Al-Mustaqbal University
College of Science
Artificial Intelligence Department
First Stage

جامعة المستقبل
AL MUSTAQBAL UNIVERSITY

كلية العلوم
قسم الذكاء الاصطناعي

Lecture (2)

REAL AND COMPLEX NUMBER

المادة : الذكاء الاصطناعي

المرحلة : الاولى

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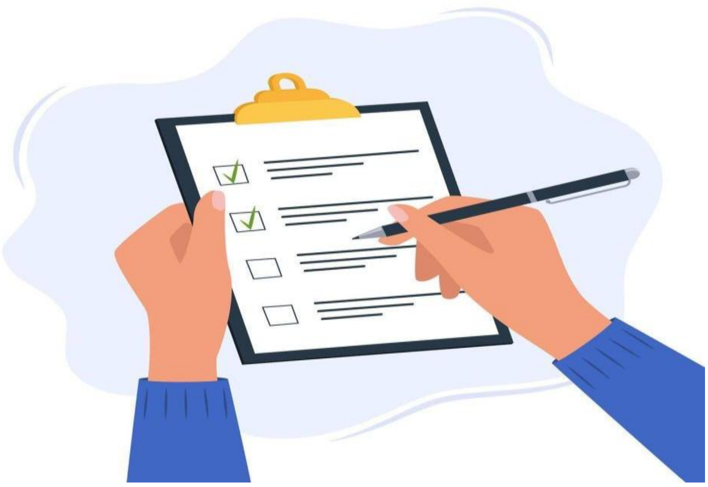

The General Aim

The general aim of studying complex analysis is to explore the properties of functions of a complex variable and apply their powerful theoretical and practical tools to solve problems in mathematics, physics, and engineering.

The Behavioral objectives

By the end of the lecture, the student will be able to:

- ✓ Define a complex number and state its general form.
- ✓ Explain the meaning of the real part and imaginary part of a complex number.
- ✓ Perform addition and subtraction of complex numbers correctly.
- ✓ Determine the conjugate of a given complex number.
- ✓ Divide two complex numbers using the conjugate method.
- ✓ Distinguish between the different operations on complex numbers and analyze their results.

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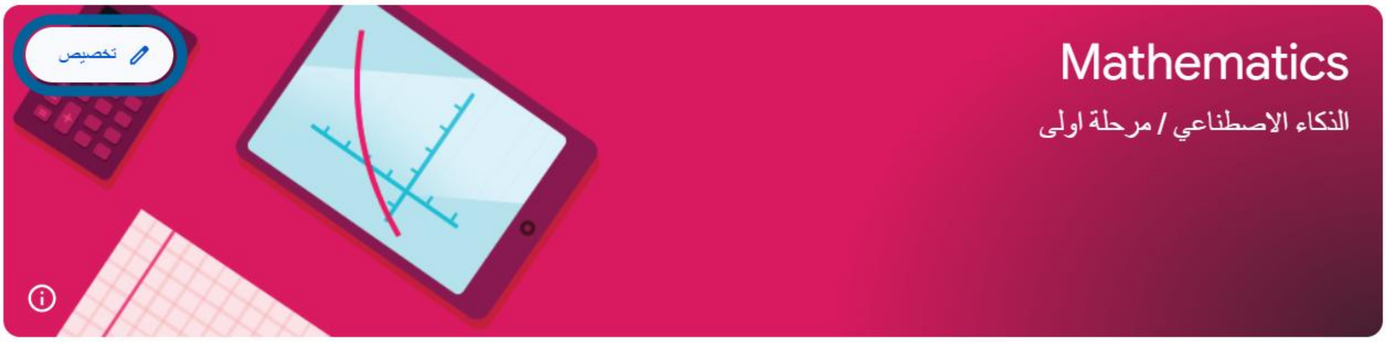

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I. Complex analysis importance

Complex analysis has not only transformed the world of mathematics, but surprisingly, we find its application in many areas of physics and engineering.

For example, we can use complex numbers to describe the behavior of the electromagnetic field.

In atomic systems, which are described by quantum mechanics, complex numbers and complex functions play a central role.

II. What is a Complex Number?

It's a solution of the equation:

$$x^2 + 1 = 0$$

$$x = \pm\sqrt{-1}$$

So we see that $i^2 = -1$, and we can solve equations like $x^2 + 1 = 0$.

III. The Algebra of Complex Numbers

More general complex numbers can be written down. In fact, using real numbers X and Y we can form a complex number:

$$Z = x + iy$$

We call X the real part of the complex number Z and refer to Y as the imaginary part of Z.

A. Addition of Complex number

If you have two complex numbers:

$$Z_1 = x + iy \quad \textit{and} \quad Z_2 = x + iy$$

where x is the real part, and y is the imaginary part.

then their sum is:

$$Z_1 + Z_2 = (x_1 + x_2) + (y_1 + y_2)i$$

Example:

If $Z_1 = 3 + 2i$, $Z_2 = 5 + 7i$ Find the addition.

Sol//

$$Z_1 + Z_2 = (3 + 5) + (2 + 7)i$$

$$Z_1 + Z_2 = 8 + 9i$$

Activity//

Find the Sum of: $\boldsymbol{Z_1 = 4(2 + 3i), \quad Z_2 = 5(6 + 3i)}$

B. Subtraction of Complex number

If you have two complex numbers:

$$Z_1 = x_1 + iy_1 \quad and \quad Z_2 = x_2 + iy_2$$

where x is the real part, and y is the imaginary part.

then their sub is:

$$Z_1 - Z_2 = (x_1 + x_2) - (y_1 + y_2)i$$

Example:

If $Z_1 = 3 - 2i$, $Z_2 = 5 - 7i$ Find the Sub.

Sol//

$$Z_1 - Z_2 = (3 - 5) - (-2 - 7)i$$

$$Z_1 - Z_2 = -2 + 5i$$

C. Multiplication of Complex number

If you have two complex numbers:

$$Z_1 = x_1 + iy_1 \quad and \quad Z_2 = x_2 + iy_2$$

where x is the real part, and y is the imaginary part.

then their multiplication is:

$$Z_1 * Z_2 = (x_1 + iy_1) * (x_2 + iy_2)$$

Expanding using distributive law:

$$= x_1x_2 + x_1y_2i + x_2y_1i - y_1y_2$$

$$= (x_1x_2 - y_1y_2) + (x_1y_2 + x_2y_1)i$$

Example:

If $Z_1 = 2 + 3i$, $Z_2 = 4 + 5i$ Find the Multiplication.

Sol//

$$Z_1 * Z_2 = (2 + 3i) * (4 + 5i)$$

$$= (2 * 4 + 2 * 5i + 3i * 4 + 3i * 5i)$$

$$= (8 + 10i + 12i - 15)$$

$$= -7 + 22i$$

D. Division of Complex number

If you have two complex numbers:

$$Z_1 = a + ib \quad \textit{and} \quad Z_2 = m + in$$

where x is the real part, and y is the imaginary part.

then their division is:

$$\frac{c}{k} = \frac{a+ib}{m+in}$$

$$= \frac{a+ib}{m+in}\frac{(m-in)}{(m-in)}$$

$$= \frac{am+ibm-ian+bn}{m^2+n^2}$$

$$= \frac{am+bn}{m^2+n^2} + i\frac{bm-an}{m^2+n^2}$$

Example:

If $Z_1 = 1 + 3i$, $Z_2 = 2 - 6i$ Find the Division.

Sol//

$$\frac{1+3i}{2-6i} = \frac{(1+3i)\,(2+6i)}{(2-6i)\,(2+6i)} = \frac{2+6i+6i+18i^2}{2^2+6^2} = \frac{-16+12i}{40} = \frac{-2}{5} + \frac{3}{10}i.$$

IV. Complex Conjugate

For a complex number:

$$Z = x + iy$$

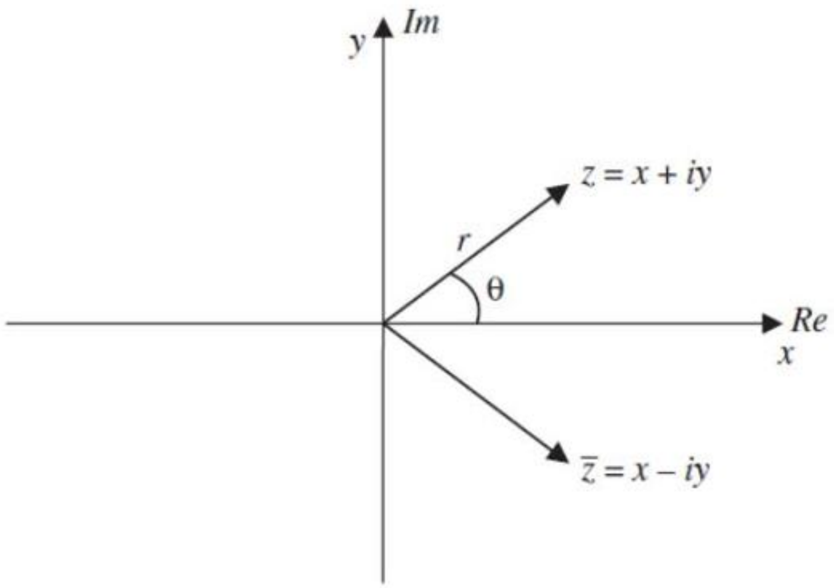
where x is the real part and y is the imaginary part, its **complex conjugate** is:

$$\overline{Z} = x - yi$$

The conjugate keeps the real part the same but changes the sign of the imaginary part.

Note that: $\bar{\bar{Z}} = Z = \overline{x - iy} = x + iy$

V. Graphical Representation of Complex Number



The complex plane, showing $z = x + iy$ and its complex conjugate as vectors.

VI. The Polar Representation

Let z = x + iy is the Cartesian representation of a complex number.

To write down the polar representation, we begin with the definition of the polar coordinates (r,θ):

$$x = r\cos\theta \quad , \quad y = r\sin\theta$$

$$r = \sqrt{x^2 + y^2}$$

We can write Z as:

$$Z = x + iy = r\cos\theta + r\sin\theta$$

$$= r(\cos\theta + i\sin\theta)$$

Very important complex transformation:

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2} \quad , \quad \sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

The modulus of a complex variable z is given by:

$$|z|^2 = x^2 + y^2 \quad \Rightarrow |z| = \sqrt{x^2 + y^2}$$

Note that r > 0 and that we have

$$tan = \frac{y}{x}$$

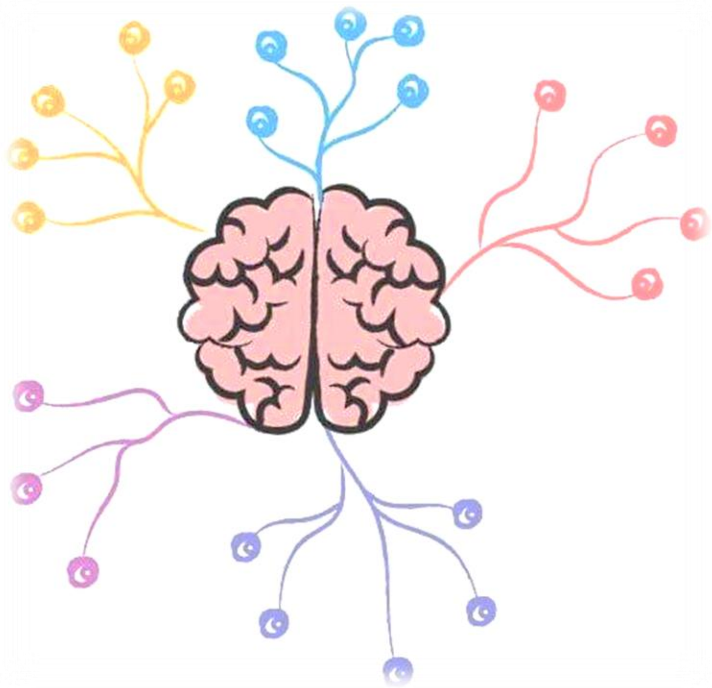
as a means to convert between polar and Cartesian representations.

The value of 0 for a given complex number is called the argument of z or arg z.

TASK:

- In Group, Calculate in the form $x + iy$, the following complex numbers:

1. $(1 + 3i) + (2 - 6i)$
2. $(1 + 3i) - (2 - 6i)$
3. $(1 + 3i)\ (2 - 6i)$



Note: The Answer must be sent to the Google Classroom