Al-Mustaqbal University
College of Science
Artificial Intelligence Department
First Stage

جامعة المستقبل
AL MUSTAQBAL UNIVERSITY

كلية العلوم
قسم الذكاء الاصطناعي

Lecture (4)

LOG FUNCTIONS

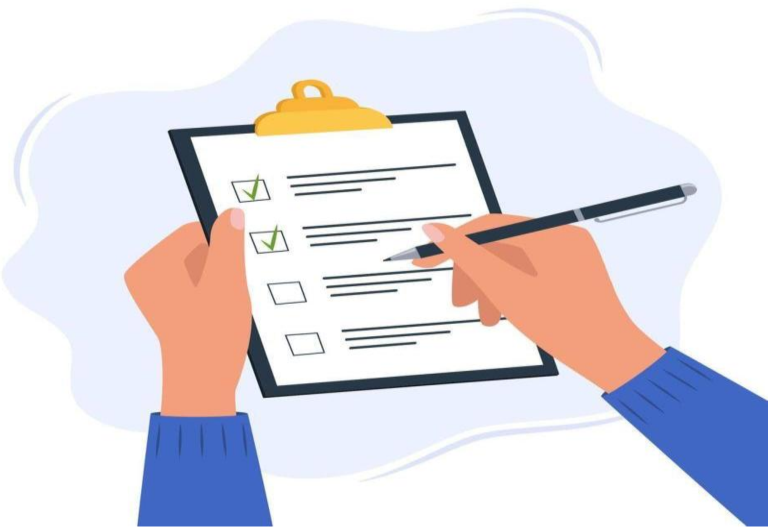
المادة : الرياضيات

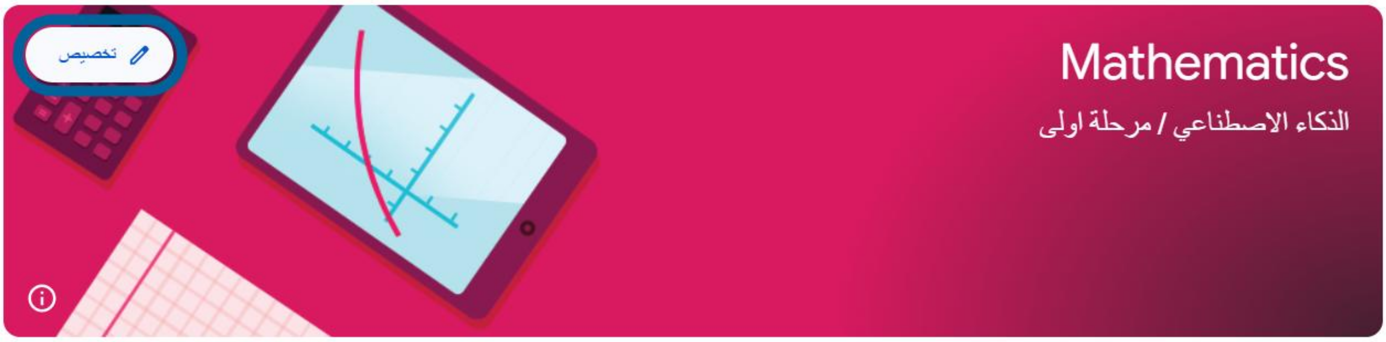
المرحلة : الاولى

اسم الاستاذ: م.م ريام ثائر احمد

Content

- Introduction
- Why do we study logarithms?
- What is a logarithm?
- The first law of logarithms
- The Second law of logarithms
- The Third law of logarithms
- The logarithm of 1





k7kgjyvz

https://classroom.google.com/c/ODM3NjM4NTg2MzY3?cjc=k7kgjyvz

I. Introduction

In this unit we are going to be looking at logarithms. However, before we can deal with logarithms, we need to revise indices. This is because logarithms and indices are closely related, and in order to understand logarithms a good knowledge of indices is required.

We know that

$$16 = 2^4$$

Here, the number **4** is the power. Sometimes we call it an exponent. Sometimes we call it an index. In the expression 2^4, the number **2** is called the base.

Example1:

We know that **64** = 8^2.

2 is the power, or exponent. The number **8** is the base.

II. Why do we study logarithms?

We study logarithms because they help simplify calculations, solve exponential equations, and explain scientific phenomena

Example 2:

16×8 can be written $2^4 \times 2^3$

This equals

$$2^7$$

Using the rules of indices which tell us to add the powers 4 and 3 to give the new power, 7. What was a multiplication sum has been reduced to an addition sum.

Example 3:

$16 \div 8$ can be written $2^4 \div 2^3$

This equals

2^1 or simply 2

Using the rules of indices which tell us to subtract the powers 4 and 3 to give the new power, 1.

Reasons for studying logarithms:

1. **Simplify calculations:**

A logarithm turns multiplication into addition and division into subtraction.

Example 4:

Log (100×10) =log (100) + log (10)

2. **Solve exponential equations:**

Used to find unknown exponents.

Example 5:

If $2^x = 8$, then $x = \log_2(8) = 3$

3. Computers and data:

Logarithms help in analyzing algorithms (like $O(\log n)$) and compressing data ranges.

III. What is a logarithm?

Consider the expression $\mathbf{16 = 2^4}$. Remember that 2 is the base, and 4 is the power. An alternative, yet equivalent, way of writing this expression is $\mathbf{\log_2 16 = 4}$. This is stated as 'log to base 2 of 16 equals 4'. We see that the logarithm is the same as the power or index in the original expression. It is the base in the original expression which becomes the base of the logarithm.

The two statements

$$\mathbf{16 = 2^4} \quad , \quad \mathbf{\log_2 16 = 4}$$

are equivalent statements. If we write either of them, we are automatically implying the other.

Example 6:

If we write down that $\mathbf{64 = 8^2}$ then the equivalent statement using logarithms is $\mathbf{\log_8 64} = 2$.

Example 7:

If we write down that $\mathbf{\log_3 27 = 3}$ then the equivalent statement using powers is $\mathbf{3^3 = 27}$.

Key Point

If $x = a^n$ then equivalently $\log_a x = n$

Key Point

$$\log_a a = 1$$

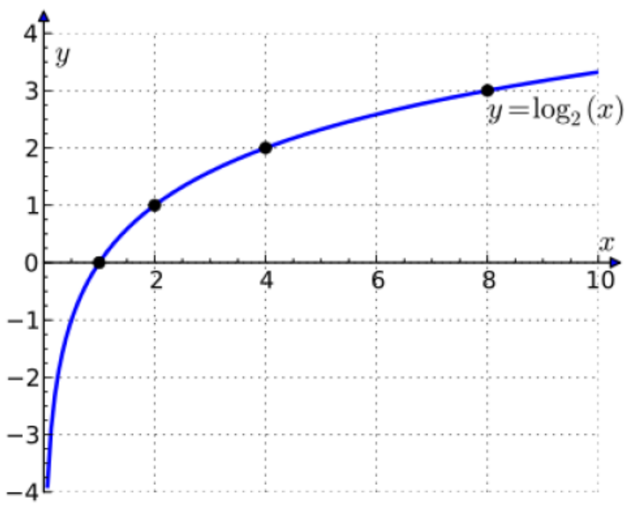
Because $10 = 10^1$ we can write the equivalent logarithmic form $\log_{10} 10 = 1$.

Similarly, the logarithmic form of the statement $2 = 2^1$ is:

$\log_2 2 = 1$

In general, for any base a, $a = a^1$ and so $\log_a a = 1$.

Here is the graph of $y = \log_2 x$.



Activity:

1. Write the following using logarithms instead of powers
   a) $8^2 = 64$ b) $3^5 = 243$ c) $2^{10} = 1024$ d) $5^3 = 125$

2. Determine the value of the following logarithms
   a) $\log_3 9$ b) $\log_2 32$ c) $\log_5 125$ d) $\log_{10} 10000$
   e) $\log_4 64$ f) $\log_{25} 5$ g) $\log_8 2$ h) $\log_{81} 3$

IV. The first law of logarithms

Suppose

$$x = a^n \quad \text{and} \quad y = a^m$$

then the equivalent logarithmic forms are

$$\log_a x = n \quad \text{and} \quad \log_a y = m \qquad \ldots.1$$

Using the first rule of indices

$$xy = a^n * a^m = a^{n+m}$$

Now the logarithmic form of the statement $xy = a^{n+m}$ is:

$$\log_a xy = n + m$$

But

$$n = \log_a x \quad \text{and} \quad m = \log_a y$$

from (1) and so putting these results together we have:

Key Point

$$\log_a xy = \log_a x + \log_a y$$

So, if we want to multiply two numbers together and find the logarithm of the result, we can do this by adding together the logarithms of the two numbers. This is the first law.

V. The second law of logarithms

Key Point

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

VI. The Third law of logarithms

Key Point

$$\log_a x^m = m \log_a x$$

VII. The logarithm of 1

Key Point

$$\log_a 1 = 0$$

NOTE:

$$\log_a 10 = 1$$

$$\log_a 100 = 2$$

$$\log_a 1000 = 3$$