Fuzzy Logic

Lecture 2:

fuzzy set

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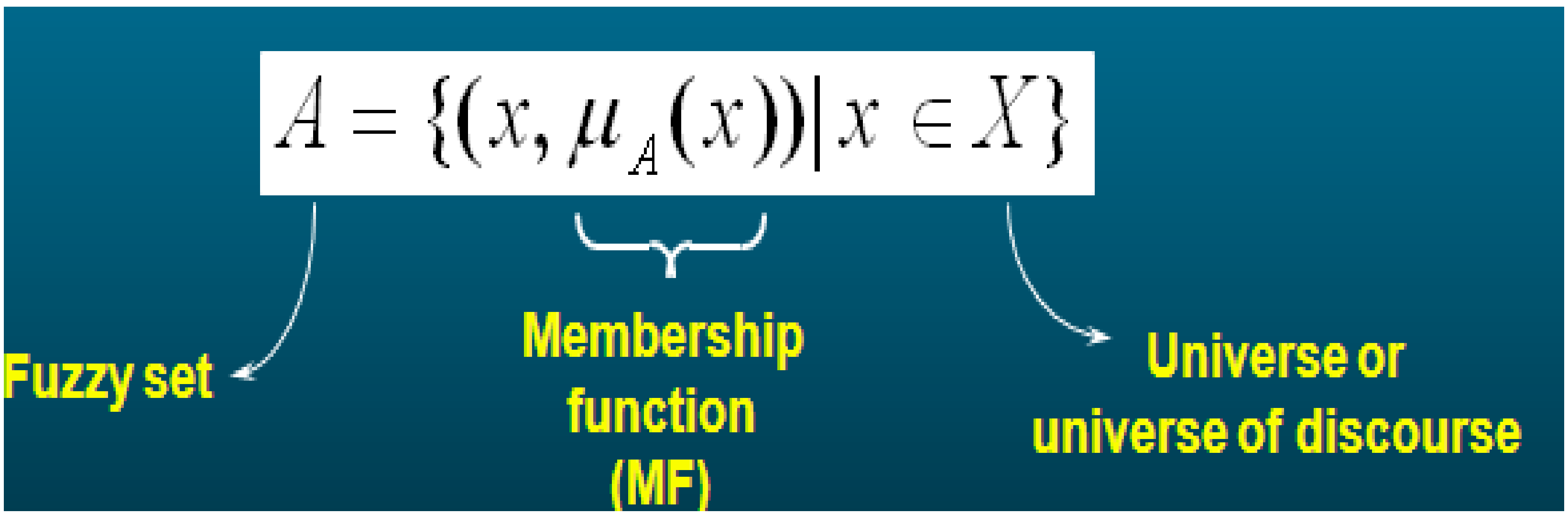
Fuzzy Sets

A fuzzy set *A* in *X* is expressed as a set of ordered pairs:

Where, x is an element in X.

$$A = \{(x, \mu_A(x)) | x \in X\}$$



Membership function

- The membership function is a graphical representation of the magnitude of participation of each input.
- It associates a weighting with each of the inputs that are processed, define functional overlap between inputs, and ultimately determines an output response.
- The rules use the input membership values as weighting factors to determine their influence on the fuzzy output sets of the final output conclusion.
- Once the functions are inferred, scaled, and combined, they are defuzzified into a crisp output which drives the system.

Continued……..

There are different membership functions associated with each input and output response. Some features to note are:

- SHAPE - triangular is common, but bell, trapezoidal, haversine and, exponential have been used. More complex functions are possible but require greater computing overhead to implement. HEIGHT or magnitude (usually normalized to 1) WIDTH (of the base of function), SHOULDERING (locks height at maximum if an outer function. Shouldered functions evaluate as 1.0 past their center) CENTER points (center of the member function shape) OVERLAP (N&Z, Z&P, typically about 50% of width but can be less).
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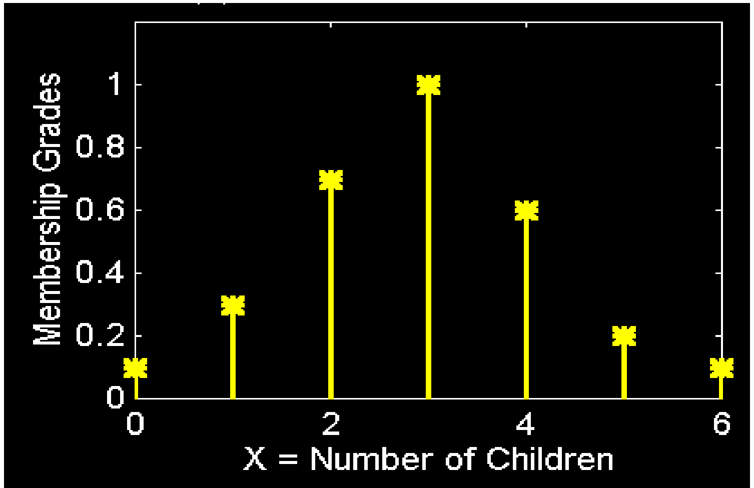
Example

- For the tumbler example, fuzzy set can be represented as
- Full={(1,1), (2,0.75), (3,0.5), (4,0.25), (5,0)}

Fuzzy Sets with Discrete Universes

- Fuzzy set C = “desirable city to live in”
  X = {SF, Boston, LA} (discrete and nonordered) C = {(SF, 0.9), (Boston, 0.8), (LA, 0.6)}
- Fuzzy set A = “sensible number of children”
  X = {0, 1, 2, 3, 4, 5, 6} (discrete universe)
  A = {(0, .1), (1, .3), (2, .7), (3, 1), (4, .6), (5, .2), (6, .1)}

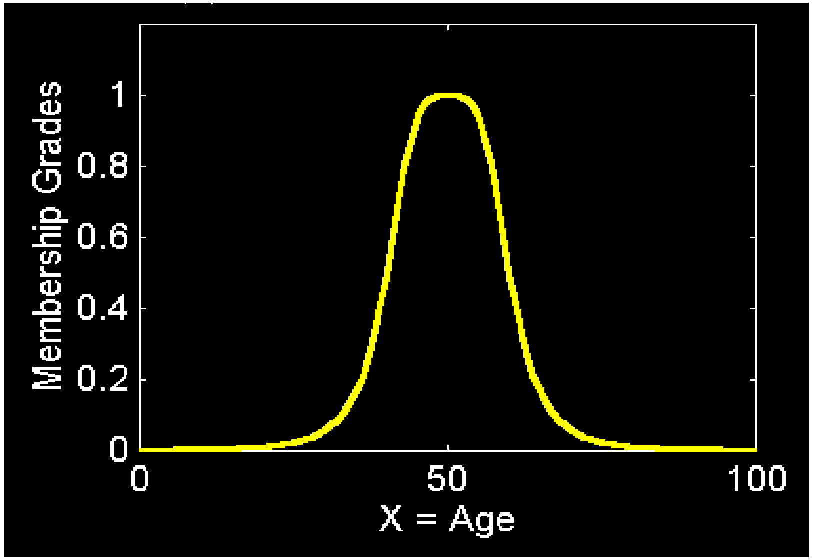


Fuzzy Sets with Cont. Universes

- Fuzzy set B = “about 50 years old”

  X = Set of positive real numbers (continuous)

  B = {(x, μ_B(x)) | x in X}



Alternative Notation

- A fuzzy set A can be alternatively denoted as follows:

X is discrete → $$A = \sum_{x_i \in X} \mu_A(x_i) / x_i$$

X is continuous → $$A = \int_X \mu_A(x) / x$$

Characteristics of Fuzzy Sets

- **Support(A)** = $\{x \in X \mid \mu_A(x) > 0\}$

  The support of fuzzy set, is the crisp set of all points in the universe of discourse U such that membership function of A is non zero.

- **Crossover(A)** = $\{x \in X \mid \mu_A(x) = 0.5\}$

  The crossover point of fuzzy set , is the element in universe of discourse U at which its membership function is 0.5.

- **Normal (A)** = $\{x \in X \mid \mu_A(x) = 1\}$

  The fuzzy set is called normal if there is at least one element in the U where the membership function is 1.

Characteristics of Fuzzy Sets

Cardinality

- The cardinality of a fuzzy set A, the so-called SIGMA COUNT, is expressed as a SUM of the values of the membership function of A, $\mu_A(x)$:

$$card_A = \mu_A(x_1) + \mu_A(x_2) + \ldots \mu_A(x_n) = \Sigma\mu_A(x_i), \quad \text{for } i=1..n$$

Example: Consider $X = \{1, 2, 3\}$ and sets A and B

$A = 0.3/1 + 0.5/2 + 1/3;$

$B = 0.5/1 + 0.55/2 + 1/3$

$card_A = 1.8$

$card_B = 2.05$

Characteristics of Fuzzy Sets

Empty Fuzzy Set

- A fuzzy set A is empty, IF AND ONLY IF:

$$\mu_A(x) = 0, \forall x \in X$$

- Example: Consider $X = \{1, 2, 3\}$ and fuzzy set

$$A = 0/1 + 0/2 + 0/3,$$

then A is *empty.*

Characteristics of Fuzzy Sets

Alpha-Cut and Strong-alpha cut

- An α-cut or α-level set of a fuzzy set $A \subseteq X$ is an ORDINARY SET $A_\alpha \subseteq X$, such that:

$$A_\alpha=\{\mu_A(x)\geq\alpha, \forall x\in X\}.$$

- Strong alpha cut is defined as,

$$A_\alpha=\{\mu_A(x)>\alpha, \forall x\in X\}.$$

- Example: Consider $X = \{1, 2, 3\}$ and set $A = 0.3/1 + 0.5/2 + 1/3$
  then: $A_{0.5} = \{2, 3\}$, $A_{0.3} = \{1, 2, 3\}$, $A_1 = \{3\}$.

Characteristics of Fuzzy Sets

- Fuzzy singleton

- Fuzzy set whose support is a single point in X with $\mu_A(x) = 1$ is called fuzzy singleton.

Characteristics of Fuzzy Sets

Equality

- Fuzzy set *A* is considered equal to a fuzzy set *B*, IF AND ONLY IF:

$$\mu_A(x) = \mu_B(x), \forall x \in X$$

- Example: *A* = 0.3/1 + 0.5/2 + 1/3, *B* = 0.3/1 + 0.5/2 + 1/3, therefore *A* = *B*.

Characteristics of Fuzzy Sets

Height

- The height of a fuzzy set ˜A is the maximum value of the membership function,

  i.e.max$\{\mu(x)\}$.

Example 1

- Let U is defined by X={1,2,3,4,5,6,7,8,9,10} & fuzzy set A={(1,0), (2,0.1), (3,0.2), (4,0.5), (5,0.3), (6,0.1),(7,0.0), (8,0.0), (9,0.0), (10,1)}
- Find the support, crossover and normal.

Solution 1

- Support={2,3,4,5,6,10}
- Crossover point x=4
- Normal point x=10

Thank you…

Any questions??

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