Al-Mustaqbal University
College of Engineering Technology
Department of Cyber Security Techniques Engineering
Class: 1st
Subject: Math
Lecturer: Dr. Hussein Ali Ameen
2nd term – Lecture: 1 – Implicit and Partial Differentiation

Indeterminate Forms and Hôpital's Rule:

Suppose that we have one of the following cases,

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0} \qquad \text{OR} \qquad \lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\pm\infty}{\pm\infty}$$

where a can be any real number, infinity or negative infinity. In these cases we have,

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Example: Find each of the following limits

a. $\lim_{x \to 0} \frac{\sin(x)}{x}$

Solution

(a) $\lim_{x \to 0} \frac{\sin(x)}{x}$

So, we have already established that this is a $0/0$ indeterminate form so let's just apply L'Hospital's Rule.

$$\lim_{x \to 0} \frac{\sin(x)}{x} = \lim_{x \to 0} \frac{\cos(x)}{1} = \frac{1}{1} = 1$$

b. $\lim_{t \to 1} \frac{5t^4 - 4t^2 - 1}{10 - t - 9t^3}$

In this case we also have a $0/0$ indeterminate form and if we were really good at factoring we could factor the numerator and denominator, simplify and take the limit. However, that's going to be more work than just using L'Hospital's Rule.

$$\lim_{t \to 1} \frac{5t^4 - 4t^2 - 1}{10 - t - 9t^3} = \lim_{t \to 1} \frac{20t^3 - 8t}{-1 - 27t^2} = \frac{20 - 8}{-1 - 27} = -\frac{3}{7}$$

c. $\lim_{x\to\infty} \frac{e^x}{x^2}$

This was the other limit that we started off looking at and we know that it's the indeterminate form ∞/∞ so let's apply L'Hospital's Rule.

$$\lim_{x\to\infty} \frac{\mathbf{e}^x}{x^2} = \lim_{x\to\infty} \frac{\mathbf{e}^x}{2x}$$

Now we have a small problem. This new limit is also a ∞/∞ indeterminate form. However, it's not really a problem. We know how to deal with these kinds of limits. Just apply L'Hospital's Rule.

$$\lim_{x\to\infty} \frac{\mathbf{e}^x}{x^2} = \lim_{x\to\infty} \frac{\mathbf{e}^x}{2x} = \lim_{x\to\infty} \frac{\mathbf{e}^x}{2} = \infty$$

Home Work

Evaluate each of the following limits

1. $\lim_{x\to 0^+} x \ln x$

2. $\lim_{x\to 2} \frac{x^3 - 7x^2 + 10x}{x^2 + x - 6}$

3. $\lim_{x\to\infty} x^{\frac{1}{x}}$

Al-Mustaqbal University
College of Engineering Technology
Department of Cyber Security Techniques Engineering
Class: 1st
Subject: Math
Lecturer: Dr. Hussein Ali Ameen
2nd term – Lecture: 1 – Implicit and Partial Differentiation

Chapter Four

Integration

Indefinite Integrals:

Definition: The set of all antiderivatives of f is the indefinite integral of f with respect to x**,** denoted by:

$$\int \boldsymbol{f(x)\,dx = F(x) + c}$$

The symbol $\int$ *is* an integral sign. The function is f the integrand of the integral, and x is the variable of integration and c is the constant of integral.

Some integration formulas:

1) $\int \frac{du}{dx} dx = u(x) + c$
2) $\int a\, u(x)\, dx = a \int u(x)$ $, a\ is\ constant.$
3) $\int [u_1(x) + u_2(x) + \cdots]dx = \int u_1(x)dx + \int u_2(x)dx + \ldots$
4) $\int u^n \frac{du}{dx}\, dx = \frac{u^{n+1}}{n+1} + c \ , n \neq -1$

Al-Mustaqbal University
College of Engineering Technology
Department of Cyber Security Techniques Engineering
Class: 1st
Subject: Math
Lecturer: Dr. Hussein Ali Ameen
2nd term – Lecture: 1 – Implicit and Partial Differentiation

1. Integration as differentiation in reverse

Suppose we differentiate the function $y = x^2$. We obtain $\frac{dy}{dx} = 2x$. Integration reverses this process and we say that the integral of $2x$ is x^2. Pictorially we can regard this as shown in Figure 1:

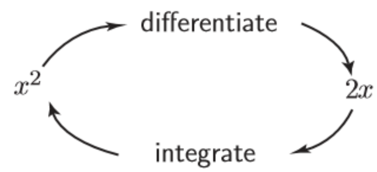
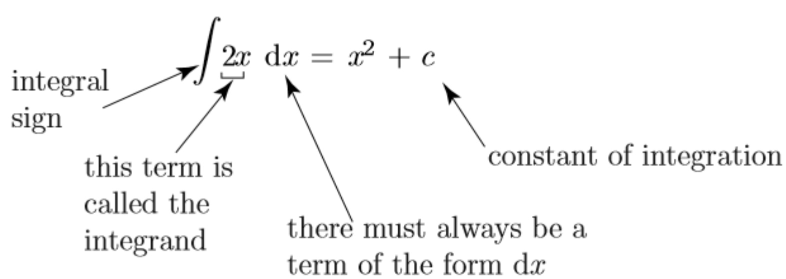


Figure 1

The symbol for integration, $\int$, is known as an **integral sign**. To integrate $2x$ we write



Exercises

1. (a) Write down the derivatives of each of: x^3, $x^3 + 17$, $x^3 - 21$

   (b) Deduce that $\int 3x^2 \, dx = x^3 + c$.

2. Explain why, when finding an indefinite integral, a constant of integration is always needed.

Answers

1. (a) $3x^2$, $3x^2$, $3x^2$ (b) Whatever the constant, it is zero when differentiated.

2. Any constant will disappear (i.e. become zero) when differentiated so one must be reintroduced to reverse the process.

hussein_awadh@uomus.edu.iq

Al-Mustaqbal University
College of Engineering Technology
Department of Cyber Security Techniques Engineering
Class: 1st
Subject: Math
Lecturer: Dr. Hussein Ali Ameen
2nd term – Lecture: 1 – Implicit and Partial Differentiation

Table 1: Integrals of Common Functions

function $f(x)$	indefinite integral $\int f(x)\,dx$
constant, k	$kx + c$
x	$\frac{1}{2}x^2 + c$
x^2	$\frac{1}{3}x^3 + c$
x^n	$\dfrac{x^{n+1}}{n+1} + c, \quad n \neq -1$
x^{-1} (or $\dfrac{1}{x}$)	$\ln\lvert x\rvert + c$
$\cos x$	$\sin x + c$
$\sin x$	$-\cos x + c$
$\cos kx$	$\dfrac{1}{k}\sin kx + c$
$\sin kx$	$-\dfrac{1}{k}\cos kx + c$
$\tan kx$	$\dfrac{1}{k}\ln\lvert\sec kx\rvert + c$
e^x	$e^x + c$
e^{-x}	$-e^{-x} + c$
e^{kx}	$\dfrac{1}{k}e^{kx} + c$

Example 1

Use Table 1 to find the indefinite integral of x^7: that is, find $\int x^7\,dx$

Solution

From Table 1 note that $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c.$ In words, this states that to integrate a power of x, increase the power by 1, and then divide the result by the new power. With $n = 7$ we find

$$\int x^7\,dx = \frac{1}{8}x^8 + c$$

Example 2

Find the indefinite integral of $\cos 5x$: that is, find $\int \cos 5x\,dx$

Solution

From Table 1 note that $\int \cos kx\,dx = \frac{\sin kx}{k} + c$

With $k = 5$ we find $\int \cos 5x\,dx = \frac{1}{5}\sin 5x + c$

Example 3

Find $\int \cos 5t\,dt$

Solution

We integrated $\cos 5x$ in the previous example. Now the independent variable is t, so simply use Table 1 and replace every x with a t. With $k = 5$ we find

$$\int \cos 5t\,dt = \frac{1}{5}\sin 5t + c$$

It follows immediately that, for example,

$$\int \cos 5\omega\,d\omega = \frac{1}{5}\sin 5\omega + c, \qquad \int \cos 5u\,du = \frac{1}{5}\sin 5u + c$$ and so on.

Example 4

Find the indefinite integral of $\frac{1}{x}$: that is, find $\int \frac{1}{x}\, dx$

Solution

This integral deserves special mention. You may be tempted to try to write the integrand as x^{-1} and use the fourth row of Table 1. However, the formula $\int x^n\, dx = \frac{x^{n+1}}{n+1} + c$ is not valid when $n = -1$ as Table 1 makes clear. This is because we can never divide by zero. Look to the fifth entry of Table 1 and you will see $\int x^{-1}\, dx = \ln|x| + c$.

Example 5

Find $\int 12\, dx$ and $\int 12\, dt$

Solution

In this Example we are integrating a constant, 12. Using Table 1 we find

$$\int 12\, dx = 12x + c \qquad \text{Similarly} \int 12\, dt = 12t + c.$$

Exercises

1. Integrate each of the following functions with respect to x:
   (a) x^9, (b) $x^{1/2}$, (c) x^{-3}, (d) $1/x^4$, (e) 4, (f) $\sqrt{x}$, (g) e^{4x}
2. Find (a) $\int t^2\, dt$, (b) $\int 6\, dt$, (c) $\int \sin 3t\, dt$, (d) $\int e^{7t}\, dt$.

ملاحظة:

في حالة (التكامل-Integration) يجب التخلص من {ظرب الدوال, الجذور, وجود متغير x في المقام ويتم العمل مثل قواعد المشتقات ف1} (عامل مشترك, فرق بين مربعين, مجموع/فرق مكعبين, تجربة, مربع كامل........)