جامعة المستقبل
AL MUSTAQBAL UNIVERSITY

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Lecture (2)

Algebra of sets and it’s proving, Power set, Classes of sets, Cardinality.

Subject: Discrete Structures

First Stage: Semester II

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(Algebra of sets)

Sets under the above operations satisfy various laws or identities which are listed below:

1- $A \cup A = A$
$A \cap A = A$

2- $(A \cup B) \cup C = A \cup (B \cup C)$ Associative laws
$(A \cap B) \cap C = A \cap (B \cap C)$

3- $A \cup B = B \cup A$ Commutativity
$A \cap B = B \cap A$

4- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ Distributive laws
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

5- $A \cup \varnothing = A$ Identity laws
$A \cap U = A$

6- $A \cup U = U$ Identity laws
$A \cap \varnothing = \varnothing$

7- $(A^c)^c = A$ Double complements

8- $A \cup A^c = U$ Complement intersections and unions

$A \cap A^c = \varnothing$

9- $U^c = \varnothing$

$\varnothing^c = U$

10- $(A \cup B)^c = A^c \cap B^c$ De Morgan's laws
$(A \cap B)^c = A^c \cup B^c$

Power set

The power set of some set S, denoted P(S), is the set of all subsets of S (including S itself and the empty set)

$P(S) = \{e : e \subseteq S\}$

Example 1:

Let A = { 1,2 3}

Power set of set A = P(A)

$=\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{\},A]$

Example 2:

$P(\{0,1\})=\{\{\},\{0\},\{1\},\{0,1\}\}$

Classes of sets:

Collection of subset of a set with some properties

Example:

Suppose A = { 1,2 3} ,

let X2 be the class of subsets of A which contain exactly two elements of A. Then

class X0 = [{}]
class X1 = [{1},{2},{3}]
class X2 = [{1,2},{1,3},{2,3}]
class X3 = [{1,2,3}]

Cardinality

The cardinality of a set S, denoted **|S|**, is simply the number of elements a set has, so

|{a,b,c,d}**| =** 4,

The cardinality of the power set

Theorem:
If |A| = n then |P(A)| = 2^n
(Every set with n elements has 2^n subsets)

Problem set

Write the answers to the following questions.

1. |{1,2,3,4,5,6,7,8,9,0}|
2. |P({1,2,3})|
3. P({0,1,2})
4. P({1})

Answers

1. 10
2. 2^3=8
3. {{},{0},{1},{2},{0,1},0,2},{1,2},{0,1,2}}
4. {{},{1}}

The Cartesian product

The Cartesian Product of two sets is the set of all tuples made from elements of two sets.

We write the Cartesian Product of two sets A and B as A × B. It is defined as:

$$A \times B = \{(a, b) | a \in A \text{ and } b \in B\}$$

It may be clearer to understand from examples;

$$\{0,1\} \times \{2,3\} = \{(0,2),(0,3),(1,2),(1,3)\}$$
$$\{a,b\} \times \{c,d\} = \{(a,c),(a,d),(b,c),(b,d)\}$$
$$\{0,1,2\} \times \{4,6\} = \{(0,4),(0,6),(1,4),(1,6),(2,4),(2,6)\}$$

Example:
If A = {1, 2, 3} and B = {x, y} then

A . B = {(1, x), (1, y), (2, x), (2, y), (3, x), (3, y)}
B . A = {(x, 1), (x, 2), (x, 3), (y, 1), (y, 2), (y, 3)}

It is clear that, the cardinality of the Cartesian product of two sets A and B is:

$$|A \times B| = |A||B|$$

A Cartesian Product of two sets A and B can be produced by making tuples of each element of A with each element of B; this can be visualized as a grid (which *Cartesian* implies) or table: if, *e.g.*,
A = { 0, 1 } and B = { 2, 3 }, the grid is

<table>
<tr><td colspan="2" rowspan="2">×</td><td colspan="2">A</td></tr>
<tr><td>0</td><td>1</td></tr>
<tr><td rowspan="2">B</td><td>2</td><td>(0,2)</td><td>(1,2)</td></tr>
<tr><td>3</td><td>(0,3)</td><td>(1,3)</td></tr>
</table>

Problem set

Answer the following questions:

1. $\{2,3,4\}\times\{1,3,4\}$
2. $\{0,1\}\times\{0,1\}$
3. $|\{1,2,3\}\times\{0\}|$
4. $|\{1,1\}\times\{2,3,4\}|$

Answers

1. $\{(2,1),(2,3),(2,4),(3,1),(3,3),(3,4),(4,1),(4,3),(4,4)\}$
2. $\{(0,0),(0,1),(1,0),(1,1)\}$
3. 3
4. 6

EXAMPLE

What is the Cartesian product $A \times B \times C$, where
$A = \{0, 1\}$,
$B = \{1, 2\}$, and
$C = \{0, 1, 2\}$?

Solution:

The Cartesian product $A \times B \times C$ consists of all ordered triples (a, b, c), where $a \in A$, $b \in B$, and $c \in C$. Hence,

$$A \times B \times C = \{(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 0), (0, 2, 1), (0, 2, 2), (1, 1, 0), (1, 1, 1), (1, 1, 2), (1, 2, 0), (1, 2, 1), (1, 2, 2)\}.$$

EXAMPLE

Suppose that $A = \{1, 2\}$. It follows that

$A^2 = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$ and

$A^3 = \{(1,1,1), (1,1,2), (1,2,1), (1,2, 2), (2,1,1), (2,1,2), (2, 2,1), (2, 2, 2)\}$.