Fuzzy Logic

Lecture 3:

Operations on Fuzzy set

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Google class room

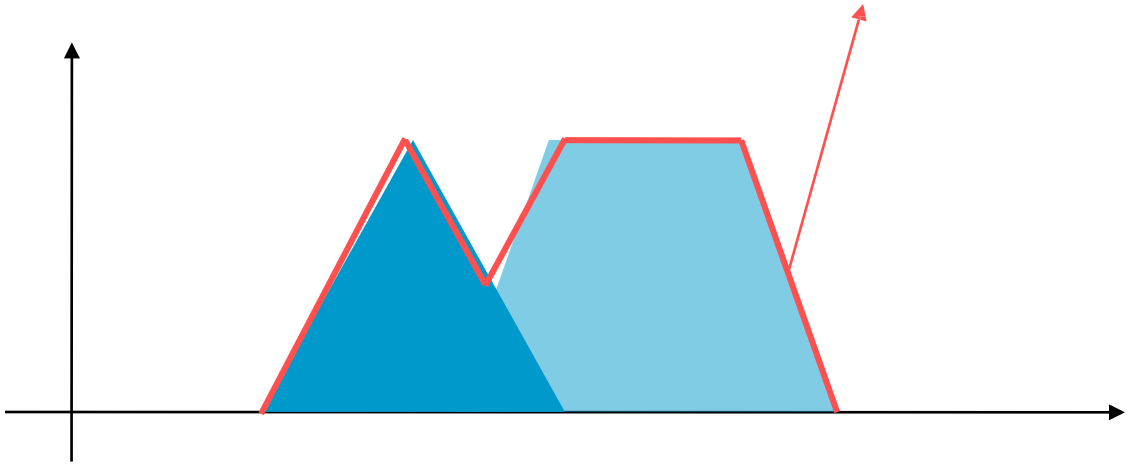
Logical Operations on Fuzzy set

- **Fuzzy Intersection**
- **Fuzzy Union**
- **Complement**

Logical Operations on Fuzzy set

- Union: The union the two sets *A* and *B* ($A \cup B$) can be defined by the membership function $\mu_{\cup}(x)$

$$\mu_{\cup}(x)=\max(\mu_A(x),\mu_B(x)),\ x \in X$$



Fuzzy Set Operations

Union

- Union of 2 sets is comprised of those elements that belong to one or both sets.

$$\mu_{A \vee B}(X) = \max(\mu_A(x), \mu_B(x)) \quad \forall x \in X$$

- Example:

  Tall = {0/5, 0.2/5.5, 0.5/6, 0.8/6.5, 1/7}

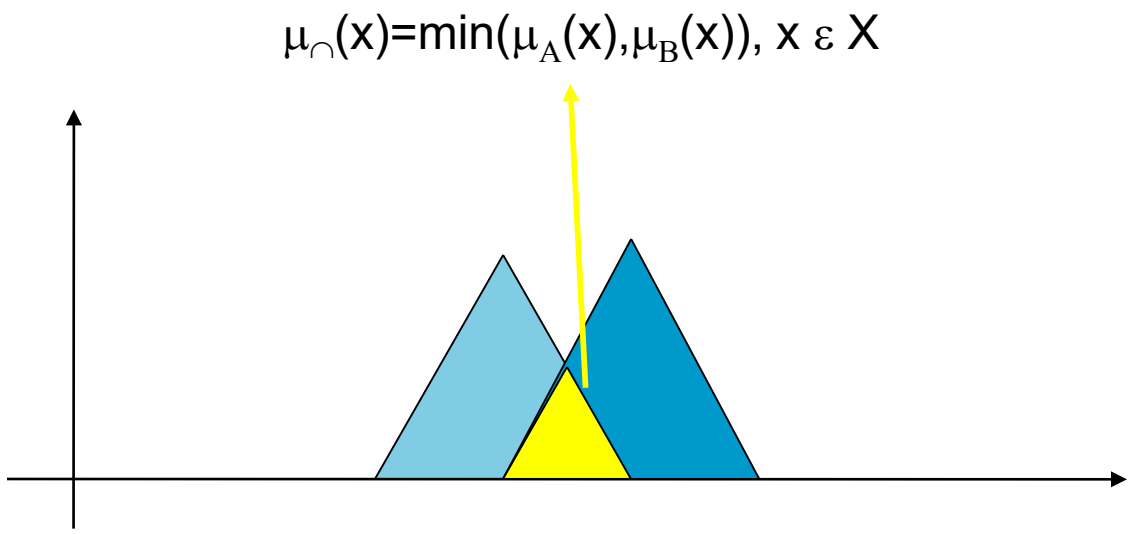
  Short = {1/5, 0.8/5.5, 0.5/6, 0.2/6.5, 0/7}

  $\mu_{\text{tall} \vee \text{short}}$ =

- Attains its highest vales at the limits and lowest at the middle.
- Tall or short can mean not medium

Logical Operations on Fuzzy set

- Intersection: the intersection of two sets *A* and *B* ($A \cap B$) can be defined by the membership function $\mu_{\cap}(x)$



Fuzzy Set Operations

Intersection

- In classical set theory, intersection of 2 sets contains elements common to both.
- In fuzzy sets, an element may be partially in both sets.

$$\mu_{A \wedge B}(X) = \min(\mu_A(x), \mu_B(x)) \qquad \forall x \in X$$

- Example:

Tall = {0/5, 0.2/5.5, 0.5/6, 0.8/6.5, 1/7}

Short= {1/5, 0.8/5.5, 0.5/6, 0.2/6.5, 0/7}

$\mu_{\text{tall} \wedge \text{short}}$ =Tall and short can mean medium

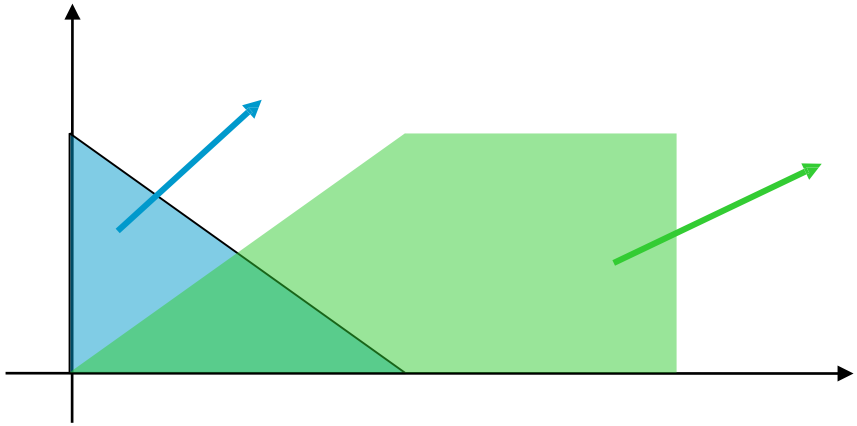
Highest at the middle and lowest at both end.

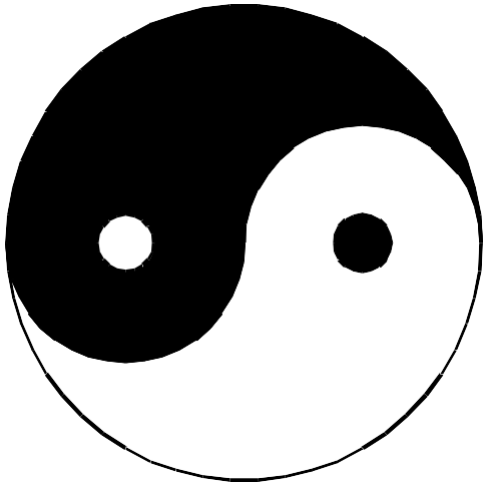
Logical Operations on Fuzzy set

- Complement: the complement of a fuzzy set *A* can be defined by the membership function $\mu_{\neg A}(x)$

$$\mu_{\neg A}(x)$$

$$\mu_{\neg A}(x) = 1 - \mu_A(x)$$





Fuzzy Set Operations

Complementation (Not)

- Find complement ~A by using the following operation:

$$\mu_{\sim A}(x) = 1 - \mu_A(x)$$

- Example:

  Short = {1/5, 0.8/5.5, 0.5/6, 0.2/6.5, 0/7}

  Not short = { /5, /5.5, /6, /6.5, /7}

Example 2

- Consider two fuzzy subsets of the set *X*, *X* = {a, b, c, d, e }

  referred to as *A* and *B*

  A = {1/a, 0.3/b, 0.2/c 0.8/d, 0/e}

  and

  B = {0.6/a, 0.9/b, 0.1/c, 0.3/d, 0.2/e}

Find out the Support, Cardinality, union, intersection and complement.

Solution

2

- Support:

  $supp(A)$ = {a, b, c, d }

  $supp(B)$ = {a, b, c, d, e }

- Cardinality:

  $card(A)$ = 1+0.3+0.2+0.8+0 = 2.3

  $card(B)$ = 0.6+0.9+0.1+0.3+0.2 = 2.1

- Complement:

  A = {1/a, 0.3/b, 0.2/c 0.8/d, 0/e}

  $\neg A$ = {0/a, 0.7/b, 0.8/c 0.2/d, 1/e}

- Union:

  $A \cup B$ = {1/a, 0.9/b, 0.2/c, 0.8/d, 0.2/e}

- Intersection:

  $A \cap B$ = {0.6/a, 0.3/b, 0.1/c, 0.3/d, 0/e}

Example 3

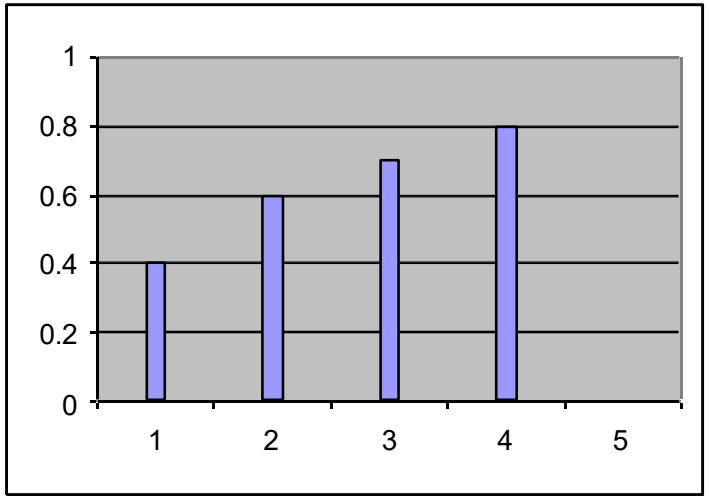
Suppose we have the following (discrete) fuzzy sets:

$$A = 0.4/1+0.6/2+0.7/3+0.8/4$$

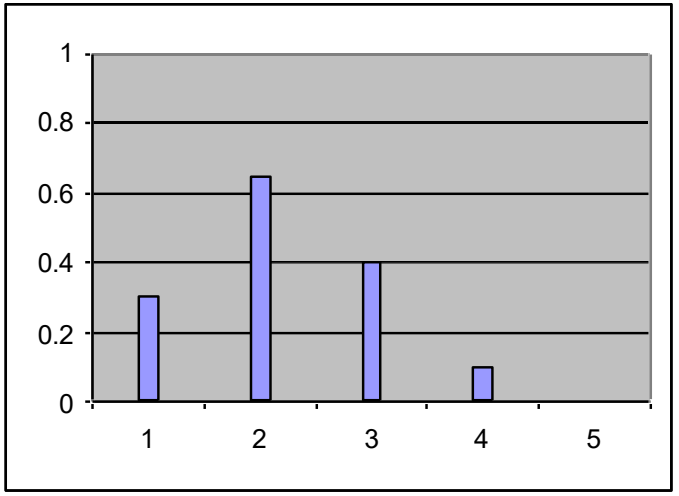
$$B = 0.3/1+0.65/2+0.4/3+0.1/4$$

a) Represent A and B fuzzy sets graphically
b) Calculate the of union of the set A and set B
c) Calculate the intersection of the set A and set B
d) Calculate the complement of the union of A and B

Solution (a)



A



B

Solution

(b)3The union of the fuzzy sets *A* and *B*

$=0.4/1+0.65/2+0.7/3+0.8/4$

(c) The intersection of the fuzzy sets *A* and *B*

$=0.3/1+0.6/2+0.4/3+0.1/4$

(c) The complement of the fuzzy set A

$=0.6/1+0.4/2+0.3/3+0.2/4$

Example 4

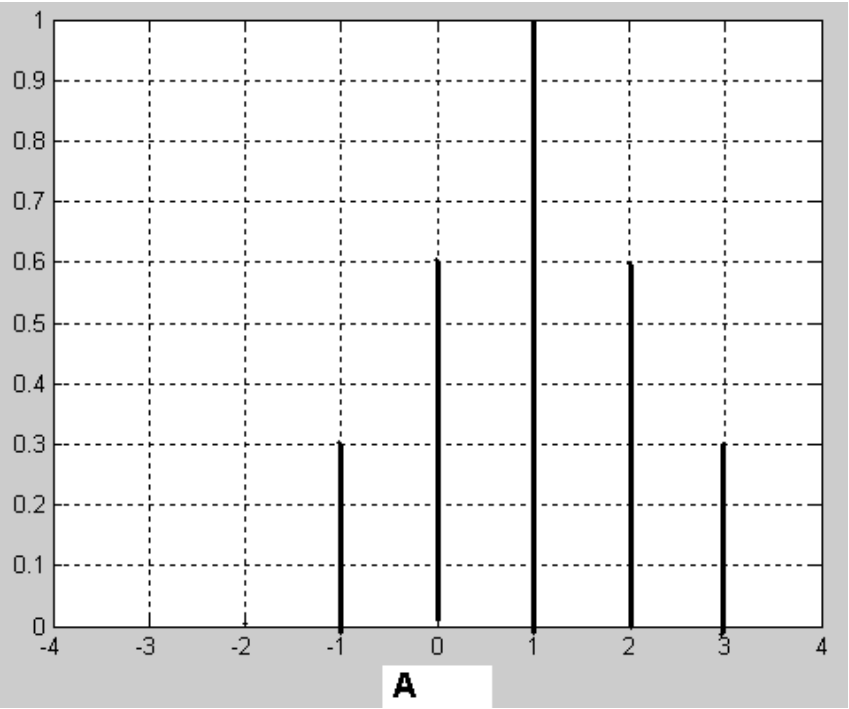
Given two fuzzy sets A and B

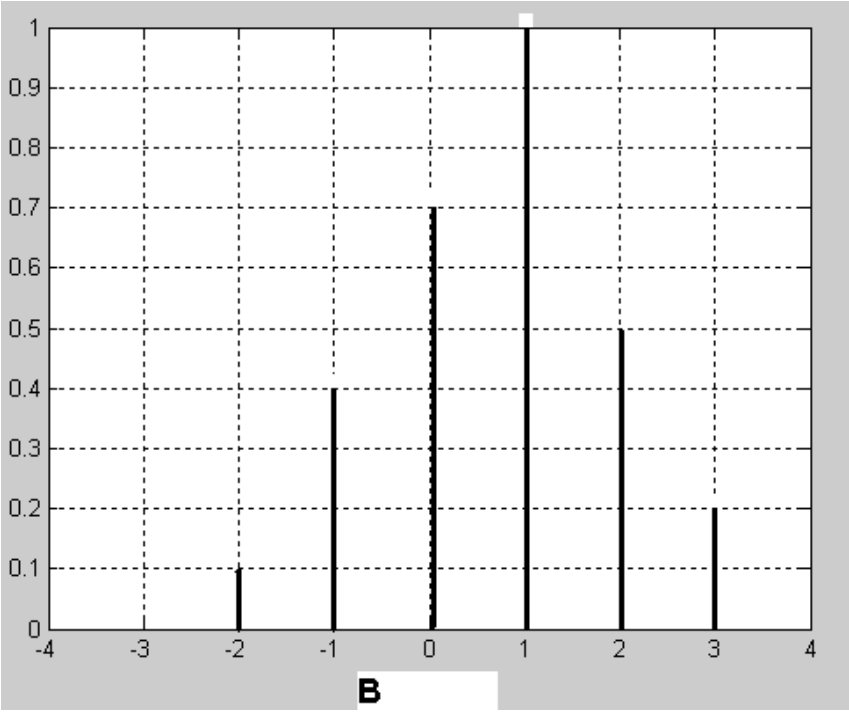
a. Represent A and B fuzzy sets graphically
b. Calculate the of union of the set A and set B
c. Calculate the intersection of the set A and set B
d. Calculate the complement of the union of A and B

$$A = 0.0/-2 + 0.3/-1 + 0.6/0 + 1.0/1 + 0.6/2 + 0.3/3 + 0.0/4$$

$$B = 0.1/-2 + 0.4/-1 + 0.7/0 + 1.0/1 + 0.5/2 + 0.2/3 + 0.0/4$$

Solution (a)


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-4
-3
-2
-1
0
1
2
3
4
A


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-4
-3
-2
-1
0
1
2
3
4
B

Solution 4

b

$$\text{Union} = \max(A, B) = 0.1/-2 + 0.4/-1 + 0.7/0 + 1.0/1 + 0.6/2 + 0.3/3 + 0.0/4$$

c

$$\text{Intersection} = \min(A, B) = 0.0/-2 + 0.3/-1 + 0.6/0 + 1.0/1 + 0.5/2 + 0.2/3 + 0.0/4$$

d

$$\text{Complement of (b)} = 1 - \max(A, B) = 0.9/-2 + 0.6/-1 + 0.3/0 + 0.0/1 + 0.4/2 + 0.7/3 + 1.0/4$$

Math Operations on Fuzzy Set

- $kA = \{k\mu_A(x), \forall x \in X\}$

  Let k =0.5, and

  A = {0.5/a, 0.3/b, 0.2/c, 1/d}

  then

  kA = {0.25/a, 0.15/b, 0.1/c, 0.5/d}

- $A^m = \{\mu_A(x)^m, \forall x \in X\}$

  Let m =2, and

  A = {0.5/a, 0.3/b, 0.2/c, 1/d}

  then

  A^m = {0.25/a, 0.09/b, 0.04/c, 1/d}

- …

Example 5

There are two fuzzy subsets of the set X = {a, b, c, d, e }:
A = {1/a, 0.3/b, 0.2/c 0.8/d, 0/e} and B = {0.6/a, 0.9/b, 0.1/c, 0.3/d, 0.2/e}.

Find the kA, A^m and α-cut. K=0.5 and m=2

Solution 5

- ***kA*:**
  for k=0.5
  kA = {0.5/a, 0.15/b, 0.1/c, 0.4/d, 0/e}

- ***A^m*:**
  for m=2
  A^a = {1/a, 0.09/b, 0.04/c, 0.64/d, 0/e}

- ***α*-cut:**
  $A_{0.2}$ = {a, b, c, d}
  $A_{0.3}$ = {a, b, d}
  $A_{0.8}$ = {a, d}
  A_1 = {a}

Example 6

$A = \{0.2/a, 0.4/b, 1/c, 0.8/d, 0/e\}$
$B = \{0/a, 0.9/b, 0.3/c, 0.2/d, 0.1/e\}$

Draw the Fuzzy Graph of *A* and *B*

Then, calculate the following:

- Support, Core, Cardinality, and Complement for *A* and *B* independently
- Union and Intersection of A and B
- the new set *C*, if $C = A^2$
- the new set *D*, if $D = 0.5 \times B$
- the new set *E*, for an alpha cut at $A_{0.5}$

Solution 6

A = {0.2/a, 0.4/b, 1/c, 0.8/d, 0/e}
B = {0/a, 0.9/b, 0.3/c, 0.2/d, 0.1/e}

Support

Supp(A) = {a, b, c, d}
Supp(B) = {b, c, d, e}

Core

Core(A) = {c}
Core(B) = {}

Cardinality

Card(A) = 0.2 + 0.4 + 1 + 0.8 + 0 = 2.4
Card(B) = 0 + 0.9 + 0.3 + 0.2 + 0.1 = 1.5

Complement

Comp(A) = {0.8/a, 0.6/b, 0/c, 0.2/d, 1/e}
Comp(B) = {1/a, 0.1/b, 0.7/c, 0.8/d, 0.9/e}

Solution 6 cont....

A = {0.2/a, 0.4/b, 1/c, 0.8/d, 0/e}
B = {0/a, 0.9/b, 0.3/c, 0.2/d, 0.1/e}

Union

$A \cup B$ = {0.2/a, 0.9/b, 1/c, 0.8/d, 0.1/e}

Intersection

$A \cap B$ = {0/a, 0.4/b, 0.3/c, 0.2/d, 0/e}

$C=A^2$

C = {0.04/a, 0.16/b, 1/c, 0.64/d, 0/e}

$D = 0.5 \times B$

D = {0/a, 0.45/b, 0.15/c, 0.1/d, 0.05/e}

$E = A_{0.5}$

E = {c, d}

Thank you…

Any questions??

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