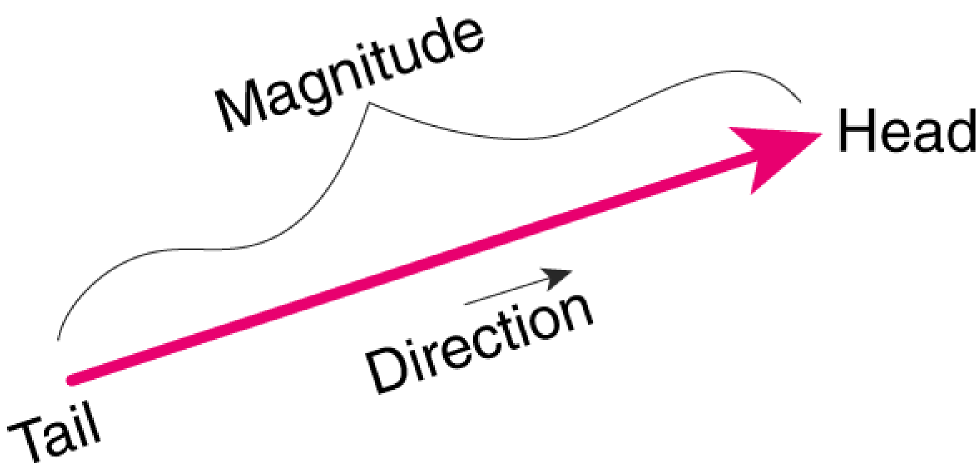
Al-Mustaqbal University
College of Engineering Technology
Department of Cyber Security Techniques Engineering
Class: 1st
Subject: Math
Lecturer: Dr. Hussein Ali Ameen
2nd term – Vectors and Vector Operations

Definition

A vector, in physics, a quantity that has both magnitude and direction. It is typically represented by an arrow whose direction is the same as that of the quantity and whose length is proportional to the quantity's magnitude. Although a vector has magnitude and direction, it does not have position. That is, as long as its length is not changed, a vector is not altered if it is displaced parallel to itself.



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Vector-Vector Operations

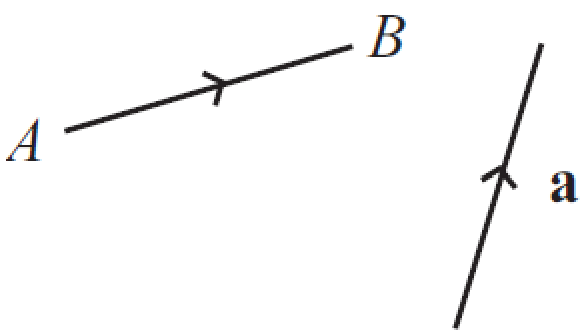
• Addition of two vectors

• Geometric representation of addition and subtraction of vectors

• Vectors and points

Representing vector quantities

We can represent a vector by a line segment. This diagram shows two vectors



We have used a small arrow to indicate that the first vector is pointing from A to B. A vector pointing from B to A would be going in the opposite direction

Key Point

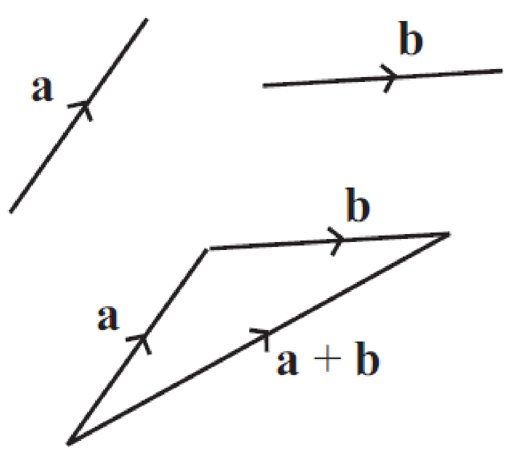
A vector has both magnitude and direction, and both these properties must be given in order to specify it. A quantity with magnitude but no direction is called a scalar.

Vector-Vector Operations

- Vector addition (and subtraction)
  - $\boldsymbol{a} + \boldsymbol{b}, \boldsymbol{a} - \boldsymbol{b}$
- Vector Multiplication
  - Dot Product: $\boldsymbol{a} \cdot \boldsymbol{b}$
  - Cross Product: $\boldsymbol{a} \times \boldsymbol{b}$

Adding two vectors

One of the things we can do with vectors is to add them together. We shall start by adding two vectors together. Once we have done that, we can add any number of vectors together by adding the first two, then adding the result to the third, and so on. In order to add two vectors, we think of them as displacements. We carry out the first displacement, and then the second. So, the second displacement must start where the first one finishes.

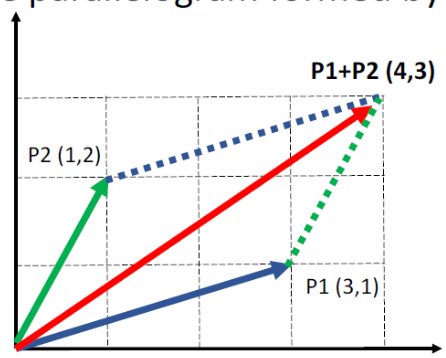


- Two vector can be added only if they have the same dimension.
- The corresponding components of the two vector are added together.
- Two vector can be subtracted in the same way of adding, by subtracting components.
- Example:
  - $\begin{bmatrix}2\\1\end{bmatrix} + \begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}3\\0\end{bmatrix}$
  - $\begin{bmatrix}1\\0\\-1\end{bmatrix} - \begin{bmatrix}\sqrt{2}\\1\\0\end{bmatrix} = \begin{bmatrix}1-\sqrt{2}\\0-1\\-1-0\end{bmatrix} = \begin{bmatrix}1-\sqrt{2}\\-1\\-1\end{bmatrix}$

When can two vectors be added?

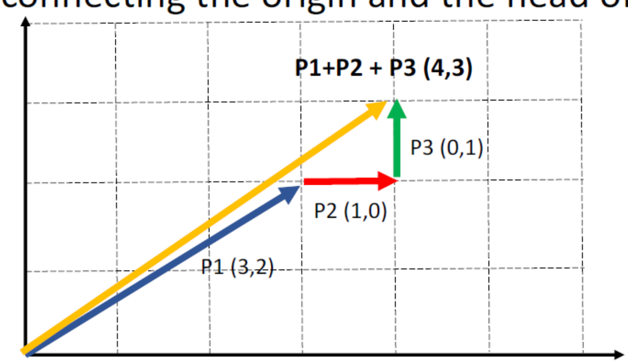
- Only if two vectors have the same dimension they can be added.
- Row vectors and column vectors of the same dimension can be added.
- Example:
  - $[1] + [0] = [1]$
  - $[1 \quad 2] + \begin{bmatrix}2\\-2\end{bmatrix} = [3 \quad 0] = \begin{bmatrix}3\\0\end{bmatrix}$
  - $\begin{bmatrix}0\\-1\end{bmatrix} + \begin{bmatrix}2\\0.5\\1\end{bmatrix} = ?$

- Draw one vector.
- Draw the other vector.
- Draw one vector along the diagonal of the parallelogram formed by P1 and P2.
- Example:
  - $\begin{bmatrix}3\\1\end{bmatrix} + \begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}4\\3\end{bmatrix}$

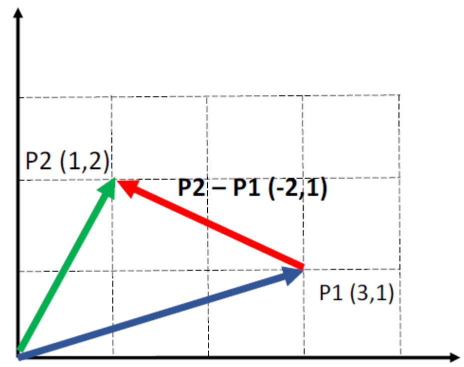


How to represent addition of three vectors graphically

- Another way to add multiple vectors graphically is to link the tail of one vector with the head of another vector as shown below.
- The final vector is obtained by connecting the origin and the head of the last vector.
- Example:
  - $\begin{bmatrix}3\\2\end{bmatrix} + \begin{bmatrix}1\\0\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}4\\3\end{bmatrix}$



- Draw one vector.
- Draw the other vector.
- Draw one vector originated at the tail of the first vector, and ends at the tail of the second vector.
- Example:



- $\begin{bmatrix}1\\2\end{bmatrix} - \begin{bmatrix}3\\1\end{bmatrix} = \begin{bmatrix}-2\\1\end{bmatrix}$