جامعة المستقبل
AL MUSTAQBAL UNIVERSITY

College of Science

Department of Forensic Evidence

Lecture (2)

Simple harmonic motion,position ,velocity ,acceleration and energy

Second stage

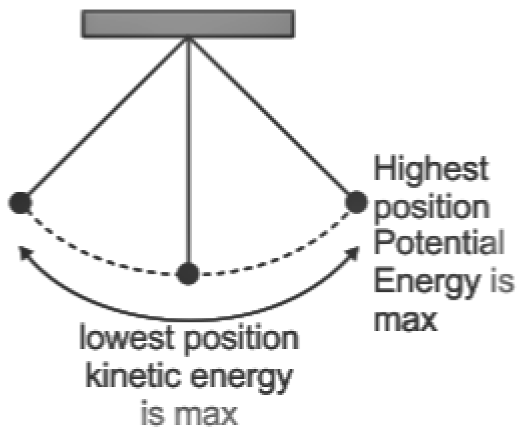
Msc. Hussein Ali

2025-2026

Simple Harmonic Motion (SHM)

Introduction:

Simple harmonic motion is considered one of the most important types of periodic motion in physics due to its wide range of applications in mechanical and physical systems such as vibrations, waves, pendulums, and spring–mass systems. This type of motion is characterized by an oscillating body moving back and forth about a fixed equilibrium position under the influence of a restoring force that is directly proportional to the displacement and opposite in direction. The study of simple harmonic motion forms a fundamental basis for understanding vibrational and wave phenomena in classical physics.



Definition of Simple Harmonic Motion:

Simple harmonic motion is a periodic oscillatory motion in which the acceleration of the body is directly proportional to its displacement from the equilibrium position and opposite in direction. This relationship is given by:

$$a = -\omega^2 x$$

where:

- a is the acceleration,
- x is the displacement from the equilibrium position,
- ω is the angular frequency.

The differential equation representing simple harmonic motion is:

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

The general solution of this equation is:

$$x = A\sin(\omega t + \theta_0)$$

the Equations Used

- Displacement:

$$x = A\sin(\omega t + \theta_0)$$

- Velocity:

$$v = \omega A\cos(\omega t + \theta_0)$$

- Acceleration:

$$a = -\omega^2 x$$

- Angular frequency:

$$\omega = \sqrt{\frac{k}{m}}$$

- Period:

$$T = \frac{2\pi}{\omega}$$

- Frequency:

$$f = \frac{1}{T}$$

- Maximum velocity:

$$v_{\max} = \omega A$$

- Maximum acceleration:

$$a_{\max} = \omega^2 A$$

Example 1 : A body oscillates in simple harmonic motion according to the equation:

x = 5 cos [2πt + π/4]

When t = 1.5 s, calculate:

1.Displacement
2.Speed of the body

Solution :

Given data:

The equation of a body performing simple harmonic motion is:

x = 5 cos [2πt + π/4]

- **Amplitude (A) = 5 metres**
- **Angular frequency ω = 2π s^{-1}**
- **Phase ϕ = π/4**
- **Time t = 1.5 seconds**

1_Displacement of the body (x):

$$x = A\cos(\omega t + \phi)$$
$$x = 5\cos(3\pi + \pi/4)$$
$$x = -5 \times 0.707$$
$$x = 3.535 \text{ m}$$

The displacement of a body in simple harmonic motion is -3.535 meters.

2_Speed of the body

The speed of the body at any point is given by:

$$v(t)=-\omega A\sin(wt+\phi)$$

$$-5\times 2\pi\ s^{-1}\ \sin[2\pi\ s^{-1}\times 1.5+\pi/4]$$

$$=-10\pi\times 0.707$$

$$= 22\ ms^{-1}$$

The speed of a body in simple harmonic motion is 22 ms^{-1}.

Restoring Force

The Restoring Force pulls or pushes the object back toward its equilibrium position. The larger the displacement, the stronger the restoring force.

Example: Hooke's Law, $F = -kx$, explains this for springs.

F is the force exerted by the spring, k is the spring constant, and x is the displacement.

Example : A spring with a spring constant $k = 5N/m$ is stretched by $x = 0.2m$. Find the restoring force.

Solution: Apply Hooke's Law, $F = -kx$

$$F = -(5N/m)(0.2m) = -1N$$

Types of Simple Harmonic Motion:

- **Linear simple harmonic motion**
- **Angular simple harmonic motion**

Linear Simple Harmonic Motion

When a particle moves to and for about a fixed point (position of equilibrium) in a straight line then it is known as linear simple harmonic motion.

Example : spring mass system.

Angular Simple Harmonic Motion

Angular Simple harmonic motion involves the to and for angular movement of a body about its mean position and the displacement is measured in terms of angular displacement.

Example: String Mass Oscillation system.

Examples of Simple Harmonic Motion

1- The Simple Pendulum:

The simple pendulum is considered one of the mechanical systems that exhibits periodic motion. It consists of a body of mass $m$suspended from a string of length L, with one end fixed, as shown in the figure. The motion occurs in a vertical plane and continues under the influence of gravity.

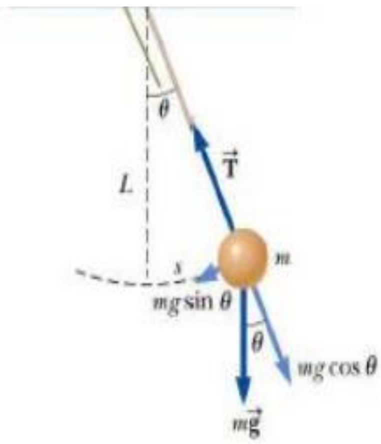
It can be demonstrated that when the angular displacement $\theta_0$is small (less than 10°), the motion of the pendulum can be approximated as simple harmonic motion (SHM).

For a simple pendulum, the period of swing of the pendulum depends on the length of the string and acceleration due to gravity.

$$T = 2\pi\sqrt{(L/g)}$$

where:

- T = period (seconds)
- L = length of the pendulum (meters)
- g = acceleration due to gravity



Example 2: **What is the length of a simple pendulum that ticks in seconds?**

Solution . The time period of a simple pendulum is given by,

$T = 2\pi\sqrt{(L/g)}$

L = gT² / 4π²

The time period of a simple pendulum that ticks in seconds will be 2 seconds.

$$T = 2\text{ s}$$
$$g = 9/8\ m/s^2$$

Substituting in the formula:

$$L = 9.8 \times 4 / 4\pi^2$$

$$L = 1\text{ meter}$$

The length of a simple pendulum that ticks in seconds is 1 meter.

Example : Calculate the period of a simple pendulum with length $L = 2m$.

- **Solution: Use the formula $T = 2\pi\sqrt{\frac{L}{g}}$,**
- $T = 2\pi\sqrt{\frac{2m}{9.8m/s^2}} \approx 2.84s$

Example 3: A body of mass $m = 25$ gm attached to a spring with a force constant $K = 400$ dyn/cm. The motion is initiated with a displacement of $x = 10$ cm to the right.

Determine the following quantities:

- Force F
- Angular frequency ω
- Velocity v
- Displacement x

- Acceleration a

Given that $v = 40\ cm/sec$

Solution:

$$X = A\sin(wt + \theta_0)$$

$$v = A\cos(wt + \theta_0)$$
$$a = -Aw^2\sin(wt + \theta_0)$$

$$f = \frac{1}{T}\ \text{where} T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{25}{400}} = 1.57\ sec$$

$$f = \frac{1}{1.57} = 0.63\ sec^{-1}$$
$$\omega = 2\pi f = 2 \times 3.14 \times 0.63 = 4\ rad/sec$$

$$A = \sqrt{\frac{2E}{K}}\ \text{where} E = \frac{1}{2}mv_0^2 + \frac{1}{2}Kx_0^2$$

$$= \frac{1}{2} \times 25 \times 40^2 + \frac{1}{2} \times 400 \times 10^2 = 40 \times 10^3\ erg$$

$$A = \sqrt{\frac{2 \times 40 \times 10^3}{400}} = 10\sqrt{2}\ cm$$

$$\sin\ \theta_0 = \frac{x_0}{A} = \frac{10}{10\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \theta_0 = \frac{\pi}{4}\ rad$$
$$x = 10\sqrt{2}\sin\left(4t + \frac{\pi}{4}\right)\ cm$$
$$v = 10\sqrt{2} \times 4\cos\left(4t + \frac{\pi}{4}\right)\ cm \cdot s^{-1}$$
$$a = -10\sqrt{2} \times (4)^2\sin\left(4t + \frac{\pi}{4}\right)\ cm \cdot s^{-2}$$

When:

$$t = \frac{\pi}{8} \Rightarrow \left(4 \times \frac{\pi}{8} + \frac{\pi}{4}\right) = \frac{3\pi}{4}$$
$$x = 10\sqrt{2}\sin\frac{3\pi}{4} = 10\ cm$$
$$v = 40\sqrt{2}\cos\frac{3\pi}{4} = -40\ cm \cdot s^{-1}$$
$$a = -160\sqrt{2}\sin\frac{3\pi}{4} = -160\ cm \cdot s^{-2}$$

Term	Definition
Amplitude	Maximum distance from equilibrium position
Equilibrium	Position where net force is zero
Restoring Force	Force that acts to bring an object back to equilibrium
Period (T)	Time to complete one cycle of motion
Frequency (f)	Number of cycles per second, $f = \frac{1}{T}$
Displacement	Distance and direction from equilibrium position
Pendulum	A weight suspended from a pivot that swings freely