كلية العلـوم

قسـم الادلة الجنائية

Lecture (10)

Partial Fractions

المـادة : **Complex Analysis**

المرحلة : الثانية

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تخصيص

Complex Analysis
الادلة الجنائية (المرحلة الثانية)

إعلان للصف

رمز الفصل الدراسي

vsp2rqlb

SCAN ME!

vsp2rqlb

https://classroom.google.com/c/ODE1MDM4MzAxMzU0?cjc=vsp2rqlb

Partial fractions:

Success in separating $\dfrac{f(x)}{g(x)}$ into a sum of partial fractions hinges on two things:-

1- The degree of $f(x)$ must be less than the degree of $g(x)$. (If this is not case, we first perform a long division, and then work with the remainder term).

2- The factors of $g(x)$ must be known. If these two conditions are met we can carry out the following steps:

Step I - let $x - r$ be a linear factor of $g(x)$. Suppose $(x - r)^m$ is the highest power of $(x - r)$ that divides $g(x)$. Then assign the sum of m partial factors to this factor, as follows:

$$\frac{A_1}{x - r} + \frac{A_2}{(x - r)^2} + \dots\dots\dots + \frac{A_m}{(x - r)^m}$$

Do this for each distinct linear factor of $f(x)$.

Step II - let $x^2 + p x + q$ be an irreducible quadratic factor of $g(x)$. Suppose $(x^2 + p x + q)^n$ is the highest power of this factor that divides $g(x)$. Then, to this factor, assign the sum of the n partial fractions:

$$\frac{B_1 x + C_1}{x^2 + p x + q} + \frac{B_2 x + C_2}{(x^2 + p x + q)^2} + \dots\dots\dots + \frac{B_n x + C_n}{(x^2 + p x + q)^n}$$

Do this for each distinct linear factor of $g(x)$.

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Step III - set the original fraction $\dfrac{f(x)}{g(x)}$ equal to the sum of all these partial fractions. Clear the resulting equation of fractions and arrange the sums in decreasing powers of x.

Step IV - equate the coefficients of corresponding powers of x and solve the resulting equations for the undetermined coefficients.

_**EX-8**_ – Evaluate the following integrals:

1) $\displaystyle\int \frac{2x+5}{x^2-9}\,dx$

2) $\displaystyle\int \frac{x\,dx}{x^2+4x+3}$

3) $\displaystyle\int \frac{x^3-x}{(x^2+1)\cdot(x-1)^2}\,dx$

4) $\displaystyle\int \frac{\sin x\;dx}{\cos^2 x - 5\cos x + 4}$

5) $\displaystyle\int \frac{2x^2-3x+2}{(x-1)^2(x-2)}\,dx$

6) $\displaystyle\int \frac{x^3+4x^2}{x^2+4x+3}\,dx$

Sol.-

1) $\displaystyle\int \frac{2x+5}{x^2-9}\,dx = \int \frac{2x+5}{(x-3)\cdot(x+3)}\,dx$

$$\frac{2x+5}{(x-3)\cdot(x+3)} = \frac{A}{x-3} + \frac{B}{x+3} \quad\Rightarrow\quad 2x+5 = A(x+3) + B(x-3)$$

$$at \quad x=3 \quad\Rightarrow\quad 6A = 6+5 \quad\Rightarrow\quad A = \frac{11}{6}$$

$$at \quad x=-3 \quad\Rightarrow\quad -6B = -6+5 \quad\Rightarrow\quad B = \frac{1}{6}$$

$$\int \frac{2x+5}{x^2-9}\,dx = \int \left(\frac{11/6}{x-3} + \frac{1/6}{x+3} \right) dx = \frac{11}{6}\ln(x-3) + \frac{1}{6}\ln(x+3) + c$$

2) $\int \dfrac{x\,dx}{x^2 + 4x + 3} = \int \dfrac{x\,dx}{(x+3)(x+1)}$

$\dfrac{x}{(x+3)(x+1)} = \dfrac{A}{x+3} + \dfrac{B}{x+1} \quad \Rightarrow \quad x = A(x+1) + B(x+3)$

$at \quad x = -3 \quad \Rightarrow \quad A = \dfrac{3}{2} \quad and \quad at \quad x = -1 \quad \Rightarrow \quad B = -\dfrac{1}{2}$

$\int \dfrac{x\,dx}{x^2 + 4x + 3} = \int \left(\dfrac{3/2}{x+3} + \dfrac{-1/2}{x+1} \right) dx = \dfrac{3}{2} ln(x+3) - \dfrac{1}{2} ln(x+1) + c$

3) $\int \dfrac{x^3 - x}{(x^2 + 1)(x-1)^2}\,dx = \int \dfrac{x(x-1)(x+1)}{(x^2 + 1)(x-1)^2}\,dx = \int \dfrac{x^2 + x}{(x^2 + 1)(x-1)}\,dx$

$\dfrac{x^2 + x}{(x^2 + 1)(x-1)} = \dfrac{Ax + B}{x^2 + 1} + \dfrac{C}{x-1} \quad \Rightarrow \quad x^2 + x = (Ax + B)(x-1) + C(x^2 + 1)$

$x^2 + x = (A+C)x^2 + (-A+B)x + (-B+C)$

$\left. \begin{array}{l} A + C = 1 \ ......(1) \\ -A + B = 1 \ ......(2) \\ -B + C = 0 \ ......(3) \end{array} \right\} \Rightarrow \quad A = 0 \ , \ B = 1 \ , \ C = 1$

$\int \dfrac{x^3 - x}{(x^2 + 1)(x-1)^2}\,dx = \int \left(\dfrac{1}{x^2 + 1} + \dfrac{1}{x-1} \right) dx = tan^{-1} x + ln(x-1) + c$

4) $let \quad y = cosx \quad \Rightarrow \quad dy = -sinx\,dx$

$\int \dfrac{sin x\,dx}{cos^2 x - 5\cos x + 4} = -\int \dfrac{dy}{y^2 - 5y + 4} = -\int \dfrac{dy}{(y-4)(y-1)}$

$\dfrac{dy}{(y-4)(y-1)} = \dfrac{A}{y-4} + \dfrac{B}{y-1} \quad \Rightarrow \quad 1 = A(y-1) + B(y-4)$

$at \quad y = 4 \quad \Rightarrow \quad A = \dfrac{1}{3} \quad and \quad at \quad y = 1 \quad \Rightarrow \quad B = -\dfrac{1}{3}$

$\int \dfrac{sin x\,dx}{cos^2 x - 5\cos x + 4} = -\int \left(\dfrac{1/3}{y-4} + \dfrac{-1/3}{y-1} \right) dy$

$= -\dfrac{1}{3} ln(y-4) + \dfrac{1}{3} ln(y-1) + c = -\dfrac{1}{3} ln(cos\,x - 4) + \dfrac{1}{3} ln(cos\,x - 1) + c$

5) $\dfrac{2x^2 - 3x + 2}{(x-1)^2(x-2)} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x-2}$

$2x^2 - 3x + 2 = A(x-1)(x-2) + B(x-2) + C(x-1)^2$

$\left.\begin{array}{l} A + C = 2 \quad.....................(1) \\ -3A + B - 2C = -3 \quad......(2) \\ 2A - 2B + C = 2 \quad...........(3) \end{array}\right\} \Rightarrow \quad A = -2 \;,\; B = -1 \;,\; C = 4$

$\displaystyle\int \dfrac{2x^2 - 3x + 2}{(x-1)^2(x-2)}\, dx = \int \left(\dfrac{-2}{x-1} + \dfrac{1}{(x-1)^2} + \dfrac{4}{x-2} \right) dx$

$\qquad\qquad = -2\ln(x-1) + \dfrac{1}{x-1} + 4\ln(x-2) + C$

6) $\dfrac{x^3 + 4x^2}{x^2 + 4x + 3} = x - \dfrac{3x}{(x+3)(x+1)}$

$\qquad\qquad x^2 + 4x + 3 \;\overline{)\;\begin{array}{l} x \\ x^3 + 4x^2 \\ \mp x^3 \mp 4x^2 \mp 3x \\ \hline -3x \end{array}}$

$\dfrac{3x}{(x+3)(x+1)} = \dfrac{A}{x+3} + \dfrac{B}{x+1} \;\Rightarrow\; 3x = A(x+1) + B(x+3)$

$at \;\; x = -3 \Rightarrow A = \dfrac{9}{2} \;\; and \;\; at \;\; x = -1 \Rightarrow B = -\dfrac{3}{2}$

$\displaystyle\int \dfrac{x^3 + 4x^2}{x^2 + 4x + 3}\, dx = \int \left(x - \dfrac{9/2}{x+3} + \dfrac{3/2}{x+1} \right) dx$

$\qquad\qquad = \dfrac{x^2}{2} - \dfrac{9}{2}\ln(x+3) - \dfrac{3}{2}\ln(x+1) + c$