Ministry of Higher Education

& Scientific Research
Al-Mustaqbal University
Department of Artificial Intelligence
Statistics and Probability
First Year

Lecture Five

Measures of Central Tendency

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Graphical Representation

Graphical representation is a clear and effective way of presenting data that facilitates understanding, interpreting, and comparing the values of a phenomenon. In graphical data presentation, the following methods are commonly used:

- Histogram
- Frequency Polygon
- Frequency Curve

Measures of Central Tendency

Most values of natural phenomena usually cluster around the center or near it. Measures of central tendency can be defined as statistical measures that aim to estimate a single value around which the majority of the data are concentrated. This central or representative value is a single number that summarizes the entire data set. The most important measures of central tendency are:

- Arithmetic Mean
- Geometric Mean
- Harmonic Mean
- Quadratic Mean (Root Mean Square)
- Median
- Mode

Arithmetic Mean (Mean)

The arithmetic mean (or average) of a set of values is obtained by dividing the sum of the values by their number. It is denoted by the symbol Y¯\bar{Y}Y¯.

Methods of Calculating the Arithmetic Mean

1. Ungrouped Data

If we have n observations or values
$Y_1, Y_2, \ldots, Y_n$
then the arithmetic mean is given by:

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}$$

Example:

The following data represent the annual rainfall (in millimeters) for a city over five years:
520, 350, 450, 380, 400

Find the average annual rainfall during this period.

Solution:

$$\bar{Y} = \frac{\sum Y_i}{n} = \frac{520 + 350 + 450 + 380 + 400}{5}$$

$$= \frac{2100}{5} = 420\, mm$$

Exercise:

Calculate the arithmetic mean of wheat yield for 40 farms (previously given) using a frequency distribution table.

2. Grouped Data

If y_1 ,y_2 , …. , y_n represent the class midpoints in a frequency distribution table, and f_1 , f_2 , … , f_n represent their corresponding frequencies,
then the arithmetic mean is calculated as:

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i\, f_i}{f_i}$$

Steps for Finding the Arithmetic Mean for Grouped Data

1. Determine the class midpoints y_i
2. Multiply each class midpoint by its corresponding frequency $f_i\, y_i$
3. Divide the sum of $f_i\, y_i$ by the total frequency.

Example:

Find the arithmetic mean of plant heights using the following frequency distribution table.

classes	f_i	(y_i)	$f_i\, y_i$
31 – 40	1	35.5	35.5
41 – 50	2	45.5	٩١
51 – 60	5	55.5	277.5
61 – 70	15	65.5	982.5
71 – 80	25	75.5	1887.5
81 – 90	20	85.5	1710.0
91 – 100	12	95.5	1146.0
$\sum$	80		6130.0

$\bar{y} = \frac{\sum_{i=1}^{n} y_i f_i}{f_i}$ =76.62 mm

Properties of the Arithmetic Mean

- The sum of deviations of the observations from their arithmetic mean equals zero.
    - For ungrouped data:

$$\sum(Y_i - \bar{Y}) = 0$$

    - For grouped data:

$$\sum f_i\,(Y_i - \bar{Y}) = 0$$

- The sum of squared deviations from the arithmetic mean is the minimum possible; that is, it is smaller than the sum of squared deviations from any other value. In other words:

  $\sum(Y_i - \bar{Y})^2$ is minimum

Weighted Arithmetic Mean

If each observation yiy_iyi has a specific weight wiw_iwi that reflects its importance, then the weighted arithmetic mean is defined as:

$$\bar{Y}_w = \frac{\sum w_i \, y_i}{\sum w_i}$$

Example:

The following values represent the exam scores of a student in a statistics course, where each exam has a specific weight:

Exam	y_i	$(w_i)^{/}$	$w_i \, y_i$
1	*70*	10%	*700*
2	*60*	30%	*1800*
3	*80*	10%	*750*
4	*55*	50%	*2750*
$\sum$		100%	*6000*

Exercise:

Four sections of first-year students consist of 30, 35, 40, and 25 students, respectively. If their average scores in the statistics exam are 80, 75, 60, and 90, respectively, find the overall average score for all sections.