جامعة المستقبل
AL MUSTAQBAL UNIVERSITY

كلية العلـوم
قســـم الادلة الجنائية

Lecture (2)

Limit in Complex Functions

المادة : Complex Analysis

المرحلة : الثانية

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Content

- The general Aim
- The Behavioral objectives
- Limit of a Complex Function
- Definition of Limit in Complex Function
- Methods for solving Limit Problems

  Direct compensation

  Analysis

  Tracks

  L'Hopital

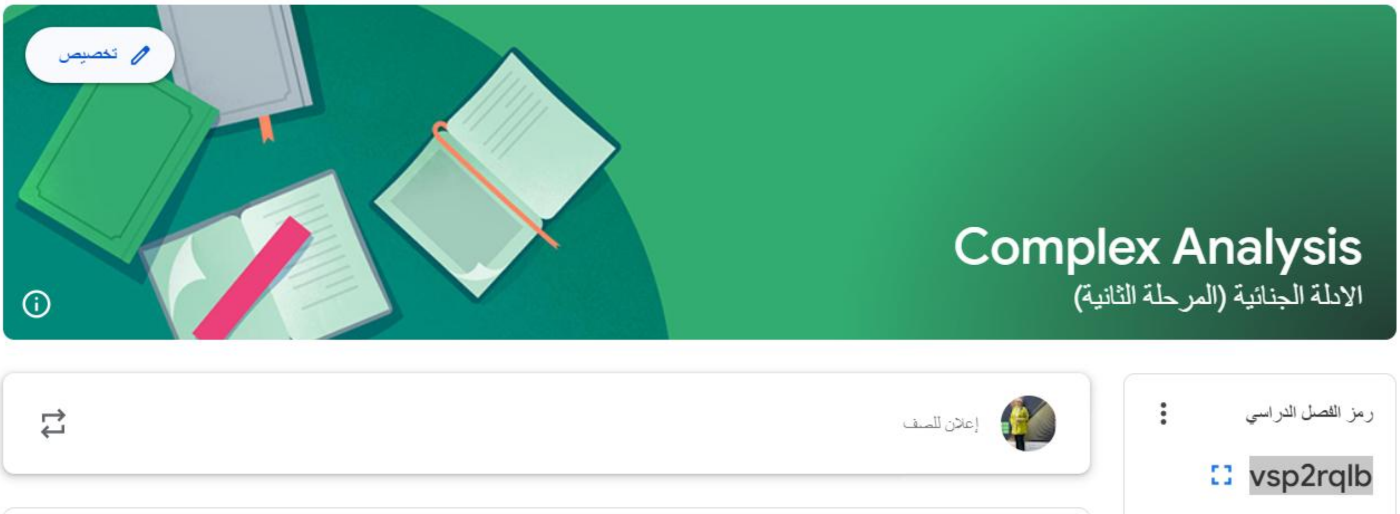
The General Aim

The general aim of studying complex analysis is to explore the properties of functions of a complex variable and apply their powerful theoretical and practical tools to solve problems in mathematics, physics, and engineering.

The Behavioral objectives

By the end of the lecture, the student will be able to:

- ✓ Define the concept of limit in complex functions and explain its formal definition.
- ✓ Apply different methods for solving limits such as direct substitution and analysis.
- ✓ Test the existence of limits using different paths.
- ✓ Use L'Hôpital's Rule to solve indeterminate forms.



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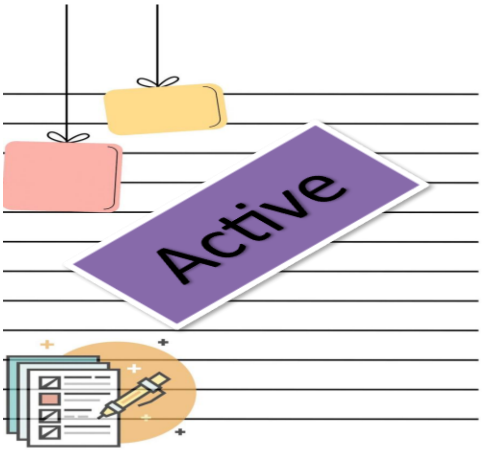


In Week 1's lecture, we discussed Basic of a Complex Analysis. Based on that, please answer the following question:

Convert the complex number (3+3i) into its exponential form

I. Limit of a Complex Function

In real analysis, the concept of a limit forms the foundation for continuity, differentiability, and integration. Similarly, in complex analysis, limits are crucial for defining continuity and analyticity of complex functions.

When dealing with complex variables, we must remember that the input variable z can approach a point not only from the left or right (like in real numbers) but from infinitely many directions in the complex plane. This makes limits in complex functions more delicate.

II. Definition of Limit in Complex Function

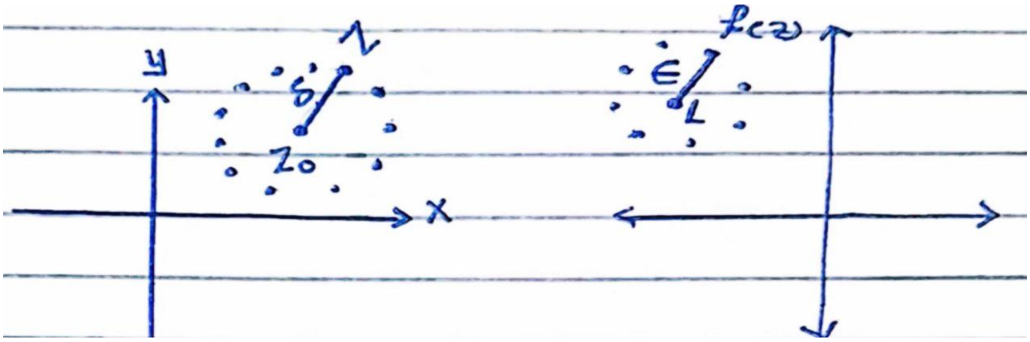
Let $f(z)$be a complex function defined in some neighborhood of a point z_o. We say that:

$$\lim_{z \to z_o} f(z) = L$$

if for every ε > 0, there exists δ > 0 such that whenever

$$0 < |z - z_o| < \delta$$

$$|f(z) - L| < \varepsilon$$



Here: z, z_o, L ∈ ℂ. The condition ensures that $f(z)$ gets arbitrarily close to L as z gets close to z_0, no matter from which direction in the complex plane.

For real functions, we only check from left and right. For complex functions, since z can approach z_0 along infinitely many paths, the limit must be the same regardless of the path of approach.

If different paths give different values, then the limit does not exist.

Example1:

for ($z \to 1+i$) find (z^2) if exist or not.

Solution:

$z^2 = (1+i)^2 = 1 + 2i + i^2 = 1 + 2i - 1 = 2i$ ($\therefore$ *Lim exist*)

Example 2:

Check weather $\lim_{z\to 0} \frac{Z}{\bar{Z}}$ exist or not.

Solution:

$Z = x + iy$

Z

$\lim_{z\to 0} \frac{Z}{\bar{Z}} = \frac{x+iy}{x-iy}$

1. $\lim_{z\to 0} \frac{Z}{\bar{Z}} = \frac{x+0}{x-0} = \frac{x}{x} = 1$

2. $\lim_{z\to 0} \frac{Z}{\bar{Z}} = \frac{0+iy}{0-iy} = \frac{iy}{-iy} = -1$

$x \neq y$ $\therefore$ *Lim dose not exist*

III. Methods for solving Limit Problems

$$\lim_{z \to z_o} f(z) = \lim_{x,y \to 0} f(x, y)$$

- Direct compensation (التعويض المباشر)

Example: Check if $\lim_{z \to 1} \frac{2z}{z+i}$ exist or not.

Solution:

$$\lim_{z \to 1} \frac{2z}{z+i} = \frac{2(1)}{1+i}$$

$$= \frac{2}{1+i} * \frac{1-i}{1-i}$$

$$= \frac{2(1-i)}{1+1}$$

$$= (1-i)$$

$\therefore$ *Lim exist*

- Analysis (تحليل البسط او المقام)

Example: Check if $\lim_{z \to 2i} \frac{z^2+4}{z-2i}$ exist or not.

Solution:

$$\lim_{z\to 2i} \frac{(2i)^2}{2i-2i} = \frac{0}{0}$$

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$$= \lim_{z\to 2i} \frac{z^2+4}{z-2i}$$

$$= \lim_{z\to 2i} \frac{(z+2i)(z-2i)}{z-2i}$$

$$= \lim_{z\to 2i} (z+2i)$$

$$= \lim_{z\to 2i} (2i+2i)$$

$$= 4i$$

$\therefore$ *Lim exist*

- Tracks (المسارات):

Let $y = mx$, $\therefore \lim_{x\to x_o} f(x) = L(m)$

$\therefore$ *Lim dos not exist*

يجب ان تكون درجة الاس بالبسط =درجة المقام

Example: Check if $\lim_{z\to 0} \frac{xy}{x^2+y^2}$ exist or not.

y=mx

درجة البسط=2

درجة المقام=2

Solution:

$$\lim_{x\to 0} \frac{x(mx)}{x^2+(mx)^2}$$

$$= \lim_{x \to 0} \frac{mx^2}{x^2(1+m^2)}$$

$$= \lim_{x \to 0} \frac{m}{(1+m^2)}$$

$\therefore$ *Lim does not exist*

Activity:

Check if $\lim_{z \to 0} \frac{2xy^3}{x^2+y^6}$ exist or not.

- L'Hopital

$$\lim_{z \to z_o} \frac{f(z)}{g(z)}$$
$$= \lim_{z \to z_o} \frac{f'(z)}{g'(z)}$$

نفاضل البسط و المقام ثم نعوض مرة اخرى

Example: Check if $\lim_{z \to 3i} \frac{z^4-81}{z^2+9}$ exist or not.

y=mx

درجة البسط=2

درجة المقام=2

Solution:

$$\lim_{z \to 3i} \frac{4z^3}{2z}$$

$$= \lim_{z \to 3i} 2z^2$$

$$= 2(3i^2)$$

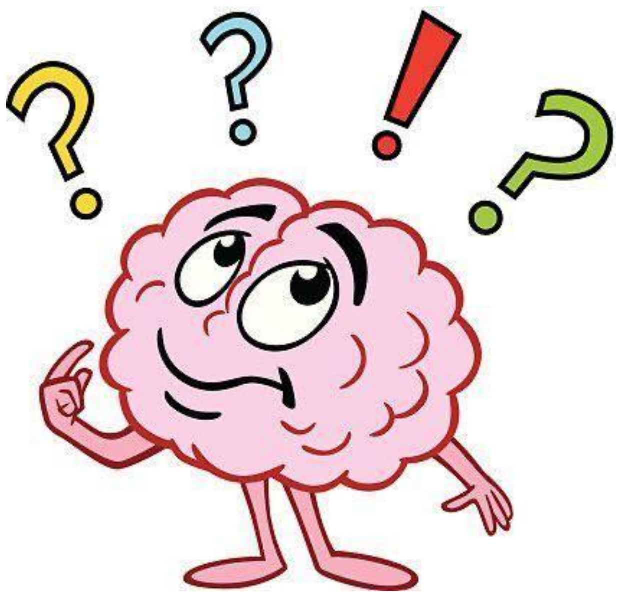
$$= 18$$

$\therefore$ *Lim exist*

TASK:

- In Group, Solve the following equations:

1. Evaluate $\lim\limits_{z \to 1+\sqrt{2}i} \frac{z^2-2z+3}{z-1-\sqrt{2}i}$



Note: The Answer must be sent to the Google Classroom

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