i$

$$\sum X_i Y_i = X_1 Y_1 + X_2 Y_2 + \cdots + X_n Y_n$$

Whereas the product of the sums of two variables is given by:

$$\left(\sum X_i\right)\left(\sum Y_i\right) = (X_1 + X_2 + \cdots + X_n)(Y_1 + Y_2 + \cdots + Y_n)$$

Clearly,

$$\left(\sum X_i\right)\left(\sum Y_i\right) \neq \sum X_i Y_i$$

Example

Suppose the values of variable Y are:

$Y_i = 3,9,6,2$

and the values of variable X are:

$X = 4,2,3,7$

Find the following:

$(\sum X_i)(\sum Y_i)$ ، $\sum X_i Y_i$ ، $(\sum Y_i)^2$ ، $\sum_{i=1}^{n} Y_i$ ، $\sum_{i=1}^{3} Y_i$ ، $\sum_{i=1}^{4} Y_i^2$

Solution

a) $\sum \boldsymbol{Y_i} = Y_1 + Y_2 + Y_3 + Y_4$

= 3 +9+6+ 3

= 20

b) $\sum_{\boldsymbol{i=2}}^{\boldsymbol{3}} \boldsymbol{Y_i} = Y_2 + Y_3$

= 9 + 9 =15

c) $\sum \boldsymbol{Y_i}^{\boldsymbol{2}} = Y_1^2 + Y_2^2 + Y_3^2 + Y_4^2$

= 9 +81+36+4

= 130

d) $\left(\sum \boldsymbol{Y_i}\right)^{\boldsymbol{2}} = (Y_1 + Y_2 + Y_3 + Y_4)^2$

= (3+9+6+ 2)

$=20^2$

= 400

e) $\sum \boldsymbol{X_i Y_i} = X_1Y_1 + X_2Y_2 + X_3Y_3 + X_4Y_4$

=(4)(3) + (2)(9)+ (3)(6)+(7)(2)

=62

f) $(\sum \boldsymbol{Y})$ **(**$\sum \boldsymbol{X}$)

$=(X_1 + X_2 + X_3 + X_4)(Y_1 + Y_2 + Y_3 + Y_4)$

=320

Useful Rules of Summation

1. If C is a constant, then:

$$\sum_{i=1}^{n} C = n.c$$

Proof

$$\sum_{i=1}^{n} C = C_1 + C_2 + \cdots + C_n$$

$$\sum CY_i = C \sum Y_i$$

$$\sum CY_i = CY_1 + CY_2 + \cdots + CY_n$$

$$= C(Y_1 + Y_2 + \cdots + Y_n)$$

$$= C \sum Y_i$$

2. The sum of two or more variables is equal to the sum of each variable separately:

$$\sum (X_i + Y_{i)} = \sum X_i + \sum Y_i$$

Proof

$$\sum (X_i + Y_{i)} = (X_1 + Y_1) + (X_2 + Y_2) + \cdots + (X_n + Y_1)$$

$$= (X_1 + X_2 + \cdots + X_n) + (Y_1 + Y_2 + \cdots + Y_n)$$

$$= \sum_{1}^{n} X_i + \sum_{1}^{n} Y_i$$

Distinguishing Between Similar Statistical Expressions

1. $\sum = \frac{X_i}{Y_i} = \frac{X_1}{Y_1} + \frac{X_2}{Y_2} + \cdots + \frac{X_n}{Y_n}$

2. $\frac{\sum X_i}{\sum Y_i} = \frac{X_1 + X_2 + \cdots + X_n}{Y_1 + Y_2 + \cdots + Y_n}$

3. $\sum (X_i - 3) = \sum X_i - n(3)$

which is different from $\sum X_i - 3$

<u>Example</u> : Given the values

$X_i = 2,6,3,1$

$Y_i = 3,9,6,2$

Find the following expressions:

1. $\sum (Y_i - X_i)^2$
2. $\sum (X_i - 3)(Y_i - 5)$
3. $\sum X_i Y_i^2$
4. $\sum (Y_i - 3)$
5. $\sum Y_i - 3$
6. $\sum \frac{X_i + 2}{Y_i}$
7. $\frac{\sum (X_i + 2)}{\sum Y_i}$
8. $\sum Y_i^2 - \frac{(\sum Y_i)^2}{n}$

9. $\sum X_i Y_i - \frac{(\sum X_i)(\sum Y_i)}{\text{n}}$

solution

1. $\sum (\boldsymbol{Y_i} - \boldsymbol{X_i})^{\boldsymbol{2}} = (Y_1 - X_1)^2 + (Y_2 - X_2)^2 + \cdots + (Y_n - X_n)^2$

$$= (3-2)^2 + (9-6)^2 + (6-3)^2 + (2-1)^2$$

$$= 1^2 + 3^2 + 3^2 + 1^2$$

$$= 20$$

أو

$\sum (Y_i - X_i)^2 = \sum (Y_i{}^2 - 2X_iY_i + X_i{}^2)$

$= \sum Y_i{}^2 - 2\sum X_iY_i + \sum X_i{}^2$

$= (3+9+6+2)^2 - 2(2+6+3+1)(3+9+6+2) + \cdots$

2. $\sum (\boldsymbol{X_i} - \boldsymbol{3})(\boldsymbol{Y_i} - \boldsymbol{5})$

$= (X_1 - 3)(Y_1 - 5) + (X_2 - 3)(Y_2 - 5) + (X_3 - 3)(Y_3 - 5) + (X_4 - 3)(Y_4 - 5)$

$= (2-3)(3-5) + (6-3)(9-5) + (3-3)(6-5) + (1-3)(2-5)$

$= (-1)(-2) + (3)(4) + 0(1) + (-2)(-3)$

$= 2 + 12 + 0 + 6$

$= 20$

or

$\sum (X_i - 3)(Y_i - 5)$

$= \sum (X_i Y_i - 5X_i - 3\, Y_i + 15)$

$= \sum X_i Y_i - 5 \sum X_i - 3 \sum Y_i + 4(15)$

$= 80 - 5(12) - 3(20) + 60$

$= 20$

3. $\sum \boldsymbol{X_i Y_i}^{\mathbf{2}} \;\; = X_1 {Y_1}^2 + X_2 {Y_2}^2 + X_3 {Y_3}^2 + X_4 {Y_4}^2$

$= 2(3)^2 + 6(9)^2 + 3(6)^2 + 1(2)^2$

$= 616$

4. $\sum (\boldsymbol{Y_i} - \mathbf{3}) = \sum Y_i - \sum (3)$

$= \sum Y_i - 4(3)$

$= (3 + 9 + 6 + 2) - 12$

$= 20 - 12 = 8$

5. $\sum \boldsymbol{Y_i} - \mathbf{3} = 20 - 3 = 17$

$$\mathbf{6.} \sum \frac{\boldsymbol{X_i + 2}}{\boldsymbol{Y_i}} = \frac{X_1 + 2}{Y_1} + \frac{X_2 + 2}{Y_2} + \frac{X_3 + 2}{Y_3} + \frac{X_4 + 2}{Y_4}$$

$$= \frac{2+2}{3} + \frac{6+2}{9} + \frac{3+2}{6} + \frac{1+2}{2}$$

$$= \frac{4}{3} + \frac{8}{9} + \frac{5}{6} + \frac{3}{2}$$

$$= \frac{164}{36}$$

$$\mathbf{7.} \quad \frac{\sum(\boldsymbol{X_i + 2})}{\sum \boldsymbol{Y_i}} = \frac{\sum X_i + (n)(2)}{\sum Y_i}$$

$$= \frac{12+8}{20} = 1$$

$$\mathbf{8.} \sum \boldsymbol{Y_i}^{\mathbf{2}} - \frac{(\sum \boldsymbol{Y_i})^{\mathbf{2}}}{\mathbf{n}} = \left(Y_1{}^2 + Y_2{}^2 + Y_3{}^2 + Y_4{}^2\right) - \frac{Y_1 + Y_2 + Y_3 + Y_4}{4}$$

$$= 3^2 + 9^2 + 6^2 + 2^2 - \frac{(3+9+6+2)}{4}$$

$$= 130 + \frac{(20)^2}{4} = 130 - 100 = 30$$

$$\mathbf{9.} \ \sum \boldsymbol{X_i Y_i} - \frac{(\sum \boldsymbol{X_i})(\sum \boldsymbol{Y_i})}{\mathbf{n}}$$

$$= X_1Y_1 + X_2Y_2 + X_3Y_3 + X_4Y_4 - \frac{(\sum X_i)(\sum Y_i)}{\mathrm{n}}$$

$$= 2(3) + 6(9) + 3(6) + 1(2) - \frac{(12)(20)}{4}$$

$$= 80 - \frac{(12)(20)}{4}$$

$$= 20$$