جامـــــعـة المـــــســتـقـبـل
AL MUSTAQBAL UNIVERSITY

كلية العلـوم
قســـم الادلة الجنائية

Lecture (3)

LIMITS

المادة : حساب التفاضل والتكامل

المرحلة : الاولى

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Al-Mustaqbal University
College of Science
Forensic Evidence Department

First Stage

zrl5hulb

https://classroom.google.com/c/ODQ5NzM0ODQ2MjAx?cjc=zrl5hulb

Limit of a Function:.

If f is a function, then we say:

A is the limit of $f(x)$ as x approaches a if the value of $f(x)$ gets arbitrarily close to A as x approaches a. This is written in mathematical notation as:

$$\lim_{x\to a} f(x) = A$$

For example:

$$\lim_{x\to 3} x^2 = 9$$

$$\lim_{x\to 0} x = 0$$

$$\lim_{x\to 2} 4 = 4$$

The limit law:.

If L, M, c and k are real numbers and

$$\lim_{x\to c} f(x) = L \quad \text{and} \quad \lim_{x\to c} g(x) = M, \quad \text{then}$$

1. *Sum Rule*: $\lim_{x\to c}(f(x) + g(x)) = L + M$

The limit of the sum of two functions is the sum of their limits.

2. *Difference Rule*: $\lim_{x\to c}(f(x) - g(x)) = L - M$

The limit of the difference of two functions is the difference of their limits.

3. *Product Rule*: $\lim_{x\to c}(f(x) \cdot g(x)) = L \cdot M$

The limit of a product of two functions is the product of their limits.

4. *Constant Multiple Rule:* $\lim_{x\to c}(k \cdot f(x)) = k \cdot L$

The limit of a constant times a function is the constant times the limit of the function.

5. *Quotient Rule*: $$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{M}, \quad M \neq 0$$

The limit of a quotient of two functions is the quotient of their limits, provided the limit of the denominator is not zero.

6. *Power Rule*: If r and s are integers with no common factor and $s \neq 0$, then

$$\lim_{x \to c} (f(x))^{r/s} = L^{r/s}$$

Example:.

(a) $\lim_{x \to c} (x^3 + 4x^2 - 3) = \lim_{x \to c} x^3 + \lim_{x \to c} 4x^2 - \lim_{x \to c} 3$ — Sum and Difference Rules

$= c^3 + 4c^2 - 3$ — Product and Multiple Rules

(b) $\lim_{x \to c} \dfrac{x^4 + x^2 - 1}{x^2 + 5} = \dfrac{\lim_{x \to c} (x^4 + x^2 - 1)}{\lim_{x \to c} (x^2 + 5)}$ — Quotient Rule

$= \dfrac{\lim_{x \to c} x^4 + \lim_{x \to c} x^2 - \lim_{x \to c} 1}{\lim_{x \to c} x^2 + \lim_{x \to c} 5}$ — Sum and Difference Rules

$= \dfrac{c^4 + c^2 - 1}{c^2 + 5}$ — Power or Product Rule

(c) $\lim_{x \to -2} \sqrt{4x^2 - 3} = \sqrt{\lim_{x \to -2} (4x^2 - 3)}$ — Power Rule with $r/s = \frac{1}{2}$

$= \sqrt{\lim_{x \to -2} 4x^2 - \lim_{x \to -2} 3}$ — Difference Rule

$= \sqrt{4(-2)^2 - 3}$ — Product and Multiple Rules

$= \sqrt{16 - 3}$

$= \sqrt{13}$

Indeterminate quantities :.

$\frac{0}{0}$	,	$\frac{\infty}{\infty}$	,	$0*\infty$	,	$\infty - \infty$	,	∞^0	,	0^0

Example:. $\lim_{x\to -2} \frac{-2x-4}{x^3+2x^2}$

Solution: $\frac{-2*-2-4}{(-2)^3+2(-2)^2} = \frac{0}{0}$, then the solution

$$\lim_{x\to -2} \frac{-2(x+2)}{x^2(x+2)} = \lim_{x\to -2} \frac{-2}{x^2} = \frac{-2}{(-2)^2} = \frac{-2}{4} = \frac{-1}{2}$$

Example:. $\lim_{x\to 2} \frac{x^2-4}{x-2}$ $\quad \frac{0}{0}$

Solution: $\lim_{x\to 2} \frac{(x-2)(x+2)}{x-2} = \lim_{x\to 2} (x+2) = 2+2 = 4$

Example:. $\lim_{x\to 0} \frac{x}{1-\sqrt{1-x}}$ $\quad \frac{0}{0}$

Solution: $\lim_{x\to 0} \frac{x(1+\sqrt{1-x})}{(1-\sqrt{1-x})(1+\sqrt{1-x})} = \lim_{x\to 0} \frac{x(1+\sqrt{1-x})}{1-(1-x)}$

$$= \lim_{x\to 0} \frac{x(1+\sqrt{1-x})}{1-1+x} = \lim_{x\to 0} \frac{x(1+\sqrt{1-x})}{x} = \lim_{x\to 0} (1+\sqrt{1-x}) = 2$$

Example:. $\lim_{x\to 0} \frac{\sqrt{x^2+100}-10}{x^2}$ $\quad \frac{0}{0}$

Solution: $\lim_{x\to 0} \frac{(\sqrt{x^2+100}-10)(\sqrt{x^2+100}+10)}{x^2(\sqrt{x^2+100}+10)}$

$$= \lim_{x\to 0} \frac{x^2+100-100}{x^2(\sqrt{x^2+100}+10)} = \lim_{x\to 0} \frac{x^2}{x^2(\sqrt{x^2+100}+10)}$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{x^2+100}+10} = \frac{1}{\sqrt{0+100}+10} = \frac{1}{10+10} = \frac{1}{20}$$

Limits of trigonometric functions:.

If a is constant.

$\lim_{x\to 0} \sin x = 0$	$\lim_{x\to 0} \cos x = 1$	$\lim_{x\to 0} \tan x = 0$
$\lim_{x\to 0} \frac{\sin x}{x} = 1$	$\lim_{x\to 0} \frac{\sin ax}{ax} = 1$	$\lim_{x\to 0} \frac{\tan x}{x} = 1$
$\lim_{x\to 0} \frac{\tan ax}{ax} = 1$	$\lim_{x\to 0} \frac{1-\cos x}{x} = 0$	

Example:. Find

1. $\lim_{x\to 0} \frac{\sin 2x}{5x} = \frac{1}{5} \lim_{x\to 0} \frac{\sin 2x}{x} = \frac{1}{5} \lim_{x\to 0} \frac{\sin 2x}{x} * \frac{2}{2}$

$$= \frac{2}{5} \lim_{x\to 0} \frac{\sin 2x}{2x} = \frac{2}{5} * 1 = \frac{2}{5}$$

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2. $\lim_{x\to 0} \frac{\sin 2x}{\sin 3x} = \lim_{x\to 0} \frac{\sin 2x \frac{2x}{2x}}{\sin 3x \frac{3x}{3x}} = \lim_{x\to 0} \frac{2x \frac{\sin 2x}{2x}}{3x \frac{\sin 3x}{3x}}$

$$= \frac{2}{3} \lim_{x\to 0} \frac{\frac{\sin 2x}{2x}}{\frac{\sin 3x}{3x}} = \frac{2}{3} \frac{\lim_{x\to 0} \frac{\sin 2x}{2x}}{\lim_{x\to 0} \frac{\sin 3x}{3x}} = \frac{2}{3} * \frac{1}{1} = \frac{2}{3}$$

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3. $\lim_{\theta\to 0} \frac{\theta}{\sin\theta} = \lim_{\theta\to 0} \frac{1}{\frac{\sin\theta}{\theta}} = \frac{1}{\lim_{\theta\to 0} \frac{\sin\theta}{\theta}} = \frac{1}{1} = 1$

4. $\lim_{\theta\to 0} \frac{\sin^2\theta}{\theta^2} = \left(\lim_{\theta\to 0} \frac{\sin\theta}{\theta} \right)^2 = (1)^2 = 1$

or $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} * \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1 * 1 = 1$

Infinite limits [as $x \to \pm\infty$] :.

The basic facts to be verified by applying the formal definition are.

$$\lim_{x\to\pm\infty} k = k \quad \text{and} \quad \lim_{x\to\pm\infty} \frac{1}{x} = 0$$

Example :. $\lim_{x\to\infty} \left(5 + \frac{1}{x}\right) = \lim_{x\to\infty} 5 + \lim_{x\to\infty} \frac{1}{x} = 5 + 0 = 5$

Example :. $\lim_{x\to-\infty} \frac{\pi\sqrt{3}}{x^2} = \lim_{x\to-\infty} \pi\sqrt{3}\,\frac{1}{x} * \frac{1}{x} = \lim_{x\to-\infty} \pi\sqrt{3} *$

$\lim_{x\to-\infty} \frac{1}{x} * \lim_{x\to-\infty} \frac{1}{x} = \pi\sqrt{3} * 0 * 0 = 0$

Example :. $\lim_{x\to\infty} \frac{5x^2+8x-3}{3x^2+2} = \lim_{x\to\infty} \frac{\frac{5x^2}{x^2}+\frac{8x}{x^2}-\frac{3}{x^2}}{\frac{3x^2}{x^2}+\frac{2}{x^2}} = \lim_{x\to\infty} \frac{5+\frac{8}{x}-\frac{3}{x^2}}{3+\frac{2}{x^2}}$

$= \frac{5+0+0}{3+0} = \frac{5}{3}$

Example :. $\lim_{x\to\infty} \frac{11x+2}{2x^3-1} = \lim_{x\to\infty} \frac{\frac{11x}{x^3}+\frac{2}{x^3}}{\frac{2x^3}{x^3}-\frac{1}{x^3}} = \lim_{x\to\infty} \frac{\frac{11}{x^2}+\frac{2}{x^3}}{2-\frac{1}{x^3}}$

$= \frac{0+0}{2-0} = \frac{0}{2} = 0$

Example :. $\lim_{x\to\infty} \sin\left(\frac{1}{x}\right) = \sin\left(\frac{1}{x}\right) = \sin(0) = 0$

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H.W: Find .

1. $\lim\limits_{x\to 2} \frac{x+3}{x+6}$	$(\frac{5}{8})$	2. $\lim\limits_{x\to 2/3} 3x(2x-1)$	$(\frac{2}{3})$
3. $\lim\limits_{x\to 5} \frac{x-5}{x^2-25}$	$(\frac{1}{10})$	4. $\lim\limits_{x\to -3} \frac{x+3}{x^2+4x+3}$	$(\frac{-1}{2})$
5. $\lim\limits_{t\to 1} \frac{t^2+t-2}{t^2-1}$	$(\frac{3}{2})$	6. $\lim\limits_{u\to 1} \frac{u^2-1}{u^3-1}$	$(\frac{2}{3})$
7. $\lim\limits_{x\to 0} (2\sin x - 1)$	(-1)	8. $\lim\limits_{x\to 0} \sec x$	(1)
9. $\lim\limits_{x\to 0} (x^2 - 1)(2 - \cos x)$	(-1)	10. $\lim\limits_{x\to \pi} \sqrt{x+4}\, \cos(x+\pi)$	$(\sqrt{\pi+4})$
11. $\lim\limits_{x\to 0} \frac{1+x+\sin x}{3\cos x}$	$(\frac{1}{3})$	12. $\lim\limits_{x\to 0} \frac{\sin(1-\cos x)}{1-\cos x}$	(1)
13. $\lim\limits_{\theta\to 0} \frac{\sin\theta}{\sin 2\theta}$	$(\frac{1}{2})$	14. $\lim\limits_{\theta\to 0} \theta\cos\theta$	(0)
15. $\lim\limits_{x\to \pi} \sin(x - \sin x)$	(0)	16. $\lim\limits_{t\to 0} \cos\left(\frac{\pi}{\sqrt{19-3\sec 2t}}\right)$	$(\frac{1}{\sqrt{2}})$
17. $\lim\limits_{x\to \infty} \sqrt{\frac{8x^2-3}{2x^2+x}}$	(2)	18. $\lim\limits_{x\to \infty} \left(\frac{1-x^3}{x^2+7x}\right)^5$	(∞)
19. $\lim\limits_{x\to \infty} \frac{2\sqrt{x}+x^{-1}}{3x-7}$	(0)	20. $\lim\limits_{x\to \infty} \frac{\sqrt[3]{x}-\sqrt[5]{x}}{\sqrt[3]{x}+\sqrt[5]{x}}$	(1)
21. $\lim\limits_{x\to \infty} \frac{3-x}{\sqrt{4x^2+25}}$	$(1/2)$	22. $\lim\limits_{x\to \infty} \left(\frac{3}{x^2} - \cos\frac{1}{x}\right)\left(1+\sin\frac{1}{x}\right)$	(-1)